SOME ASPECTS OF ESTIMATING VARIANCES BY HALF-SAMPLE REPLICATION IN CPS

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1. INTRODUCTION

The focus of this paper is an empirical study of certain aspects of variance estimation using a replication approach for the Current Population Survey (CPS). The CPS is a monthly labor force survey of approximately 60,000 U.S. households drawn from a multistage stratified design, with one primary sampling unit (PSU) per stratum.

There have been several previous studies of variance estimators which used data from complex surveys. For example, in Frankel (1971) and Bean (1975), CPS data and Health Interview Survey data were used respectively. The approach taken in this paper has at least one fundamental difference from the previous studies. In the works cited, the sample from the complex survey was treated as if it were the population of interest. Samples were selected from the full sample and variance estimates computed from the subsamples. In this paper, the full CPS sample is viewed, as it actually is, a sample from a national population. Consequently, the variance estimates computed here are for the full sample.

The two approaches each have advantages and disadvantages. The chief advantage of the first approach described is that since a known population is assumed, such key information as estimates of biases in the variance estimators can be directly computed, while in this paper it cannot. On the other hand, the results in the previous studies only apply directly to the relatively small samples choosen from the artificial populations. It is generally not evident how well the results also apply to variance estimates for the full sample.

The following are some of the principal areas investigated in this study.

A. A comparison of reweighting each replicate as opposed to using the parent sample weights for all replicates.

B. The constants to be used in the collapsed stratum estimator to reduce the bias of this estimator.

C. A comparison of random replication and partially balanced replication.

D. The effect of the number of replicates on the precision of the variance estimates.

The items just listed, along with other aspects to be studied, are described in detail in Section 2. This section also includes a description of the form of the variance estimator considered here. The numerical results are presented and analyzed in Section 3.

Due to lack of space some portions of the complete paper have been omitted here. Among the omissions are all but a brief summary of the results on C and D above; an analytical result providing some insight on A; and the list of references. The complete paper is available from the authors.

2. TOPICS OF STUDY

The general form of the variance estimator studied in this paper is explained in Section 2.1. In the remaining subsections each of the specific aspects to be studied is described.

2.1 The Replicated Variance Estimator

For one PSU per stratum designs like CPS, a collapsed stratum variance estimator is generally employed as explained in Wolter (1985). We begin by reviewing this form of variance estimation, using the notation of Wolter for the most part, and then explain how it is used in this paper in conjunction with a replicate variance estimator.

The first step in using a collapsed stratum estimator is the partitioning or "collapsing" of the set of all strata into groups of two or more strata. Then consider a population total Y that is estimated by a linear estimator of the form

$$\hat{Y} = \sum_{g=1}^{G} \hat{Y}_g = \sum_{g=1}^{G} \sum_{h=1}^{L_g} \hat{Y}_{gh},$$

where G denotes the number of groups of collapsed strata; L_g the number of strata in the g-th group; $\hat{Y}_g$ the estimator of total for the g-th group; and $\hat{Y}_{gh}$ the estimator of total for the h-th stratum in the g-th group. The general form of the collapsed stratum variance estimator is then

$$v_{cs}(\hat{Y}) = \sum_{g=1}^{G} \frac{L_g}{L_g-1} \sum_{h=1}^{L_g} (\hat{Y}_{gh} - \frac{A_{gh}}{A_g} \hat{Y}_g)^2, \tag{2.1}$$

where A_{gh} is a known measure associated with the gh-th stratum that tends to be well correlated with $\hat{Y}_{gh}$, and $A_g = \sum_{h=1}^{L_g} A_{gh}$. Commonly used values for A_{gh} include $A_{gh}=1$ for all g,h and $A_{gh} = p_{gh}$ where p_{gh} is the population of the gh-th stratum from the most recent census. The terms $v_{cs}(\hat{Y})$ and A_{gh} will be discussed further in Section 2.3.

In the CPS there are 379 nonself-representing strata, which we partitioned into 188 pairs of strata and one group of three strata. There are also 350 self-representing strata. To take into account the variability arising from sampling from these strata, the sample in each of them is divided into two panels, with the assignment of ultimate sampling units alternating between the panels. In applying (2.1), the two panels corresponding to each self-representing strata are treated as if they constituted a pair of nonself-representing strata collapsed together. Thus, G = 539 for the entire sample, with L_g=3 for one group and $L_g=2$ for all other groups.

