SAMPLE SIZE SELECTION FOR A CLASS OF RATIO ESTIMATES: BRAND SHARE ESTIMATION

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One of the most important applications of ratio estimates is in measuring brand share or market share and in detecting changes in share. A model of consumer behavior is used to derive a simple approximation for the variance of such ratio estimates. The approximation is a function of the parameters of purchase incidence, purchase size, and brand loyalty distributions. In survey or panel design, these parameters are easier to determine, apriori, than those normally required to determine the variance of a ratio.

1. INTRODUCTION

The estimation of market share is important to every company. The precision of these estimates is often crucial, since it may be important to detect small (for example: less than 1%) shifts in brand share over a given time period. Certainly the strategic importance of market share is well known and documented (Schoeffler(1977) and Buzzel et al(1975)).

Market share may have many definitions; it is basically a proportion. In a defined time period, it is the proportion of the total purchases (as measured by rolls, exposures, ounces, dollars, etc) that are a particular brand. In this paper, the brand of interest will be called Brand Y. If it were possible to take a random sample of purchases, estimation of market share would simply be one of estimating a proportion using simple random sampling. Variance estimates would be trivial and planning experiments (picking an appropriate sample size) would also be a simple task. In reality, the simplest possible sampling scheme to estimate share involves a cluster sample since purchase is not a convenient sampling unit. The sampling unit is usually an individual or household in consumer studies and a firm or establishment in commercial/industrial studies.

In cluster sampling the estimate of market share is:

$$(1) \qquad \hat{R} = \sum_{i=1}^{n} Y_i \Big/ \sum_{i=1}^{n} X_i$$

where X is the number of purchases of the product class by an individual, Y is the number of purchases of brand Y, and n is the sample size (number of individuals). For large samples, the variance of the estimate can easily be calculated from:

$$(2) \quad V(\hat{R}) = \frac{(1-f)}{n\bar{X}^2} V(Y-R*X) = \frac{(1-f)}{n\bar{X}^2}\sigma_c^2$$

$$= \frac{(1-f)}{n\bar{X}^2}\left[(V(Y)+R^2V(X)-2\ R\ COV(XY))\right]$$

where (1-f) is the finite population correction factor, if necessary. Once the data is collected, there is really no difficulty in computing the variance of the estimate and confidence bounds. There are some issues regarding the bias of the ratio estimate and the approximation involved in the use of (2) but these issues are well understood; see Cochran (1977)). In many brand-share estimation studies these issues are not of practical importance, since a large sample will be used.

If the interest is in a change in market-share, estimates of the difference between two ratio estimates $\hat{R}_1$ and $\hat{R}_2$ is also well-known. The variance of the difference is given by Cochran (1977):

$$(3) \quad V(\hat{R}_2 - \hat{R}_1) = V(\hat{R}_2) + V(\hat{R}_1) - 2\ COV(\hat{R}_1\hat{R}_2)$$

For two independent samples, the covariance is zero and a simple sum of the variances is appropriate. A more common way of estimating share is by the use of consumer panels (see Sudman and Ferber (1979) or Parfitt and Collins (1968)). In this case, the covariance term becomes important. In reality, because of panel "turnover", the sample at a second point in time consists of individuals from the first time period and individuals who are entering the panel for the first time. In this case, the variance can be modified in a manner similar to that discussed by Tam (1984).

In the planning of a study, the determination of sample size requires data from a previous study, since the variance σ_c^2 must be known. If this is available, there is no problem in picking an appropriate sample size. If only one period of data is available from a panel, it will not be possible to plan a sample size to give a specified precision on the variance of a difference, since it is necessary to estimate the covariance between samples over the two time periods.

In this paper, we develop an approximation for the variance of a proportion under cluster sampling. This approximation is based on a model of consumer behavior. The model is relatively simple, but has enough structure to adequately represent the variances and covariances in equations (2) and (3). Although developed for a behavior for frequently purchased low-cost product classes, empirical evidence indicates that it may be useful in a wider variety of applications.

The use of a model to evaluate ratio estimates is not new. Ratio estimation is typically based on a regression model in which the mean of the the ith response Y is proportional to the level X of an explanatory variable. The variance is generally assumed to be proportional to X . These simple models have been used to compare ratio-type estimates (see Rao (1969)). Whitmore(1986) has considered more complicated models. Models, to our knowledge, have not been used for the same purpose as in this paper: to assist in sample-size determination when there is little apriori data available.

