MULTI-PURPOSE SURVEY SAMPLING

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SUMMARY

This paper describes a methodology for multipurpose sampling that generalizes results of Bryant, Hartley, and Jessen (1960). Given several univariate stratifications and sample sizes, we allocate samples satisfying convex constraints to multi-purpose cells in populations having structural zeros. Allocation to cells is by Dykstra's Generalized Iterative Fitting Procedure (GIFP) (1985). Determination of a random nonnegative integer matrix having expected value equal to the fractional components determined by GIFP is by a controlled allocation. It is similar to the controlled rounding used by Causey, Cox, and Ernst (1985) but does not satisfy the strong restraints imposed by them. Estimates of the parameter and its variance are determined by Horvitz-Thompson methods in which the approximate inclusion and joint inclusion probabilities have been determined by bootstrap estimation.

1. INTRODUCTION

While much of the sampling literature deals with procedures for sampling using one variable, many surveys need to control the accuracy of two or more (possibly uncorrelated) variables. This paper will present a strategy for sampling two or more variables that places controls on the sample size and the coefficients of variation of each variable.

In this paper we assume that we have a population containing two or more variables. Using some method of sampling that generalizes stratified simple random sampling (srs), we wish to minimize the sample size while the coefficient of variation of each sample meet a priori bounds. A stratification that is efficient (i.e., has strata boundaries that allow minimizing sample size for a given bound on the coefficient of variation) for one variable may not be efficient for another. Consequently, we stratify using each variable separately and consider the table of cross-strata (multi-purpose) cells determined by two or more individual stratifications.

One approach to dealing with two or more variables is to sample independently across the cross-strata defined by two or more variables. Nonlinear programming techniques introduced by Kokan (1963) are applicable (see also Kokan and Khan, 1967). The advantage of the Kokan techniques is their ease of implementation. Bethel (1985) provides an algorithm that works well in practice. The disadvantage of the Kokan techniques is that sample size cannot be adequately controlled. For instance, to compute variances, the sampling scheme must allocate two elements to each cross-strata cell. As the number of cross-strata increase, sample size can rise dramatically (Bryant, Hartley, and Jessen, 1960, p. 124).

This paper deals with the problem in which sample size determined by individual variables is controlled and sampling is not independent across cross-strata. Various aspects of this problem have been addressed by a variety of authors. Frankel and Stock (1942) introduced the notion of deep stratification. Deep stratification refers to those sample designs in which the Latin square principle can be used to reduce the number of sample units representing all cross-strata cells.

Tepping, Hurwitz, and Deming (1943) used deep stratification for designs in 2-way cases in which blocks were of approximately equal size. They observed that variances under deep stratification were generally, but not always, less than under srs. Goodman and Kish (1950) introduced the notion of controlled selection. Their probabilistic model of sampling allowed selection of preferred combinations of sample elements with greater probability than under simple or stratified random sampling. They also observed that variances were generally, but not always, less than under srs.

Bryant, Hartley, and Jessen (1960) (hereafter denoted BHJ) introduced the notion of 2-way stratification for complete data patterns (those patterns of cross-stratification in which all resulting cells contain population members). As the BHJ paper provides a useful model for describing later research and the techniques introduced in this paper, we will describe the BHJ approach more fully.

The BHJ model consists of three parts. First, they determine the sample counts needed for two univariate samples and consider the matrix of counts determined by the 2-way stratification that has margins equal the univariate sample sizes. They then devise a method of allocating the marginal sample counts to individual cells. Such an initial allocation may have fractional values. Second, given the fractional cell values, they determine a random mechanism for assigning nonnegative integer values to individual cells such that the expected values of the random integers are equal the fractional cell values and have the same margins as the margins determined by the fractional values. Third, given the random mechanism for assigning integers to cells, they sample randomly without replacement in cells and determine estimators of the desired population total (or mean) and its variance based on the marginal constraints.

The basic BHJ approach allocated the sample to 2-way cells without regard to the distribution of population counts within those cells. BHJ recognized that, if sample proportions deviated from population proportions, variances for samples satisfying the fixed marginal constraints could increase unnecessarily. Consequently, they introduced a way of allocating, with certainty, a portion of the sample to cells so that deviations from allocations proportional to population size within rows (or columns) were reduced.