Returning now to (2.1), it can be shown that this is algebraically equivalent to

$$v_{cs}(\hat{Y}) = d \sum_{g=1}^{G} \frac{1}{L_g} \sum_{h=1}^{L_g} [(1 + \frac{L_g}{(L_g-1)^{1/2} d^{1/2}} (1 - \frac{A_{gh}}{A_g})) \hat{Y}_{gh} + \sum_{\substack{t=1 \\ t\neq h}}^{L_g} (1 - \frac{L_g}{(L_g-1)^{1/2} d^{1/2}} \frac{A_{gh}}{A_g}) \hat{Y}_{gt} - \hat{Y}_g]^2, \quad (2.2)$$

where d is a parameter introduced by Fay, with different notation (see Dippo, Fay and Morganstein (1984)), that leads to a more general form of the replicate variance estimator than the standard form for which d=1. This parameter is discussed in Section 2.4. The form of the replicate variance estimator, $v_k(\hat{Y})$, considered in this paper is

$$v_k(\hat{Y}) = \frac{d}{k} \sum_{\alpha=1}^{k} (\hat{Y}_\alpha^R - \hat{Y})^2, \quad (2.3)$$

where k is the number of replicates and each replicate estimate $\hat{Y}_\alpha^R$ is obtained as follows. Corresponding to each α and each group g, a stratum gh is selected from the g-th group. Then

$$\hat{Y}_\alpha^R = \sum_{g=1}^{G} [(1 + \frac{L_g}{(L_g-1)^{1/2} d^{1/2}} (1 - \frac{A_{gh}}{A_g})) \hat{Y}_{gh} + \sum_{\substack{t=1 \\ t\neq h}}^{L_g} (1 - \frac{L_g}{(L_g-1)^{1/2} d^{1/2}} \frac{A_{gh}}{A_g}) \hat{Y}_{gt}]. \quad (2.4)$$

Now provided that for all g, each stratum in the g-th group is selected k/L_g times, (2.3) reduces to (2.2) plus a sum of cross-product terms involving the bracketed portion of (2.4) from pairs of groups. If additionally, each pair of strata gh and g′h′ from two groups are selected together $k/L_g L_{g'}$ times then the cross-product terms cancel and (2.4) reduces to (2.2). These assertions are all explained in Borack (1971) and Wolter (1985) for the case when the L_g are the same for all g, but the concept is not limited to only that case. A set of replicates satisfying these conditions is said to be in full orthogonal balance.

For linear estimators, there is no particular advantage to computing variance estimates using (2.3), since (2.1) can be computed directly just as readily. However, as explained in Section 2.2, CPS estimators using the final weights are nonlinear estimators even for estimates of totals. Expressions such as (2.3) are used to estimate variances for nonlinear estimators also. The previous empirical studies cited in the Introduction support the use of this approach as do certain asymptotic results, such as those of Krewski and Rao (1981).

The particular topics to be studied here derive from the many specific forms that (2.3) and (2.4) can take for CPS data. For estimators of total using the final CPS weights, $\hat{Y}_\alpha^R$ can be computed in several ways, as explained in Section 2.2. The different possible values for A_{gh} and d are discussed in Sections 2.3 and 2.4, respectively. Finally, in Section 2.5, two less expensive alternatives to a fully balanced set of replicates, partially balanced replication and random replication are considered, along with the question of number of replicates to be used. Section 2.5 is omitted here, but appears in the complete paper.

2.2 Weighting the Replicates

The final weights used in CPS are obtained by beginning with the reciprocal of probability of selection for each sample unit, which we will refer to as the base weight, and then subjecting the set of weights to three successive adjustments: the noninterview adjustment, the first-stage ratio adjustment and the second-stage ratio adjustment. Of these adjustments, the second-stage ratio adjustment generally has the largest impact on both the expected values and the variances of the estimates (Hanson 1978). The adjustment for the population 16 years and older, which is the one of interest here, uses the following procedure (Jones 1984). First the sample weights after the first-stage adjustment are ratio adjusted to obtain estimates that agree with independently derived estimates of the total population for that month in each of the 50 states and the District of Columbia. The resulting weights are then further ratio adjusted to obtain agreement with independently derived national estimates in 16 age/Hispanic ethnicity/sex cells. Finally, these weights are adjusted again to obtain agreement with independent national estimates in 70 age/race/sex cells. Note that each successive adjustment destroys the agreement with the independent estimates controlled to in the previous adjustment. The entire procedure is therefore repeated five more times. This repeated iteration of the procedure, a process known as iterative proportional fitting or "raking," results in a set of final weights which yields estimates in near agreement with all three sets of controls.