2. A SIMPLE ONE-PERIOD CONSUMER MODEL

Probabilistic models of market behavior for frequently purchased low-cost product classes have been well studied by such authors as Ehrenberg (1972), Chatfield and Goodhardt (1973), Stewart (1979) and many others. Models have been proposed for the number of units purchased of a given brand/package/size, the number of purchase occasions, combinations of timing and purchase size (see, for example, Jeuland, Bass, and Wright (1980)). These models seem to work well in describing data (Schmittlein, Bemmaor, and Morrison (1985)), even though at first glance they seem very simple. The model proposed here is similar in spirit to many of these. It is being used for a different purpose. We assume that consumer purchase occasions for a particular product class over a time period t occur according to a Poisson process N(t) with rate λ. The rate λ varies across a population of individuals or households according to some arbitrary distribution. At each purchase occasion, the consumer buys a random number, S, of items in the product class. All S items in one given purchase occasion are the same brand. On a given purchase occasion, the probability that the S items are of Brand Y (the brand for which we want to estimate the market share), is given by P. This probability varies across individuals (or households) according to some arbitrary distribution. We assume that P, S, and N(t) are independent random variables.

Although some of these assumptions may be criticized on "mechanistic" grounds they lead to simple expression for the variance of a ratio estimate. This approximate variance can be verified empirically in many situations. It is shown in the Appendix that under these assumptions, a bound on the variance of the ratio estimate (1) is given by:

$$(4)\qquad V(\hat{R}) \le \frac{\mu_p(1-\mu_p)}{n\mu_x^2}\,[\sigma_x^2+\mu_x^2] \le \frac{\mu_p(1-\mu_p)}{n}\left\{\left[\frac{\sigma_x^2}{\mu_x^2}\right]+1\right\}$$

This bound is a function of the unknown market share, R, and the square of the coefficient of variation of the Number of Purchases of the product class by an individual over some time period, t. The first term in equation (4) is the approximate binomial sampling variance; the second term is an inflation factor to account for the cluster sampling. Therefore, for planning a study, one must approximate the market share (or use 50% as a worst case) and the mean and variance of the number of items purchased by an individual over a time period. These can generally be estimated apriori. The required mean and variance combine the information in our model on the number of purchase occasions and the size, S, of the purchase for the individuals.

3. DISCUSSION OF THE APPROXIMATION

Sample size determination for a market share estimation using the results of this paper is quite easy:

1. Select the appropriate sample as if one was doing binomial sampling.
2. Modify this sample size by multiplying by the inflation factor:

$$(5)\qquad \cdot\left[\frac{\sigma_x^2}{\mu_x^2}+1\right]$$

The result is the number of clusters (individuals, households, establishments, etc) that should be sampled. It is the author's experience that many people use the binomial formula in planning a brand-share survey. Cochran (1977) gives an example (page 66) of the improper use of this formula. In that example, he compares the ratio formula (2) with the binomial formula based on a sample size n which is the denominator of equation 1. In a market-share estimate this would be the total number of purchases in the sample. Clearly, this is inappropriate, because each purchase is not an independent observation. A more common mistake is to assume that the sample size is the number of clusters and again apply the binomial formula. Some even feel that this approach is conservative. This is definitely not true under the model proposed here. In every case, improper use of the binomial formula would underestimate the variance and therefore underestimate the required sample size.

Table 1 shows the results of applying equation (4) to several market-share estimation situations. Because of the proprietary nature of these markets, they are not described in detail here. The first market is a frequently purchased, low-cost, consumer product. The model was developed to specifically handle this type of product. The second market is a professional product; and the third market is a commercial product. In the latter, the 10 cases are strata from one industrial market. In each situation, the variance calculated from survey data using equations (2) and (4) may be compared. In some of the strata, particularly strata 7, the approximate variance does not appear to be a bound. In this case the sample size is only 39 and the variance using equation (2) is only an estimate. In addition, one could easily criticize some of the assumptions in the model for this particular market. In fact, the author was surprised that the approximation worked as well as it does in this case. Several other examples similar to the first market have shown excellent agreement between the approximation (4) and formula (2).

In no circumstances do we recommend the use of a short-cut procedure based on equation (4) to compute an estimate of the variance. Equation (2) presents no calculation problems with today's computers. The value of the approximation given in this paper is in planning a survey; specifically in selecting the sample size. Approximate apriori estimates of market share, mean purchases per individual, and variance of purchases per individual (perhaps estimated using a range) can usually be determined. In the next section we discuss some of the situations where we would expect the approximation to work well and some where the assumptions may not hold up.