There are two limitations to the BHJ approach. First, their approach does not allow determination of a probabilistic mechanism for determining integers in populations having empty cross-strata cells. Second, their method of adjusting for skewed population distributions only works well in the case of slightly skewed populations. With moderately or highly skewed populations, a more rigorous approach is needed.

Recognizing the first limitation, Jessen (1970) introduced a heuristic method of defining a probabilistic mechanism for determining the integer components associated with the within-cell fractions. As Jessen's approach was not rigorous, Ernst (1981) (see also Causey, Cox, and Ernst, 1985) introduced a formal algorithm for determining the within-cell integers when those integers are at most one. Neither Jessen nor Causey, Cox, and Ernst showed how to obtain the within-cell fractional cell allocations, nor did they not develop closed form estimators of the variance based on the marginal counts as BHJ did. Jessen, however, did indicate that if joint inclusion probabilities could be computed, then estimates could be obtained using the Horvitz-Thompson estimator.

The second section provides notation and provides examples of some of the difficulties that can arise. The remainder of this paper addresses the three components of the model for multi-purpose sampling as defined by BHJ.

The third section contains procedures that determine the required fractional cell counts, given the fixed margins and the patterns of missing data (structural zeros). When simple iterative proportional fitting (IPF) yields cell allocations that exceed the population counts, an additional constraint is needed. We show that the generalized fitting procedures of Dykstra (1985a, 1985b) can be used. These procedures allow convex constraints; thus, we can introduce cell allocations that are bounded above by population size in addition to the linear constraints imposed by margins.

The fourth section contains the probabilistic mechanism for determining matrices of random integers having expected counts equal to the desired fractional counts of section 2. In the special case in which we wish to obtain binary (i.e., zero-one) solutions for 2-way matrices of fractions, our procedure agrees with the procedure of Causey, Cox, Ernst (1985). If we wish to allow the matrices of random integers to deviate by more than one from the fractional values, then our procedure generalizes the procedure of Causey, Cox, and Ernst. Our procedure also holds for arrays in three or more dimensions for which controlled allocations exist. As shown by Causey, Cox, Ernst (1985, p. 907), such controlled allocations do not always exist for arrays having a large number of structural zeros. They do, however, exist for many arrays having a small number of structural zeros.

The fifth section presents the method of obtaining estimates of the population parameters and their variances. It is based on using Horvitz-Thompson estimators for which approximate inclusion and joint inclusion probabilities have been obtained using bootstrap simulation. The simulation involves repeatedly sampling the frame data according the procedures developed in the third and fourth sections and taking the proportion of times that pairs of records within and across cells occur.

The sixth section contains a summary and discussion of the results.

2. NOTATION AND EXAMPLES

For convenience, we will use the notation of 2-way contingency tables (Bishop, Fienberg, and Holland, 1975). With the exception of LP (linear programming) results, results hold in two or more dimensions.

3. ITERATIVE FITTING

In this section, we consider methods of determining cells counts in arrays given fixed marginal counts using classical IPF and Dykstra's (1985a,b) iterative fitting procedure.

To describe better the multi-dimensional situation, we first describe how one stratifies in one dimension. The basic idea of stratification is to order the population by a measure of size and determine strata boundaries so that within-strata variation is minimized.

Methods of determining univariate strata boundaries and sample sizes are due to Dalenius and Hodges, Ekman, and Sethi (see e.g., Cochran, 1977, pp. 127-131). The method of Dalenius and Hodges, for instance, was developed for ease of hand computation. Under the assumptions that the finite probability correction can be ignored, that the frequency count function is smooth and can be reasonably approximated by constants in suitably chosen intervals, and that variance in all intervals can be approximated by a fixed constant times the interval length, the Dalenius and Hodges method can be applied to determine strata boundaries.

For some populations of individuals, the method of Dalenius and Hodges provides a useful method of quickly determining strata boundaries. For other populations, say skewed populations of businesses, it may be more useful to compute variances directly without resort to approximations.

We note, however, that the methods of determining univariate strata boundaries and sample sizes generally require that sampling within strata be simple random. The method of BHJ and the method of this paper will require that sampling in a given row (column) not be independent of sampling in other rows (columns). It, thus, cannot be simple random within rows.

In cases where sampling within rows (columns) deviates from simple random, it is possible that the variances within rows (columns) increase. This is primarily due to the fact that the allocation of sample counts to cells is no longer in proportions that are consistent with the original proportions of the population (e.g., BHJ, p. 121; see also Cochran, 1977, p. 99).

