ANALYSIS OF CROSS-CLASSIFIED SCENARIOS FOR CATEGORICAL CHOICES OF ACTIONS

Eliana Heiser, Suzanne Landis, Gary Koch
Department of Biostatistics, University of North Carolina, Chapel Hill

Abstract

Some studies concerned with health services and/or social services are directed at the providers or users of services. These subjects are often asked questions about their choices among a fixed set of alternative actions under several scenarios. Moreover, these scenarios may be cross-classified according to two or more study factors. In this paper, the application of weighted least squares methods for the analysis of the correlated categorical data for the chosen responses from such studies is discussed. Extensions to stratified sampling are also considered. Two examples dealing with day care are provided for illustrative purposes.

1. Introduction

This paper is concerned with describing applications of weighted least squares methods to analyze a series of categorical responses that pertain to cross-classified scenarios related to the health services and social services areas.

Discussion is made more attractive and/or concrete by going through an application of a study of day care centers' criteria for sending sick children home. This study involves day care center staff who are asked questions of what to do, under the situations where the child may have fever, other illness symptoms, or a combination of these two factors. Here, questions of interest are:

1. How do day care center staff take care of children who are perceived to be sick;
2. What specific symptoms or signs in children prompt the day care center to act;
3. Do these actions vary with the center characteristic of being certified (i.e., allowing for partial monetary reimbursement for needy children) or not certified.

Even though staff within the same center might be viewed as clusters where they work, this paper will not account for this structure and will proceed to regard them as the unit of analysis. However, methods to account for the correlation structure of multiple staff within a day care center, with day care center as the unit of analysis, are outlined in the Appendix.

2. Methods

A straightforward framework for repeated measurements studies involves a set of subjects who often come from two or more subpopulations, with subjects in each group (or subpopulation) having their responses observed for a distinct set of conditions. These conditions can be based on a single factor or a cross-classification of two or more factors. For this framework, questions of interest are:

1. evaluation of group effects
2. evaluation of condition effects
3. description of the pattern of variation of reponse distributions across groups and conditions.

In this paper, attention is focused on responses from s=2 groups of subjects. The response for the k-th subject in the i-th group for the j-th condition is denoted by y_{ijk}; $i=1,2$; $k=1,2,\ldots,n_i$; $j=1,2,\ldots,d$. Each subject's response is classified into two possible values (a binary observation) indexed by $\ell=0$ or $\ell=1$. Weighted least squares methods, as described by Grizzle, Starmer and Koch [1969], are useful for describing the variation among groups and conditions, for certain summary statistics, such as proportions, means and ratios, through regression models. Here, attention is given to the extensions of these methods provided by Koch et al [1977] for repeated measurements studies and Koch et al [1986] for longitudinal data. They are directed at the description of the variation of the mean scores $\mu_{ij} = \sum_{\ell=0}^{1} m_\ell \phi_{ij\ell}$ with respect to $m_\ell=0$ and $m_\ell=1$ (binary case), across groups and conditions, where $\phi_{ij\ell} = \Pr\{y_{ijk}=\ell\}$. The binary response situation leads to $\mu_{ij} = \phi_{ij1}$.

An estimator for μ_{ij} is $\bar{g}_{ij}=\sum_{k=1}^{n_i} g_{ijk}/n_i$ where $g_{ijk}=m_\ell$ for $y_{ijk}=\ell$; $\bar{g}_{ij}$ is the across subjects mean of the scores $m_\ell(\ell=0,1)$ for the i-th group and j-th condition. Let $\underset{\sim}{g}_{i*k} = (g_{i1k},\ldots,g_{idk})'$ denote the (d x 1) vector of response scores for the k-th subject in the i-th group. When the groups of subjects from the respective subpopulations are conceptually equivalent to simple random samples and are sufficiently large (e.g., $n_i \geq d+25$), then $\bar{\underset{\sim}{g}}_i = \sum_{k=1}^{n_i} \underset{\sim}{g}_{i*k}/n_i$ has approximately a multivariate normal distribution. A consistent estimate of the covariance matrix is given by

$$\underset{\sim}{V}_{\bar{g},i} = \frac{1}{n_i}\sum_{k=1}^{n_i} (\underset{\sim}{g}_{i*k}-\bar{\underset{\sim}{g}}_i)(\underset{\sim}{g}_{i*k}-\bar{\underset{\sim}{g}}_i)'. \quad (1)$$