4. DISCUSSION OF ASSUMPTIONS

Careful scrutiny of the derivation given in the Appendix leads one to the conclusion that the model will provide a tight upper bound on the variance in markets where there is strong brand loyalty. The simple approximation achieved in equation (4) is the result of the observation that the variance of P (the probability that a purchase occasion will result in purchases of Brand Y) is related to the mean of P by:

$$\sigma_p^2 \leq \mu_p (1 - \mu_p) \tag{6}$$

The distribution of P is the so-called "brand loyalty" distribution which is usually modelled by a Beta distribution. The inequality in (6) becomes equality when this brand loyalty distribution is a two-point distribution with mass at 0 and 1. This occurs when a proportion of the population purchases Brand Y with probability 1 and a proportion purchases Brand Y with probability 0 (is loyal to some other brand). The inequality holds for other distributions, but may tend to be too conservative for a product class which behaves as a "commodity."

Another of our assumptions is that on a purchase occasion, S items, all of Brand Y, are purchased. This does not allow for "variety seeking" behavior which probably occurs in product classes like soft drinks, snacks, etc. The approximation in this paper has not been evaluated in markets where this is likely to occur.

Another assumption, which might be questioned, is the independence of purchase size and number of purchase occasions. In the examples studied, this does not seem to have a noticeable effect. If it turned out to be a large factor, one could stratify the survey based on light/heavy users of a product and apply the approximation within each stratum. The same comment may be appropriate if the assumption of the independence of P with S or N(t) is questionable.

5. A TWO-PERIOD MODEL: DIFFERENCE IN BRAND SHARE

In a panel study, the variance of the difference in market-share estimates between two time periods is given by equation (3). An approximation is derived in the Appendix:

$$V(\hat{R}_1 - \hat{R}_2) = \frac{1-f}{n\bar{x}^2} \mu_n t (\sigma_s^2 + \mu_s^2) [(\mu_{p_1} - \mu_{p_1}^2) + (\mu_{p_2} - \mu_{p_2}^2)]$$

where:

μ_s = Mean purchase size.

σ_s^2 = Variance of purchase size distribution.

μ_p = Mean of distribution of p_i.

The use of this approximation requires more apriori knowledge than the approximation based on the single-period model. It is not difficult to imagine that reasonable specification of these parameters is possible in planning a study. Work on this model is continuing; initial indications are that the approximation works well.

6. SUMMARY

An approximation for the variance of a ratio estimate has been presented. This approximation, based on a consumer behavior model, is particularly useful for sample-size selection in market-share estimation studies. Its main value is in the limited amount of apriori knowledge needed to apply in many situations.

Another application of the approximation is in the tradeoff of time and sample size. In a diary panel it is possible to use the approximation to determine the optimal n and t. In other types of surveys (telephone or mail) based on recall of purchases, this tradeoff is complicated by the decreasing accuracy of recall data with time. Another complication is the cyclic nature of a business, if present. This will be the subject of a future paper.

APPENDIX

A SIMPLE SINGLE PERIOD CONSUMER MODEL

Assume that consumer purchase occasions for a particular product class over a time period t occur according to a point process N(t). On each purchase occasion, a consumer buys a number of items, S, in the product class. With probability p, all the items in the purchase are Brand Y. We make the following assumptions:

1. N(t) is a poisson process with rate λ.
2. The rate λ varies across the population of individuals (or households) according to an arbitrary distribution $g(\lambda)$.
3. The purchase size S has an arbitrary distribution which does not differ between individuals.
4. p varies across individuals (or households) according to an arbitrary distribution.
5. S, p, and N(t) are independent.

The following notation will be used:

X_i = Number of purchases of product class by ith individual, $i = 1, 2, \ldots, n$

Y_i = Number of purchases of brand Y by ith individual, $i = 1, 2, \ldots, n$

$N_i(t)$ = Number of purchase occasions of product class by ith individual, $i = 1, 2, \ldots, n$ in time period t.

S_j = (Purchase size) Number of items in the product class purchased on the jth purchase occasion of the ith individual, $j = 1, 2, \ldots, N_i(t)$.

p_i = The probability that individual i's purchase will be brand Y (<u>all</u> items in the jth purchase occasion of individual i will be brand Y with probability p_i).

R = "True" market share.

R = Estimate of market share.

μ_s = Mean purchase size.

σ_s^2 = Variance of purchase size distribution.

μ_p = Mean of distribution of p_i.

σ_p^2 = Variance of distribution of p_i.

N = Population size.

n = Sample size.

1-f = Finite population correction factor.