To minimize the increase in within-row (-column) variances BHJ introduced a method of adjusting the within-cell sample allocation n_{ij}.

Their method basically consisted of allocating some of the overall sample to fixed cells. The fixed allocation caused the fraction n_{ij} to

correspond roughly to the population proportion P_{ij}/P.

With the slightly skewed population considered by BHJ, the sample size adjustment worked well. It should be noted, however, that the population considered by BHJ also had within-cell means and variances approximately equal. With moderately or highly skewed populations having two uncorrelated stratifying variables, we can expect the population counts, means, and variances to vary significantly across cells, possibly even in the same row (column).

Example 3.1 provides an example in which the BHJ procedure breaks down in skewed populations. The BHJ procedure fixed allocation procedure, which is intended to determine fixed allocations on a cell-by-cell basis, does not control the overall allocation.

Example 3.1. The data in this example are taken from an Energy Information Administration State-level sampling frame. To facilitate comparison with BHJ results, we stratify the population in a representative State so that the corresponding population matrix is complete. In example 3.2 below we will give a fuller description of the data.

The population matrix P has values

5	8	12	12	37
3	19	31	33	86
3	15	55	107	180
6	12	54	288	360
14	18	38	419	489
31	72	190	859	1152

We assume that we have a marginal (univariate) sample fractions of 4 in each row and 5 in each column. The BJH fixed allocation procedure (1960, pp. 121-123) necessitates that 6 units be allocated to cell (5,4).

To deal with skewed populations, we need a systematic way of allocating the sample to cells so that the sample proportions correspond roughly to the population proportions. To do this allocation, we propose using IPF in which the marginal values are determined by univariate sampling techniques and the initial matrix is the set of population values.

Example 3.2. The following example is more natural than example 3.1. The data are taken from the same Energy Information Administration State-level frame. The frame contains five measures of size. For convenience, we take the data associated with the two least correlated measures of size in one representative State.

The two variables are, at most, slightly correlated (r-square $\leq$ 0.2). Their distributions are moderately or highly skewed. Using each variable separately, we stratify the population and assign sample sizes within strata efficiently so that coefficients of variations meet a priori upper bounds and overall sample sizes are minimized. Sampling within univariate strata is simple random.

We modify the samples so that records chosen with certainty for one variable are chosen with certainty for the other and so that overall sample sizes for the two procedures are equal. For the purposes of this example, we assume that the number of noncertainty strata for each variable is eight and four records are sampled within each single-variable strata.

This example differs from the example 3.1 in that the population matrix contains structural zeros. The structural zeros arise because each univariate stratification is designed to minimize sample size for fixed a priori bounds on the coefficients of variation. In example 3.1, strata were collapsed so that the resultant population matrix was complete.

We wish to allocate the univariate samples to 2-way strata according to the pattern of the population values. This will minimize deviations from the allocation proportional to frequency count within rows or columns that would be determined using simple random sampling.

The 8 by 8 matrix $\underline{A}$ of population values determined by the 2-way stratification is:

-	1	1	3	1	2	1	5	14
-	1	5	5	7	1	3	17	39
-	-	2	4	5	11	3	29	54
1	-	-	6	3	10	16	67	103
-	1	2	3	8	13	20	90	137
-	-	3	1	3	11	22	130	170
-	-	1	1	4	3	27	139	175
4	2	3	3	8	9	19	208	256
5	5	17	26	39	60	111	685	975

If we perform ordinary iterative proportional fitting with initial matrix $\underline{A}$ and marginal constraints all set equal to $\underline{4}$, we obtain matrix $\underline{B}$:

-	1.42	.37	.69	.18	.22	.06	.06
-	.69	.90	.56	.60	.05	.09	.09
-	-	.52	.65	.62	.85	.13	.23
1.28	-	-	.50	.19	.40	.36	.27
-	.54	.28	.26	.53	.54	.47	.38
-	-	.56	.12	.27	.61	.70	.74
-	-	.23	.14	.43	.20	1.04	.95
1.72	.34	.13	.08	.17	.12	.14	.28

We notice that entries in cells (4,1) and (1,2) exceed available population values of 1 unit. If we apply Dykstra's procedure, we obtain matrix $\underline{C}$:

-	1.00	0.75	1.23	0.36	0.42	0.12	0.12
-	1.44	1.03	0.56	0.70	0.06	0.10	0.11
-	-	0.72	0.79	0.87	1.12	0.18	0.32
1.00	-	-	0.80	0.36	0.69	0.65	0.50
-	1.15	0.33	0.27	0.64	0.60	0.55	0.46
-	-	0.77	0.14	0.37	0.79	0.93	1.01
-	-	0.31	0.17	0.59	0.26	1.37	1.30
3.00	0.41	0.09	0.05	0.11	0.07	0.09	0.19

The fitted matrices determined by IPF contain fractional components in the cells that sum to integer margins. We need a procedure for determining random nonnegative integer matrices with the same fixed margins possessing the property that their within-cell expected values equal the fractional component on the fitted matrices.

4. PROBABILISTIC MECHANISM FOR CELL COUNTS

This section contains our method of determining random nonnegative integer matrices having margins equal the sum of the fractional components determined in section 3. The procedure parallels results given by Causey, Cox, and Ernst (1985), pp. 905-906. When 0-1 integer solutions exist for the defined matrices, our procedure agrees with theirs. Although we use the notation of 2-dimensional arrays for convenience, the basic algorithm holds in two or more dimensions.

Let $m_{i.}$, $i = 1, \ldots, I$ and $m_{.j}$, $j = 1, \ldots, J$ be fixed margins determined by univariate sampling strategies. Let N_{ij} be the population counts in cross-strata cell (i.j.) determined by the two univariate strategies. Apply Dykstra's IPR to N_{ij} to obtain a fitted matrix $\{g_{ij}\}$ with the desired margins $m_{i.}$, $m_{.j}$. Assume $g_{ij} \le N_{ij}$ for all i,j.

Let [.] be the roundoff function, and let a_{ij} be the integer component of g_{ij} for all i,j.

We wish to find a sequence of matrices M_{ijk} ($k = 1, \ldots, t$) having nonnegative integer entries and margins $m_{i.}$, $m_{.j}$ for all i,j and positive constants p_k, $k = 1, \ldots, t$, such that

$$(4.1) \qquad \sum_{k=1}^{t} p_k = 1 \quad , \quad \sum_{k=1}^{t} p_k M_{ijk} = g_{ij}.$$

We assume that, for any matrix $\underline{Q} = (q_{ij})$ having integer margins, we can find a nonnegative integer matrix $R = (r_{ij})$ having the same margins as $\underline{Q}$ and such that, if q_{ij} is an integer, then r_{ij} is an integer. We call such an R a <u>controlled allocation</u>. Although controlled allocations can generally not be found in three or more dimensions (say, using integer-LP algorithms), they may often be obtainable in typical sampling situations (see example 4.2).

<u>Proof of</u> (4.1). We define the sequence of matrices M_{ijk} and positive constants p_k inductively.

For k = 1, let $A_1 = A = (a_{ij})$ and M_{ij1} be the controlled allocation of A_1 and

$$p_1 = 1 - d_1/[d_1 + 0.5]$$

where $d_k \equiv \max_{ij} |a_{ijk} - m_{ijk}|$ for all k.

(4.5) For $k \ge 2$, let $p_k =$

$$\left(1 - \sum_{i=1}^{k-1} p_i\right)(1 - e_k)$$

where $e_k = [d_k + 0.5]/d_k$.

If $d_k > 0$, define A_{k+1} by

$$(4.6) \qquad a_{ijk+1} = m_{ijk} + (a_{ijk} - m_{ijk})\, e_k \text{ and}$$

let N_{k+1} be the controlled allocation of A_{k+1}.

At the end of this proof, we will show that all controlled allocations N_{k+1} exist.

Note that $a_{ijk} \ge 0$ for all i, j, k. If $a_{ijk} - m_{ijk} = d_k$ then a_{ijk+1} is the smallest integer larger than a_{ijk}. If $m_{ijk} - a_{ijk} = d_k$ then a_{ijk+1} is the largest integer smaller than a_{ijk}. If a_{ijk} becomes an integer for some k, then it remains an integer for subsequent values of k. As there exist only I x J cells, there necessarily exists a smallest integer t such that $d_t = 0$. Note that $\sum_{k=1}^{t} p_k = 1$.