A general class of linear models for describing the variation among the μ_{ij} for the cross-classification of the s=2 groups and d conditions can be expressed as $\underset{\sim}{\mu}=\underset{\sim}{X}\underset{\sim}{\beta}$ where $\underset{\sim}{\mu}=(\mu_{11},\mu_{12},\ldots,\mu_{1d},\mu_{21},\mu_{22},\ldots,\mu_{2d})'$; $\underset{\sim}{X}$ is a known (2d x t) design matrix of full rank $t<2d$ and $\underset{\sim}{\beta}$ is the unknown (t x 1) vector of model parameters.

The weighted least squares estimators for $\underset{\sim}{\beta}$ are given by

$$\underset{\sim}{b} = (\underset{\sim}{X}'\underset{\sim}{V}_{\bar{g}}^{-1}\underset{\sim}{X})^{-1}\underset{\sim}{X}'\underset{\sim}{V}_{\bar{g}}^{-1}\bar{\underset{\sim}{g}} \quad (2)$$

where $\bar{\underset{\sim}{g}}=(\bar{\underset{\sim}{g}}_1',\bar{\underset{\sim}{g}}_2')'$ is the composite vector of means $\bar{g}_{ij}$ for all (group x condition) combinations; also $\underset{\sim}{V}_{\bar{g}}$ is the (2d x 2d) block diagonal matrix with $\underset{\sim}{V}_{\bar{g},i}$ as diagonal blocks and serves as a consistent estimate for the covariance matrix of $\bar{\underset{\sim}{g}}$.

The goodness of fit for the model can be assessed through the Wald statistic

$$Q_W = (\bar{\underset{\sim}{g}}-\underset{\sim}{X}\underset{\sim}{b})'\underset{\sim}{V}_{\bar{g}}^{-1}(\bar{\underset{\sim}{g}}-\underset{\sim}{X}\underset{\sim}{b})=\bar{\underset{\sim}{g}}'\underset{\sim}{W}'(\underset{\sim}{W}\underset{\sim}{V}_{\bar{g}}\underset{\sim}{W}')^{-1}\underset{\sim}{W}\bar{\underset{\sim}{g}} \quad (3)$$

for which $\underset{\sim}{W}$ is a full rank orthocomplement to $\underset{\sim}{X}'$; the statistic Q_W has an approximate chi-square distribution with (2d-t) degrees of freedom (d.f.). For models considered appropriate, the linear hypothesis $H_0:\underset{\sim}{C}\underset{\sim}{\beta} = \underset{\sim}{0}_c$ can be tested with the statistic $Q_C=\underset{\sim}{b}'\underset{\sim}{C}'(\underset{\sim}{C}\underset{\sim}{V}_b\underset{\sim}{C}')^{-1}\underset{\sim}{C}\underset{\sim}{b}$ which has approximately a chi-square distribution with d.f.=c under the null hypothesis.

Finally, predicted values for $\underset{\sim}{\mu}$ are defined by $\hat{\underset{\sim}{\mu}}=\underset{\sim}{X}\underset{\sim}{b}$ and a consistent estimate for their covariance matrix is given by $\underset{\sim}{V}_{\hat{\mu}}=\underset{\sim}{X}\underset{\sim}{V}_b\underset{\sim}{X}'$ where $\underset{\sim}{V}_b$ is the estimated covariance matrix for $\underset{\sim}{b}$.

Extensions of the previously described methods can be specified for situations with s>2 subpopulations and for functions other than the mean scores μ_{ij} . For other functions of the first order marginal distributions $\{\phi_{ij}\}$ or of higher order marginal distributions, see Koch et al [1986]. The application of weighted least squares methods to the mean scores μ_{ij} for responses by day care center staff to questions concerning children with illness symptoms is illustrated with examples in Section 3.

3. Examples

Applications of the weighted least squares methodology will be illustrated with a study of day care center (DCC) criteria to send sick children home.