The population parameter to be estimated is the market share:

$$R = \frac{\text{total \# of Brand Y purchased}}{\text{total \# in product class purchased}}$$

$$R = \frac{\sum_{i=1}^{N} Y_i}{\sum_{i=1}^{N} X_i}$$

The estimate of market share is given by:

$$\hat{R} = \frac{\sum_{i=1}^{n} Y_i}{\sum_{i=1}^{n} X_i} \qquad (1)$$

From Theorem 2.5 of Cochran [1977] the variance of the estimate is approximately:

$$V(\hat{R}) = \frac{(1-f)}{n\bar{x}^2} V(Y_i - RX_i)$$

Over a time period t, the number of purchases of the product class by individual i is:

$$X_i = \sum_{j=1}^{N_i(t)} S_j$$

and the number of purchases of brand Y by individual i is:

$$Y_i = \sum_{j=1}^{N_i(t)} \tau_j S_j$$

where: τ_j = 1 if the jth purchase is brand Y
= 0 if the jth purchase is not brand Y

or: τ_i = 1 with prob p_i
= 0 with prob $1 - p_i$

Therefore:

$$V[Y_i - RX_i] = V\left[\sum_{j=1}^{N_i(t)} (\tau_i - R) S_i\right] \qquad (2)$$

Assume that the number of purchases $N_i(t)$ and the purchase size S_i are independent. In addition, assume that τ_i and S_i are independent. Let $N_i(t)$ be a Poisson random variable with parameter λt. This is not a crucial assumption in the following derivation, but it leads to a simplification in the final result. To find the variance of the market share estimate, we first condition on p and λ. For convenience, we eliminate use of the subscript i.

$$E[(Y_i - RX_i) \mid \lambda, p] = E[N \cdot E[(\tau - R)S]]$$

$$V[(Y_i - RX_i) \mid \lambda, p] = E[N \cdot V[(\tau - R)S]] + V[N \cdot E[(\tau - R)S]]$$

and therefore:

$$E[(Y - RX) \mid \lambda, p] = \lambda t (p - R) \mu_s$$

$$V[(Y - RX) \mid \lambda, p] = \lambda t [(\sigma_s^2 + \mu_s^2)[(p(1-p) + (p-R)^2]]$$

We remove the condition on λ by using the results for conditional expectations and variances:

$$E[Z] = E[E(Z \mid \lambda)]$$

$$V[Z] = V[E(Z \mid \lambda)] + E[V(Z \mid \lambda)]$$

Then, noting that $\mu_n = E[\lambda]$ and $\sigma_\lambda^2 = V(\lambda)$:

$$E[(Y - RX) \mid p] = \mu_n \mu_s (p - R)t$$

$$V[(Y - RX) \mid p] = \mu_n [(\sigma_s^2 + \mu_s^2)[p(1-p) + (p-R)^2]]t + \sigma_\lambda^2 \mu_s^2 (p-R)^2 t^2$$

Remove condition on p in a similar manner:

$$E(Y - RX) = \mu_n \mu_s (\mu_p - R)\, t = 0$$

since $R = \mu_p$ in this model. After some minor manipulations:

$$(3)\quad V(Y - RX) = \mu_n [(\sigma_s^2 + \mu_s^2)[\mu_p - \mu_p^2]]\, t + \mu_s^2 \sigma_p^2 [\sigma_\lambda^2 + \mu_n^2]\, t^2$$

Therefore:

$$(4)\quad V(\hat{R}) = \frac{(1-f)}{n\, \bar{x}^2} \mu_n [(\sigma_s^2 + \mu_s^2)[\mu_p - \mu_p^2]\, t + \mu_s^2 \sigma_p^2 [\sigma_\lambda^2 + \mu_n^2]\, t^2]$$

This result may be useful in its own right since it expresses the variance of the market share estimate in terms of parameters that may be "guessed" apriori. It is easy to show that for a poisson process:

$$(5)\quad \sigma_\lambda^2 = \frac{\sigma_n^2 - \mu_n t}{t^2}$$

Equation 4 may be simplified to an upper bound on the variance of R which may be more useful in practice.

First note that:

$$E(X \mid \lambda) = \lambda t \mu_s$$
$$V(X \mid \lambda) = \lambda t [\sigma_s^2 + \mu_s^2]$$

Then:

$$(6)\quad \mu_x = E[X] = \mu_n \mu_s t$$
$$\sigma_x^2 = V(X) = \mu_n [\sigma_s^2 + \mu_s^2]\, t + \mu_s^2\, t^2\, \sigma_\lambda^2$$