For the replication method of estimating variances, each replicate is subject to the same weighting procedures as the parent sample. That is, to obtain a final value for $\hat{Y}_\alpha^R$, first compute (2.4) using the base weights to obtain estimates of strata totals and then perform the same ratio adjustments that are done for the parent sample. As one might expect from the complexity of the second-stage adjustment just described, this can require extensive computer time. A short cut would be to use the final weights from the parent sample for each replicate; that is, $\hat{Y}_\alpha^R$ would be computed directly from (2.4) using the final weights to obtain the estimates for the strata totals. The effectiveness of this short cut has been studied previously by a number of authors, including

Bean (1975), who found it produced little loss in accuracy, and Lemeshow (1979), who found evidence of greater bias and lower precision for variance estimates computed using the parent sample weights.

For this part of the study, variance estimates were computed using three different approaches to account for the weighting. The first two are the Reweighting method and the Parent Sample Weights method that we have been discussing. (Actually to simplify matters for the Reweighting method, only the second-stage weights are replicated; that is, the computation of a replicate estimate begins by computing (2.4) using the first-stage weights from the parent sample). The final method, the Base Weights method, simply uses the base weights in the replicate estimates in order to allow for a comparison of variance estimates using unadjusted weights to those based on the other two procedures.

For the Reweighting method, 6 cycles of raking are used. Since some cost savings would ensue if fewer cycles were used, variances estimates were also obtained for 1, 2, and 3 cycles for the purpose of determining if the variances estimates would be substantially affected by fewer cycles.

The question of reweighting versus not reweighting replicate estimates is one area where analytic results that provide some insight into the problem can be presented. In the complete paper, but omitted here, it is established that under simplified conditions whatever gains in precision arise from the weighting adjustments are lost in the variance estimation whenever a replicate variance estimator together with the Parents Sample Weights method is used.

2.3 Values for A_{gh}

The collapsed stratum variance estimator, like any variance estimator for one PSU per stratum designs, is biased. In Hansen, Hurwitz and Madow (1953), Volume II, Chapter 9, it is established that for a linear estimator $\hat{Y}$ with $v_{cs}(\hat{Y})$ as in (2.1),

Bias $[v_{cs}(\hat{Y})]$

$$= \sum_{g=1}^{G} \left[\frac{L_g}{L_g-1} \sum_{h=1}^{L_g} \left(\frac{A_{gh}^2}{A_g^2} - \frac{2\, A_{gh}\, \sigma_{gh}^2}{A_g\, \sigma_g^2}\right) + \frac{1}{L_g-1}\right] \sigma_g^2$$

$$+ \sum_{g=1}^{G} \frac{L_g}{L_g-1} \sum_{h=1}^{L_g} \left(Y_{gh} - \frac{A_{gh}}{A_g} Y_g\right)^2, \qquad (2.7)$$

where $\sigma_{gh}^2 = \mathrm{Var}(\hat{Y}_{gh})$, $\sigma_g^2 = \sum_{h=1}^{L_g} \sigma_{gh}^2$, $Y_{gh} = E(\hat{Y}_{gh})$ and $Y_g = E(\hat{Y}_g)$. Two commonly used values for A_{gh} for the nonself-representing strata for surveys such as CPS are $A_{gh} = 1$ and $A_{gh} = p_{gh}$, where p_{gh} is the population of the gh-th stratum from the most recent census. $A_{gh} = 1$ is the natural choice if, ignoring the original stratification, the L_g PSUs in the g-th group are treated as independent selections from a single stratum. In this case only the second term in (2.7) is present; that is, the bias would consist only of a between strata component. If Y_{gh} is well correlated with p_{gh}, then the second term in (2.7) can generally be reduced by the use of $A_{gh} = p_{gh}$ and would disappear if Y_{gh} is proportional to p_{gh}. The first term, however, would no longer be zero. Furthermore with $A_{gh} = 1$, the bias must always be upward, while with $A_{gh} = p_{gh}$ it is possible for the bias to be downward since the first term can be negative. For a nonlinear estimator computed using (2.2) and (2.3), no such blanket statements can be made about the direction of the bias.