All licensed DCC's located in three North Carolina counties, as of January 1985, identified from the North Carolina Day Care Licensing Board, were stratified by their "level" into two groups: certified centers (C) and uncertified centers (NC). Thirty three of 126 uncertified centers were randomly selected, and all 29 certified centers not previously used in a pretest were included in the study. All staff working in each of the selected DCC's were included as the study population. Each staff member completed a self-administered questionnaire. This questionnaire asked how the staff member would handle mildly ill children given the presence or absence of fever and presence or absence of specific symptoms/signs. Only a subset of the questions asked will be addressed here. The responses to questions of temperature alone, symptoms/signs alone and combinations of temperature and symptoms were: "Do nothing", "Tell parent at the end of the day", "Call parent to tell them", and "Call parent for immediate pick-up". In total, the data comprise information from 302 questionnaires. Some questions were specific to care of children in a certain age bracket. Staff members of DCC's that did not handle children of that specific age left those questions blank.

Although it is recognized that staff within the same center may tend to provide relatively similar responses, those subjects are viewed as independent in the discussions here. An outline of the methodology to account for the correlation structure of multiple staff within a day care center, with day care center as the unit of analysis, is presented in the appendix.

3.1 Choice for Immediate Pick-Up When Type of Day Care Center, Age of Child and Level of Temperature are Taken into Account

The first example is concerned with staff's choice of calling the parent for immediate pick-up for a child with diarrhea, taking into consideration both types of DCC's (C and NC), scenarios encompassing two age groups (age<2, 2<age<5) and three levels of temperatures (99°-99.9°F, 100°-100.9°F, 101°-101.9°F).

Analysis of the proportion of staff choosing immediate pick-up is done by first transforming the responses to the indicator

$$g_{ijk} = \begin{cases} 1 & \text{if the response } y_{ijk} \text{ is call parent for pick up} \\ 0 & \text{if otherwise} \end{cases} \quad (4)$$

then calculating the means $\bar{g}_{ij} = \sum_{k=1}^{n_i} g_{ijk}/n_i$, $n_1=95$, $n_2=69$, for each (temperature x DCC level x child's age) combination and the (12x12) covariance matrix $V_{\bar{g}}$;

$$V_{\bar{g}} = \begin{bmatrix} V_{\bar{g},1} & 0_{6,6} \\ 0_{6,6} & V_{\bar{g},2} \end{bmatrix} \quad (5)$$

where $V_{\bar{g},1}$ and $V_{\bar{g},2}$ are the estimated covariance matrices for the two types of DCC's obtained from (1). The vector of means $\bar{g}$ for the respective (temperature x DCC level x age of child) combinations and the corresponding standard errors are given in Table 1.

In the absence of an apriori model for the means $\bar{g}_{ij}$, the cell mean (or identity) model $X_1=I_{12}$ is used to obtain a preliminary assessment of the sources of variation across the two subpopulations (levels of DCC's) and the six scenarios.

Table 2 shows preliminary hypotheses of interest, with their corresponding contrast matrices relative to the cell mean model X_1. The first three entries address respectively the null hypotheses of no variation among (i) the two subpopulations, (ii) the two age groups and (iii) the three levels of temperature; their test statistics are significant ($\alpha=0.05$). Tests for the interaction of age x DCC level, age x temperature level and age x temperature x DCC level show non-significant results ($\alpha=0.05$). However, for the null hypothesis H_5, no DCC level x temperature level interaction, the Wald statistic of 21.2 (d.f.=2) is significant with P<0.01. Because of this significant interaction, a model including main effects and the DCC level x temperature interaction is fit to the data. The design matrix X_2 along with β_2 the vector of parameters have the form

$$X_2 = \begin{bmatrix} 1&1&1&1&1&1&1&1&1&1&1&1 \\ 0&0&0&0&0&0&1&1&1&1&1&1 \\ 0&0&0&1&1&1&0&0&0&1&1&1 \\ 0&1&0&0&1&0&0&1&0&0&1&0 \\ 0&0&1&0&0&1&0&0&1&0&0&1 \\ 0&0&0&0&0&0&0&1&0&0&1&0 \\ 0&0&0&0&0&0&0&0&1&0&0&1 \end{bmatrix} \quad (6)$$

$$\beta_2 = [\beta_{2,1},\beta_{2,2},\beta_{2,3},\beta_{2,4},\beta_{2,5},\beta_{2,6},\beta_{2,7}]' \quad (7)$$

where $\beta_{2,1}$ = reference cell parameter corresponding to age<2, NC DCC and temperature of 99 -99.9 F.