For a variate p restricted to the range [0, 1] it is easy to show that:

$$(7)\quad \sigma_p^2 \leq \mu_p (1 - \mu_p)$$

Therefore, from (4):

$$V(\hat{R}) \leq \frac{1-f}{n\, \mu_x^2} [(\mu_p)(1 - \mu_p)[\mu_n (\sigma_s^2 + \mu_s^2)\, t + \mu_s^2\, t^2 [\sigma_\lambda^2 + \mu_n^2]]]$$

and using (6), we obtain:

$$V(\hat{R}) \leq \frac{\mu_p (1-\mu_p)}{n\, \mu_x^2} \left[\sigma_x^2 + \mu_x^2\right]$$

$$V(\hat{R}) \leq \frac{\mu_p (1-\mu_p)}{n} \left[\frac{\sigma_x^2}{\mu_x^2} + 1\right]$$

We would have arrived at same conclusion if we originally had assumed a single purchase $\mu_n = 1$, $\sigma_\lambda^2 = 0$, for each individual (i.e., sampled purchases rather than households but allowed purchase sizes to vary across the population as reflected in μ_s and σ_s^2).

VARIANCE OF THE DIFFERENCE BETWEEN TWO MARKET SHARE ESTIMATES FROM PAIRED SAMPLES: A TWO PERIOD MODEL

Assuming x is the same in the two periods $x_1 = x_2$ and that the distribution of S does not change:

$$V(\hat{R}_1 - \hat{R}_2) = \frac{1-f}{n\, \bar{x}^2}\; V[Z_1 - Z_2]$$

where:

$$Z_i = \sum_{j=1}^{N_i(t)} (\tau_j - R_i)\, S_i$$

It is easier to deal with the sum of the random variables Z_1 and Z_2. Therefore, we use:

$$(8)\quad V[Z_1 - Z_2] = 2[V(Z_1) + V(Z_2)] - V(Z_1 + Z_2)$$

Under the poisson assumption and conditional on λ and p, we obtain:

$$E[(Z_1 + Z_2) \mid \lambda, p_1, p_2] = \lambda t (p_1 - R_1)\mu_s + \lambda t (p_2 - R_2)\mu_s$$

$$V((Z_1+Z_2) \mid \lambda, p_1, p_2) = \lambda t (\sigma_s^2 + \mu_s^2)[p_1(1-p_1) + p_2(1-p_2) + (p_1 - R_1)^2 + (p_2 - R_2)^2]$$

The subscripts on the p and R indicate samples 1 and 2, respectively, on the same individuals. We are assuming that the rate of purchase and the purchase size distributions do not change in the two samples. The method, however, could be easily extended to account for differences in these distributions, if desired.

Removing the condition on λ using conditional expectations and conditional variance formulas as before and removing condition on p, noting that $E(p_1) = R_1$ and $E(p_2) = R_2$:

$$E[Z_1 + Z_2] = 0$$

$$V[Z_1 + Z_2] = \mu_n t (\sigma_s^2 + \mu_s^2)[(\mu_{p_1} - \mu_{p_1}^2) + (\mu_{p_2} - \mu_{p_2}^2)] + \mu_s^2 \sigma_\lambda^2 t^2 [\sigma_{p_1}^2 + \sigma_{p_2}^2 + 2\,\mathrm{cov}(p_1, p_2)] + \mu_n^2 t^2 \mu_s^2 [\sigma_{p_1}^2 + \sigma_{p_2}^2 + 2\,\mathrm{cov}(p_1, p_2)]$$

Since $\mathrm{cov}(p_1, p_2) = \rho\, \sigma_{p_1} \sigma_{p_2}$, if we assume $\sigma_{p_1} \approx \sigma_{p_2}$ and that $\rho \approx 1$, then:

$$V[Z_1 - Z_2] \approx \mu_n t (\sigma_s^2 + \mu_s^2)[(\mu_{p_1} - \mu_{p_1}^2) + (\mu_{p_2} - \mu_{p_2}^2)]$$

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Table 1

Product	Sample Size n	Std. Dev. Formula (2)	Approx. Std. Dev. Formula (4)	Binomial Std. Dev.
Consumer Product 1	1,700	.0168	.0175	.010
Consumer Product 2	1,700	.0157	.0179	.010
Prof. Prod. 1	204	.034	.045	.024
Prof. Prod. 2	175	.053	.050	.029
Prof. Prod. 2	210	.032	.050	.024
Industrial Products				
Strata 1	99	.036	.036	.035
Strata 2	23	.089	.113	.094
Strata 3	95	.055	.055	.046
Strata 4	182	.034	.041	.032
Strata 5	102	.064	.066	.047
Strata 6	122	.051	.056	.044
Strata 7	39	.087	.075	.062
Strata 8	165	.041	.053	.036
Strata 9	49	.046	.057	.046
Strata 10	294	.033	.047	.026