For the self-representing strata, $A_{gh} = 1$ is always used, since the two panels corresponding to each such stratum have the same expected size.

In this paper variance estimates are computed using both $A_{gh} = 1$ and $A_{gh} = p_{gh}$ for nonself-representing strata, and compared.

2.4 Values for d

The standard form of the replicate variance estimator, as presented in Wolter (1985), only considers expressions like (2.3) for $d = 1/(L_g - 1)$. The more general form was introduced in Dippo, Fay and Morganstein (1984), with the following motivation. In (2.2) the factor multiplying the estimated $\hat{Y}_{gt}$ if the gh-th stratum is selected, $h \neq t$, is

$$1 - \frac{L_g}{(L_g-1)^{1/2} d^{1/2}} \frac{A_{gh}}{A_g}. \qquad (2.8)$$

For d = 1, this factor is 0 with $L_g = 2$ and $A_{gh} = 1$, and can be negative for other combinations of L_g and A_{gh}. A negative value for (2.8) can result in negative values for replicate estimates computed using the Reweighting method even when the full sample estimate cannot be negative, an undesirable situation. Furthermore, as noted in Dippo, Fay and Morganstein, (2.8) must be strictly positive to ensure that complex functions built from ratios would be defined for each replicate whenever the function could be computed for the whole sample. To avoid these difficulties, Fay suggests d = 4 as an alternative. For d = 4, $L_g = 2$, (2.8) is positive for any set of positive A_{gh}. For d = 4, $L_g = 3$, (2.8) is positive for $A_{gh} = 1$, and also for $A_{gh} = p_{gh}$ as long as $A_{gh} < 2^{3/2} A_g/3$ for all g and h, as it is in this study.

Variance estimates obtained from (2.3) are clearly the same for all d for linear estimators. Furthermore, even for nonlinear estimators, under appropriate conditions, the variance estimators, treated as a function of d, asymptotically converge to the same estimators for all d.

In this paper the effects of different d on the variance estimates for the characteristics of interest are studied for the Reweighting method only, since variance estimates obtained using the Base Weights and Parent Sample Weights methods are identical for all d. Variance estimates were computed for d=1, 4, 100 and 10,000. d=100 and d=10,000 are included to provide some insight on the effects of large values of d.

3. EMPIRICAL RESULTS

We first describe the variance estimates that were computed. As detailed in the previous section, the following were varied.

1. Weighting methods: Reweighting (with 1, 2, 3, and 6 raking cycles), Parent Sample Weights, Base Weights.

2. A_{gh}: 1, p_{gh}

3. d: 1, 4, 100, 10,000

4. Set of replicates methods: Partial balancing, random replication.

5. k: 12, 24, 48

For the Parent Sample Weights and Base Weights methods, variance estimates were computed for each combination of the other aspects listed, with the exception that only one value of d was used, since variance estimates for these weighting methods are independent of d. For each combination, 50 estimates were obtained, with different groupings of the strata for the partially balanced method, and different random replications. In addition, for these two weighting methods, the variance estimates corresponding to a fully balanced set of replicates were computed directly from (2.1) for both sets of A_{gh}.

For the Reweighting method, the combinations for which variance estimates were computed are presented in Table 1. For each of the indicated combinations, 10 estimates were obtained. The principal reason that all combinations were not considered for the Reweighting method and that more estimates were not computed for each combination is simply that it is much more expensive to compute variance estimates for this method. Also, combinations for which $A_{gh} = p_{gh}$ and d=1 were omitted because of the potential problems discussed in Section 2.4.

The estimates for which variance estimates were computed are all estimates of population totals. The specific characteristics estimated are the same for all aspects of the study, and are listed in Tables 2-7.

The first comparisons are for the three weighting methods, with the computations summarized in Table 2 for each weighting method and A_{gh} combination. For the Parent Sample Weights and Base Weights methods, the variance estimates listed are those computed directly from (2.1), so that the variability in the replicate variance estimates that would otherwise arise from the cross-product terms has been eliminated. For the Reweighting method, the variance estimates listed for $A_{gh} = p_{gh}$ are the simple average of the twenty repetitions for which k=24, d=4 and either partial balancing or random replication was used. For $A_{gh} = 1$, the estimates are averaged over the 10 repetitions for which d=4. (Refer to Table 1.) Variance estimates from other possible combinations were not used in computing the average, because they were not independent of the repetitions that were used. The standard errors of the variance estimates arising from the choice of the set of replicates for the Reweighting method for each set of A_{gh} is also presented in Table 2. For A_{gh}=1 the estimates of the standard errors of the variance estimates were computed by considering the 10 repetitions to be independent, equal probability selections, while for $A_{gh} = p_{gh}$, the sets of partially balanced and randomly selected replications were considered separate strata in this computation.