$\beta_{2,2}$ = increment for C DCC's for temperature of 99 -99.9 F

$\beta_{2,3}$ = increment for 2<age<5

$\beta_{2,4}$ = increment for temperature of 100 -100.9 F for NC DCC's

$\beta_{2,5}$ = increment for temperature of 101 -101.9 F for NC DCC's

$\beta_{2,6}$ = interaction increment for C DCC's and temperature of 100 -100.9 F

$\beta_{2,7}$ = interaction increment for C DCC's and temperature of 101 -101.9 F

The use of this model is supported by the non-significance ($\alpha=0.25$) of the Wald statistic $Q_W=3.32$ (d.f.=5).

Statistical tests for model parameters indicate that the model X_2 can be simplified by removal of $\beta_{2,2}$ and $\beta_{2,6}$. Thus, consideration is given to the final model X_3 and its parameters β_3 where

$$X_3 = \begin{bmatrix} 1&1&1&1&1&1&1&1&1&1&1&1 \\ 0&0&0&1&1&1&0&0&0&1&1&1 \\ 0&1&0&0&1&0&0&1&0&0&1&0 \\ 0&0&1&0&0&1&0&0&1&0&0&1 \\ 0&0&0&0&0&0&0&0&1&0&0&1 \end{bmatrix} \quad (8)$$

$$\beta_3 = [\beta_{3,1},\beta_{3,2},\beta_{3,3},\beta_{3,4},\beta_{3,5}]' \quad (9)$$

for which $\beta_{3,1}$, $\beta_{3,2}$, $\beta_{3,3}$, $\beta_{3,4}$, $\beta_{3,5}$ have the same interpretation as their respective counterparts $\beta_{2,1}$, $\beta_{2,3}$, $\beta_{2,4}$, $\beta_{2,5}$, $\beta_{2,7}$ for the model X_2. The use of the model X_3 is supported by the non-significance (α=0.25) of the Wald statistic Q_W=5.12 with d.f.=7. The estimated model parameters are b_3 = [0.136, -0.076, 0.215, 0.517, 0.222] and their estimated covariance matrix is

$$V_{b_3} = \begin{bmatrix} 44.21 & -27.22 & 1.31 & -5.82 & -16.27 \\ -27.22 & 44.64 & -1.88 & -8.52 & 13.55 \\ 1.31 & -1.88 & 69.00 & 39.44 & -24.39 \\ -5.83 & -8.52 & 39.44 & 149.98 & -134.62 \\ -16.27 & -13.55 & -2.44 & -134.62 & 214.29 \end{bmatrix} \times 10^{-5} \qquad (10)$$

Predicted values along with standard errors for this final model are presented in the last two columns of Table 1.

We conclude from this example that age and temperature of the perceived ill child are factors that contribute significant information toward staff's choice for immediate pick-up. Predicted percentages of pick-up for sick children of age less than two years are higher than those for children between ages of two and five years. As for temperature level, predicted percentages increase with temperature. It is interesting to note that staff's choice for immediate pick-up predicted percentages for temperature bracket of 101°-101.9°F are higher for certified centers than for uncertified centers.

3.2 Choice for Immediate Pick-Up for Children Between Ages of 2 and 5 Years, Three Symptoms/Signs of Illnesses and Elevated or Normal Temperature

In this example, weighted least squares methods are applied to data involving only children between ages of 2 and 5 years, with one of the three symptoms crankiness, diarrhea or conjunctivitis and in the presence (100°-100.9°F) or absence (<100°F) of elevated temperature. In order to study the proportion of staff choosing immediate pick-up under this new set of conditions, we first dichotomize the responses of interest as in (4) of example 3.1. Second, since the 133 staff responses from NC DCC's and the 169 from C DCC's have some missing data, an estimation procedure for dealing with multivariate situations in which missing data occur at random is applied. The problem is then to find an estimator μ_{ij} under this new condition. A method of estimation convenient for missing data situations was suggested by Stanish, Gillings and Koch [1978]; it works with additional random variables h_{ijk} which account for the missing data pattern. The indicator h_{ijk} has the value 1 if y_{ijk} is observed and the value 0 otherwise, thus the observed variables are $f_{ijk} = y_{ijk}h_{ijk}$ where "missing" is assigned the value zero.