Note that the estimates of the standard errors of the variance estimates reflect the variability in the variance estimates for the Reweighting method arising from the variability in the chosen set of replicates, but does not reflect any of the other possible sources of error in the computation of the variance estimates. For example, the bias in the collapsed stratum variance estimator, and the variability in the variance estimates that would result from a different CPS sample, are not measured. Furthermore, these sources of error in the variance estimates affect all three weighting methods. Consequently, the results in the tables must be interpreted with caution.

The following are key observations from Table 2 concerning the weighting methods. For those characteristics possessed either by a large proportion of the total population, or a large proportion of a demographic subgroup which is controlled to in the second-stage adjustment, the variance estimates appear to be much lower for the Reweighting method than the Parent Sample Weights method. This includes total, black and teenage employed, and in labor force. This is in accord with the results in Section 2.2. For other characteristics, such as the unemployment characteristics, for which the proportion of the total population or the indicated demographic subgroup possessing the characteristic is small, differences between the variance estimates computed with the two weighting methods are generally not as dramatic.

The Parent Sample Weights and Base Weights methods were also compared. For each A_{gh} and characteristic combination, the entry in Table 2 for the Base Weights method is lower than for the Parent Sample Weights method. If this is indicative of significant differences between these two methods, it may be do to the following. As noted in Section 2.2, the gains in actual variances arising from the second-stage adjustment, may not be reflected in the variance estimates when the Parent Sample Weights methods is used. In fact, variance estimates for this method are computed in the same manner as the Base Weights method, but the weights used with the Parent Sample Weights method are more variable due to the second-stage adjustment, and generally larger due to the undercoverage that the second-stage adjustment seeks to correct. More variable and larger weights tend to increase variance estimates, although in the case of larger weights, not necessary relative variances. Thus, ironically, by performing the second-stage adjustment, which has increased precision of the estimates as one of its goals, and then using the Parent Sample Weights methods to compute variance estimates, larger variance estimates may result than if the second-stage adjustment had not been done at all.

The results when using the Reweighting method with fewer than six cycles of raking are presented in Table 3, which appears only in the complete paper. To summarize this table, none of the variance estimates computed with one and two cycles of raking differed from the corresponding variance estimates with six cycles by more than 2% and .3% respectively. Thus, the variance estimates for two cycles and even possibly for one cycle appear to be close enough to the variance estimates for six cycles to be viable approximations.

We next consider the effect of the choice of A_{gh} on the variance estimates. Examining Table 2 again, we note that most of the entries for $A_{gh} = p_{gh}$ are lower than the corresponding entries for $A_{gh} = 1$. For the Reweighting method, however, the differences would generally not be significant, even if the standard errors of the variance estimates given in Table 2 are assumed to be the only source of error. We suspect that this is at least partly due to the small number of repetitions done for the Reweighting method.

For the Base Weights method, an estimator of total is a linear estimator, and consequently (2.7) is an exact expression for the bias of the variance estimator. If the variance estimates are actually smaller for $A_{gh} = p_{gh}$ and (2.7) is positive, then $A_{gh} = p_{gh}$ does result in lower biases than $A_{gh} = 1$. Furthermore, for estimates for which it is additionally true that the second-stage adjustment does lower the variances, but for which this is not reflected in the variance estimates computed with the Parent Sample Weights method, $A_{gh} = p_{gh}$ results in smaller biases for this weighting method also.

There is a further complication in comparing the two sets of A_{gh}. Different sets of collapsed strata were used for the two sets of A_{gh} for the variance estimates summarized in Table 2. This arose because collapsing was done in an attempt to minimize an average over several key characteristics of the bias expression (2.7). This is described fully in Ernst, Huggins and Grill (1986). Since (2.7) involves A_{gh}, different A_{gh} lead to different optimal collapsings. Consequently, Table 2 reflects not only the effect of the different A_{gh} but also the different sets of collapsed strata.