The mean μ_{ij} is estimated by the mean of the data present, i.e., the ratio estimator

$$R_{ij} = \left[\sum_{k=1}^{n_i} y_{ijk}h_{ijk}/n_i\right] \Big/ \left[\sum_{k=1}^{n_i} h_{ijk}/n_i\right] = \bar{f}_{ij}/\bar{h}_{ij}. \qquad (11)$$

The pertinent quantities are included in composite vectors

$$m_{i*k} = (f_{11k},\ldots,f_{1dk},h_{11k},\ldots,h_{1dk}) \qquad (12)$$

for which the corresponding mean vectors are

$\bar{m}_i = \frac{1}{n_i}\sum_{k=1}^{n_i} m_{i*k}$. The multivariate ratio estimator of μ_i is expressed as $R_i = \exp[A \log_e \bar{m}_i]$ (13)

where log transforms each element of a vector to its natural logarithm, exp transforms each element of a vector to its antilogarithm, and $A=[I_d,-I_d]$.

A consistent estimate of the covariance nature of R_i denoted by $V_{R,i}$, is given by

$$V_{R,i} = D_{R_i} A D^{-1}_{\bar{m}_i} V_{\bar{m},i} D^{-1}_{\bar{m}_i} A' D_{R_i} \qquad (14)$$

where $V_{\bar{m},i}$ is the estimated covariance matrix for $\bar{m}_i$ with form similar to $V_{\bar{g},i}$ in (1) and $D_{\bar{m}_i}$ is a diagonal matrix with elements of $\bar{m}_i$ on the main diagonal. For the composite vector of ratio estimates $R=[R_1',R_2']'$ for the two types of day care centers, the estimated covariance matrix V_R has block diagonal structure like (5), but with the $V_{R,i}$ as the diagonal blocks.

In terms of the example being discussed, the elements of the vector R for the two levels of DCC are as follows:

R_{i1} = no elevated temperature but cranky
R_{i2} = no elevated temperature but diarrhea
R_{i3} = no elevated temperature but conjunctivitis
R_{i4} = elevated temperature alone
R_{i5} = elevated temperature and cranky
R_{i6} = elevated temperature and diarrhea
R_{i7} = elevated temperature and conjunctivitis.

The vector of means R_{ij}, for the (temperature x symptoms) combinations, by level of DCC are shown in Table 3 together with their corresponding estimated standard errors.

Since an apriori model is not available, the cell mean (or identity) model $X=I_7$ is used to obtain a preliminary assessment of the statistical significance for the sources of variation concerning the cross-classification of (temperature x symptoms) for each of the two levels of DCC's separately.

Table 4 shows the preliminary hypotheses of interest along with C matrices for both levels of DCC's. It can be seen that there is no statistically significant difference between elevated temperature alone and crankiness nor between the signs of diarrhea and conjunctivitis in the presence of elevated temperature (α=0.01) for neither type of DCC. In the absence of elevated temperature there is a suggested difference between diarrhea and conjunctivitis for NC DCC's (α=0.01). In view of these findings, the following tentative model is examined for goodness of fit, for each type of day care center separately

$$E[R] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \beta_{2,1} \\ \beta_{2,2} \\ \beta_{2,3} \\ \beta_{2,4} \end{bmatrix} = X_2\beta_2. \qquad (15)$$

The parameter $\beta_{2,1}$ represents the predicted reference value for crankiness and no elevated temperature, $\beta_{2,2}$ represents the effect of diarrhea and conjunctivitis under no elevated temperature, $\beta_{2,3}$ represents the effect of elevated temperature alone and crankiness in the presence of elevated temperature and $\beta_{2,4}$ represents the effect of diarrhea and conjunctivitis in the presence of elevated temperature. The use of this model is supported by the non-significance (α=.10) of the weighted least squares (or Wald) statistic Q_W,

$$Q_W = \begin{cases} 5.399 & \text{(d.f.=3) (NC)} \\ 0.071 & \text{(d.f.=3) (C)} \end{cases} \qquad (16)$$

where the estimated model parameters and their estimated covariance matrices are

$$\underset{\sim}{b}_{(NC)} = \begin{bmatrix} 0.029 \\ 0.504 \\ 0.305 \\ 0.633 \end{bmatrix} \quad \underset{\sim}{V}_{b(NC)} = \begin{bmatrix} 2.48 & -1.85 & -3.03 & -2.06 \\ -1.85 & 15.18 & 7.14 & 9.35 \\ -3.03 & 7.14 & 16.99 & 10.09 \\ -2.06 & 9.35 & 10.09 & 14.14 \end{bmatrix} \times 10^{-4} \qquad (17)$$

$$\underset{\sim}{b}_{(C)} = \begin{bmatrix} 0.027 \\ 0.677 \\ 0.477 \\ 0.841 \end{bmatrix} \quad \underset{\sim}{V}_{b(C)} = \begin{bmatrix} 1.76 & -1.47 & -1.48 & -1.96 \\ -1.47 & 10.51 & 2.65 & 4.62 \\ -1.48 & 2.65 & 12.94 & 4.60 \\ -1.96 & 4.62 & 4.60 & 7.17 \end{bmatrix} \times 10^{-4} \qquad (18)$$

Predicted values of the proportions for choosing immediate pick-up are obtained from $\hat{\underset{\sim}{\Pi}}=Xb$. These quantities along with their estimated standard errors are shown in the last two columns of Table 3. There, it can be seen that the predicted percentage for staff's choice of immediate pick-up for NC DCC's is 66.3% when child presents symptoms/signs of diarrhea and conjunctivitis in the presence of elevated temperature; 53.4% for these symptoms but no elevated temperature; 33.5% when child has elevated temperature alone and elevated temperature plus crankiness and only 3% for the symptom of crankiness alone. Thus, the symptoms/signs of diarrhea and conjunctivitis with a temperature of less than 100 F are perceived by the staff as a more serious illness situation than temperature of 100°-100.9°F alone, and when elevated temperature accompanies these two symptoms, the proportion of staff personnel that opts for calling parents for immediate pick-up of the child is even higher. When predicted percentages for C DCC's are examined, a similar pattern is observed to that of NC DCC's but with higher predicted percentages except for the symptom of crankiness in the absence of elevated temperature. These predicted percentages are 86.8% for diarrhea and conjunctivitis in the presence of elevated temperature and 70.3% when temperature is lower than 100°F; 50.4% when child has elevated temperature alone or with crankiness and only 2.7% for crankiness alone. So, except for crankiness alone, C day care centers show much higher percentages for choosing to call parents for immediate pick-up of the perceived ill child than the NC day care centers.

4. Summary

Day care centers staff's criteria for sending sick (or perceived sick) children home is evaluated through two examples, with respect to what actions they report for situations where the child may have fever, other illness symptoms or a cross-classification of these two conditions.

Day care center staff seem to act differently with regard to taking care of children perceived to be sick, depending on the age and level of temperature of the child and also on type of day care center. When only these three factors are taken into consideration, age of the child seems to contribute significant information for staff's choice of calling parent or not for immediate pick-up; while levels of center and temperature interact with each other.

When three specific symptoms/signs, crankiness, diarrhea and conjunctivitis, are taken into account in the presence and absence of elevated temperature for children between the ages of 2 and 5 years, the results obtained are straightforward to interpret. Temperature of 100°F-100.9°F with no symptoms is considered a less serious symptom/sign of illness than diarrhea and conjunctivitis; elevated temperature of 100°F-100.9°F along with any of these two symptoms shows the highest percentage for calling parent for immediate pick-up. The sign of elevated temperature alone and in the presence of crankiness are similar, and finally crankiness with temperature of less than 100°F leads only to a relatively small proportion of staff to call parent for immediate pick-up.

With respect to variation in action by the two types of day care centers, the attitude of staff members under the situation where age and temperature of child are taken into account does not vary significantly with level of center except for the temperature of 101°-101.9°F when percentage for choosing to call parents are higher for certified centers than for uncertified. When an analysis of symptoms and normal/elevated temperature is considered, except for crankiness alone, certified centers show larger percentages for choosing to call parents for immediate pick up than uncertified centers.

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Appendix

The study of day care center criteria to send sick children home involves a data structure with some degree of clustering where day care center is viewed as a cluster and staff members as the subunits of a cluster.