In an attempt to learn something about this matter, variance estimates were also computed with the A_{gh} and the sets of collapsed strata reversed, with the results presented in Table 4. That is, variance estimates were obtained with $A_{gh} = p_{gh}$ for the collapsed strata optimal for $A_{gh} = 1$ and vice versa. Comparing Tables 2 and 4 for the Base Weights and Parent Sample Weights methods, the most striking observation is that for characteristics possessed by a large proportion of the total population, that is total employed, and total in labor force, the entries in Table 4 for $A_{gh} = 1$ are much larger than the corresponding entries in Table 2 for $A_{gh} = p_{gh}$. That is, for these characteristics at least, the substitution of $A_{gh} = 1$ for $A_{gh} = p_{gh}$ with the set of collapsed strata optimal for $A_{gh} = p_{gh}$ may produce variance estimates that are severely biased upward. An explanation for this is that the optimal collapsing for $A_{gh} = p_{gh}$ tends to group strata together with total populations that vary more than the optimal collapsing for $A_{gh} = 1$, since the use of $A_{gh} = p_{gh}$ in the variance estimates can compensate for the biases that otherwise would result from the grouping of strata with different population totals. That is, for fixed g, the variability of Y_{gh}/A_{gh} with h, which is reflected in the second term of (2.7), will only arise for $A_{gh} = p_{gh}$ from differences in the proportions of the population possessing the characteristic among the strata collapsed together, not any differences in total population (assuming the strata populations remain in the same proportion from the point in time that p_{gh} was computed). However, when $A_{gh} = 1$ is used instead, the possibly large variability in the population of the strata collapsed together can increase the variability of the Y_{gh}/A_{gh}, and hence increase (2.7), particularly for characteristics possessed by a large proportion of the total population.

For the same two weighting methods, the effect of the opposite substitution, that is the use of $A_{gh} = p_{gh}$ instead of $A_{gh} = 1$ with the collapsing optimal for $A_{gh} = 1$, is not at all apparent. In fact, for many characteristics the substitution of $A_{gh} = p_{gh}$ for $A_{gh} = 1$ results in lower values for the entries in Table 4 with $A_{gh} = p_{gh}$ than for the corresponding entries in Table 2 with $A_{gh} = 1$.

Thus, it appears that for these weighting methods, Table 4 provides some evidence that it is the $A_{gh} = p_{gh}$ rather than the particular set of collapsed strata that lowers the variance estimates.

For the Reweighting method, the large variances of the estimated variances again severely limits what can be inferred from comparing Tables 2 and 4. There is, however, no evidence of any large increase in the variance estimates with this method when $A_{gh} = 1$ is substituted for $A_{gh} = p_{gh}$, as there is with the other weighting methods. A possible explanation is that the increase in the variability of the estimate of the total population that occurs for the other two weighting methods as a result of this substitution is completely removed by the reweighting.

We next consider the effects of different values of the d parameter on the variance estimates for the Reweighting method, with the results summarized in Table 5. Each entry in this table is obtained by taking the simple average of the ten repetitions for which k=12 and partial balancing was used. For $A_{gh} = 1$ the table entries are all lower for d=4 than d=1. For $A_{gh} = p_{gh}$ the entries are lower for d=100 than d=4, while the entries for d=10,000 are close to d=100. Although these differences are generally not significant, it appears that the variance estimates are generally decreasing functions of d which converge to positive limits as d approaches ∞. This is consistent with the findings in Judkins (1987) who provides an explanation for this relationship.

The results for the effects of partial balancing versus random replication and the number of replicates, k, on the population variances of the variance estimates, including several tables, are presented in full in the complete paper. In summary the variance of the variance estimates, as expected, do generally decrease as the number of replicates increase, although they remain relatively high even for 48 replicates.

Somewhat surprisingly, the data does not appear to support the generally held belief that the variances of the variance estimates are higher for random replication than partial balancing. Neither method seemed clearly superior in this respect.

* This paper reports the general results of research undertaken by Census Bureau staff. The views expressed are attributable to the authors and do not necessarily reflect those of the Census Bureau.