If day care center is considered as the unit of analysis, methods to account for the correlation structure of multiple staff within a day care center can be applied. The estimation methods described in Section 2 are revised to focus on day care center as the primary unit of analysis, and staff members as the secondary units of analysis. For this analysis, the response for the k-th subject in the h-th cluster, i-th group and j-th condition is denoted by y_{hijk}, with $h=1,2,\ldots,N_i$; $i=1,2$; $j=1,2,\ldots,d$ and $k=1,2,\ldots,n_{hi}$. An estimator for μ_{ij}, the mean score, is given by

$$\tilde{g}_{*ij} = \frac{\sum_{h=1}^{N_i} \bar{g}_{hij}}{N_i} = \frac{\sum_{h=1}^{N_i} \sum_{k=1}^{n_{hi}} (y_{hijk}/n_{hi})}{N_i} \quad ;$$

its estimated covariance matrix is given by

$$V_{\tilde{g}} = \frac{1}{N_i^2} \sum_{h=1}^{N_i} [(\bar{g}_{hi*} - \tilde{g}_{*i*})(\bar{g}_{hi*} - \tilde{g}_{*i*})']$$

where g_{hi*} is the response vector of mean scores $\{\bar{g}_{hij}\}$ for the i-th group and h-th day care center and

$$\tilde{g}_{*i*} = \frac{\sum_{h=1}^{N_i} \bar{g}_{hi*}}{N_i}$$

Accounting for the clustered structure of the data in the analysis is relevant so that applicable variances are not underestimated (or overestimated). When such analysis (not presented here) was done for the data in Example 1, the estimates for the mean proportions obtained were very similar to the results presented in Table 1. This indicates that the correlation among cluster subunits (i.e. staff) for the day care center study is relatively negligible and that viewing the subunits (staff) as independent of one another gives reasonable results.

Table 1. Observed and Predicted Staff's Choice of Immediate Pick Up Proportions and Standard Errors for Example 3.1

	Level	Age	Temperature	Observed Proportion	Observed S.E.	Final Model Predicted Proportion	Final Model Predicted S.E.
NC	0	<2	99°-99.9°F	0.126	0.034	0.136	0.021
	0	<2	100°-100.9°F	0.347	0.049	0.351	0.036
	0	<2	101°-101.9°F	0.642	0.049	0.653	0.043
	0	2-5	99°-99.9°F	0.074	0.027	0.060	0.019
	0	2-5	100°-100.9°F	0.242	0.044	0.275	0.032
	0	2-5	101°-101.9°F	0.579	0.051	0.578	0.040
C	2	<2	99°-99.9°F	0.166	0.039	0.136	0.021
	2	<2	100°-100.9°F	0.420	0.059	0.351	0.036
	2	<2	101°-101.9°F	0.885	0.039	0.826	0.031
	2	2-5	99°-99.9°F	0.058	0.028	0.060	0.019
	2	2-5	100°-100.9°F	0.290	0.055	0.275	0.032
	2	2-5	101°-101.9°F	0.812	0.047	0.799	0.031

NC: uncertified; C: certified

Table 2. Results of Hypothesis Tests from Cell Mean Model for Staff's Choice of Immediate Pick Up Proportions for Example 1

Hypothesis	C Matrix	Q_C	d.f.	Approximate p-value
H_1: No difference between the 2 DCC levels	[1 1 1 1 1 1 -1 -1 -1 -1 -1 -1]	5.29	1	0.022
H_2: No difference between the 2 age groups	[1 1 1 -1 -1 -1 1 1 1 -1 -1 -1]	12.69	1	<0.010
H_3: No difference among the 3 levels of temperature	[1 -1 0 1 -1 0 1 -1 0 1 -1 0] [1 0 -1 1 0 -1 1 0 -1 1 0 -1]	541.04	2	<0.010
H_4: No age group vs. DCC level interaction	[1 1 1 -1 -1 -1 -1 -1 -1 1 1 1]	0.087	1	0.769
H_5: No DCC level vs. temp. level interaction	[1 -1 0 1 -1 0 -1 1 0 -1 1 0] [1 0 -1 1 0 -1 -1 0 1 -1 0 1]	21.20	2	<0.010
H_6: No age group vs. temp. level interaction	[1 -1 0 -1 1 0 1 -1 0 -1 1 0] [1 0 -1 -1 0 1 1 0 -1 -1 0 1]	3.05	2	0.218
H_7: No age group vs. temp. level vs. DCC level	[1 -1 0 -1 1 0 -1 1 0 1 -1 0] [1 0 -1 -1 0 1 -1 0 1 1 0 -1]	0.077	2	0.963

Table 3. Observed and Predicted Staff's Choice of Immediate Pick Up Proportions and Standard Errors for Example 3.2

		Uncertified Day Care Centers				Certified Day Care Centers			
		Observed		Predicted		Observed		Predicted	
Fever	Symptom	Proportion	S.E.	Proportion	S.E.	Proportion	S.E.	Proportion	S.E.
No	None	-	-	-	-	-	-	-	-
No	Crankiness	0.032	0.016	0.030	0.016	0.027	0.133	0.027	0.132
No	Diarrhea	0.504	0.045	0.534	0.037	0.707	0.037	0.703	0.031
No	Conjunctivitis	0.593	0.046	0.534	0.037	0.699	0.038	0.703	0.031
Yes	None	0.308	0.042	0.335	0.037	0.500	0.042	0.504	0.034
Yes	Crankiness	0.371	0.043	0.335	0.037	0.507	0.041	0.504	0.034
Yes	Diarrhea	0.661	0.043	0.663	0.035	0.871	0.028	0.868	0.022
Yes	Conjunctivitis	0.690	0.043	0.663	0.035	0.865	0.028	0.868	0.022

Table 4. Results of Hypothesis Tests from Cell Mean Model for Staff's Choice of Immediate Pick Up Proportions for Example 3.2

		Uncertified DCC's			Certified DCC's		
Hypothesis	C-Matrix	Q_C	d.f.	p-value	Q_C	d.f.	p-value
H_1: No difference between symptoms in the absence of fever	$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}$	171.38	2	<0.01	433.24	2	<0.010
H_2: No difference between crankiness and conjunctivitis in the absence of fever	[1 0 -1 0 0 0 0]	142.38	1	<0.01	276.92	1	<0.010
H_3: No difference between diarrhea and conjunctivitis in the absence of fever	[0 1 -1 0 0 0 0]	3.34	1	0.068	0.03	1	0.867
H_4: No difference between symptoms in the presence of fever	$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$	70.75	2	<0.01	90.58	2	<0.010
H_5: No difference between crankiness and conjunctivitis in the absence of fever	[0 0 0 0 1 0 -1]	51.82	1	<0.01	75.49	1	<0.010
H_6: No difference between diarrhea and conjunctivitis in the presence of fever	[0 0 0 0 0 1 -1]	0.41	1	0.52	0.04	1	<0.010
H_7: No difference between fever alone and symptoms plus fever	$\begin{bmatrix} 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix}$	96.06	3	<0.01	121.65	3	<0.010
H_8: No difference between fever alone and crankiness plus fever	[0 0 0 1 -1 0 0]	2.10	1	0.147	0.02	1	0.883
H_9: No difference between fever alone and diarrhea plus fever	[0 0 0 1 0 -1 0]	53.72	1	<0.01	72.08	1	<0.010
H_{10}:No difference between fever alone and conjunctivitis plus fever	[0 0 0 1 0 0 -1]	54.96	1	<0.01	60.19	1	<0.010
H_{11}:No fever x symptom interaction	$\begin{bmatrix} 1 & 0 & -1 & 0 & -1 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 & -1 & 1 \end{bmatrix}$	17.36	2	<0.01	47.05	2	<0.010
H_{12}:No difference between symptoms in the presence and absence of fever	$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix}$	53.91	3	<0.01	146.12	3	<0.010
H_{13}:No difference between crankiness in the presence and absence of fever	[1 0 0 0 -1 0 0]	52.19	1	<0.01	130.89	1	<0.010
H_{14}:No difference between diarrhea in the presence and absence of fever	[0 1 0 0 0 -1 0]	15.35	1	<0.01	17.71	1	<0.010
H_{15}:No difference between conjunctivitis in the presence and absence of fever	[0 0 1 0 0 0 -1]	54.36	1	0.02	24.10	1	<0.010