Table 1. Combinations for Which Variance Estimates Computed for Reweighting Method (Indicated by "X")

	A_{gh} = 1	A_{gh} = P_{gh}			
	k = 12	k = 12		k = 24	
d	Partial Balancing	Partial Balancing	Random Replication	Partial Balancing	Random Replication
1	X				
4	X	X	X	X	X
100		X			
10,000		X			

Table 2. Variance Estimates ($x10^9$) for Each A_{gh} and Weighting Method Combination

Characteristic	A_{gh} = 1				A_{gh} = P_{gh}			
	Base Weights	Parent Sample Weights	Reweighting	Standard Error of Variance Estimates for Reweighting	Base Weights	Parent Sample Weights	Reweighting	Standard Error of Variance Estimates for Reweighting
Labor Force, Total	236.430	268.103	59.908	10.448	148.340	165.660	55.529	2.964
Black	21.606	26.717	6.736	0.843	21.904	24.940	6.625	0.452
Teenager (16-19)	10.386	13.432	4.724	0.676	8.869	10.717	4.793	0.321
Employed, Total	204.628	234.618	68.084	9.748	132.618	150.924	59.678	3.990
Black	15.812	20.519	9.016	1.082	17.065	20.830	7.722	0.521
Teenager (16-19)	8.372	10.421	5.295	0.633	7.460	8.946	5.013	0.374
Agriculture	7.492	8.531	7.912	0.821	4.910	6.180	6.519	0.453
Manufacturing wage & salary	52.675	66.430	45.912	7.309	41.634	49.799	40.405	2.604
Unemployed, Total	13.869	19.151	13.550	1.531	13.239	17.190	13.587	0.761
Black	3.234	4.345	3.406	0.503	3.021	4.145	2.967	0.154
Teenager (16-19)	2.399	3.090	2.355	0.291	2.043	2.650	2.133	0.170
15 weeks or more	4.099	5.660	3.701	0.275	3.294	4.632	3.548	0.236

Table 4. Variance Estimates ($x10^9$) for Each A_{gh} and Weighting Combination with the Sets of Collapsed Strata Reversed

Characteristic	A_{gh} = 1				A_{gh} = P_{gh}			
	Base Weights	Parent Sample Weights	Reweighting	Standard Error of Variance Estimates for Reweighting	Base Weights	Parent Sample Weights	Reweighting	Standard Error of Variance Estimates for Reweighting
Labor Force, Total	778.414	881.754	58.619	8.287	265.537	310.744	54.968	10.026
Black	28.828	34.284	7.573	1.306	20.967	24.851	6.010	0.863
Teenager (16-19)	12.239	14.378	5.879	0.849	8.878	10.737	4.616	0.570
Employed, Total	661.571	752.634	72.457	8.465	241.971	284.536	60.984	9.716
Black	20.596	26.222	7.755	1.114	14.575	18.449	7.983	0.838
Teenager (16-19)	9.820	11.551	5.741	0.818	7.013	8.347	5.213	0.579
Agriculture	10.627	12.539	13.979	1.650	4.393	5.436	5.226	0.917
Manufacturing wage & salary	75.264	89.343	52.243	5.652	49.356	59.976	42.227	7.857
Unemployed, Total	19.661	24.579	16.837	2.367	14.056	17.934	12.750	1.668
Black	3.614	4.730	3.124	0.343	3.364	4.430	3.337	0.495
Teenager (16-19)	2.278	2.873	2.295	0.344	2.074	2.731	2.309	0.225
15 weeks or more	4.323	5.694	4.518	0.543	3.233	4.535	3.432	0.363

Table 5. Variance Estimates ($x10^9$) for Each Reweighting Method Using A_{gh}, d Combinations

Characteristic	A_{gh} = 1		A_{gh} = P_{gh}		
	d=1	d=4	d=4	d=100	d=10,000
Labor Force, Total	64.462	59.908	49.080	46.994	46.698
Black	7.297	6.736	6.355	6.198	6.198
Teenager (16-19)	4.942	4.724	4.970	4.723	4.685
Employed, Total	74.665	68.804	58.636	55.795	55.381
Black	10.131	9.016	6.983	6.768	6.758
Teenager (16-19)	5.659	5.295	4.833	4.621	4.590
Agriculture	8.953	7.912	6.521	6.136	6.080
Manufacturing wage & salary	49.945	45.912	40.834	40.699	40.793
Unemployed, Total	14.623	13.550	13.908	13.556	13.531
Black	3.718	3.406	3.054	2.993	2.994
Teenager (16-19)	2.447	2.355	2.176	2.189	2.196
15 weeks or more	3.921	3.701	4.022	3.979	3.980