Now we show that $\sum_{k=1}^{t} p_k M_k = A.$

Solving (4.6) for n_{ijk} and multiplying by p_k, we obtain

$$p_k\, n_{ijk} = \frac{p_k}{1 - e_k}\,(a_{ijk} - e_k\, a_{ij,k+1})$$

which together with (4.5) yields

$p_i\, n_{ij1} +$

$(p_i - 1)\, a_{ij2}\, pk\, n_{ijk} =$

$$(1 - \sum_{s=1}^{k-1} p_s)\, a_{ijk} +$$

$$(\sum_{s=1}^{k} p_s - 1)\, a_{ij,k+1}), \quad 2 \le k \le t.$$

Consequently,

$$\sum_{k=1}^{t-1} p_k\, n_{ijk} = a_{ij1} + \sum_{s=1}^{t-1} (p_s - 1)\, a_{ijt},$$

which, in turn, yields the desired result.

Example 4.1. Three dimensional procedure. Table 1 presents the results from applying the algorithm to a 3x3x3 matrix. The population considered is similar to the population in earlier examples, but three variables are used in stratifying. Each univariate strata is assigned sample size 4. For the purpose of presentation, the following mapping reduces the 3x3x3 matrix to one dimension

(a,b,c) --> a*3**2 + b*3 + c + 1,

where a,b,c take values 0,1,2.

The first matrix (ITER=0) is the fitted matrix derived by Dykstra's Iterative Fitting Procedure. The next sixteen matrices (ITER=1 thru 16) are the integer matrices obtained by the LP procedure. The final matrix (ITER=28) is the convex sum (with column P used as coefficients) of the integer matrices.

5. BOOTSTRAP AND ESTIMATION METHODOLOGY

In this section, we propose using Horvitz-Thompson (HT) estimators of the population total and its variance. The inclusion and joint inclusion probabilities are approximations obtained using Efron's bootstrap (1979, 1981, 1985). As the strengths and weaknesses of HT estimates are adequately addressed in the literature, we merely address the procedure for obtaining estimates of inclusion probabilities. The procedure is:

1. Given fixed marginal restraints determined by two univariate stratifications and a matrix of population counts determined by the univariate stratifications, determine the fitted matrix (g_{ij}).
2. Given the matrix $\underline{A}$ of fractions (non-integer portions) of the MLE, determine positive constants p_k and nonnegative integer matrices N_k such that
$$\sum_{k=1}^{t} p_k = 1 \quad \text{and} \quad \sum_{k=1}^{t} p_k N_k = A\ .$$
3. Sample records in the frame by first sampling matrix $N_k = (n_{ijk})$ with probability proportional to size p_k and then randomly selecting $(g_{ij} - a_{ij}) + n_{ij}$ elements in cell i,j.
4. Repeat step 3 a large number of times (100 is a good number) and compute approximate single and joint inclusion probabilities by averaging the number of times a given record or record pair appears, respectively. Computation can be minimized by noting that joint inclusion probabilities are constant for all record pairs within a fixed pair of cells.
5. With each record in the frame, associate its inclusion and joint inclusion probabilities. If there are IJ cells, then each record will have (IJ + 1)IJ/2 joint inclusion probabilities associated with it.
6. Draw the actual sample using the same convex sum of nonnegative integer matrices determined in 2.
7. Perform Horvitz-Thompson estimation with inclusion probabilities replaced with the approximations determined in 4.

Example 5.1, which uses the same data and stratification as Example 3.1, shows how the BHJ estimation procedure works if no IPF adjustment for skewness is done and if one is.

Example 5.1. The data base consists of 1188 records, each with two variables. Thirty-six records are sampled with certainty. A sample of size 20 is allocated to the remaining 1122 records. The first variable has five strata each containing four records and the second has four strata each containing five records. Based on the independently obtained stratifications, the univariate cvs are .05 and .04, respectively. The basic BHJ procedure yields multi-purpose cvs of .11 and .07, respectively. The IPF-modified procedure yields multi-purpose cvs of .06 and .03, respectively. Thus, given a fixed sample size, we have exerted reasonable control of cvs.

6. SUMMARY AND DISCUSSION

We have presented a theoretically justified but computationally intensive method for sampling and estimation when sample size (i.e., margins) and variances associated with two variables must be controlled.

When we can consider 2-way sampling in slightly skewed populations for which the population matrix induced by the univariate stratifications is complete, then the procedure of Bryant, Hartley, and Jessen is preferred. If we have moderately or extremely skewed populations with complete population matrices, then modifying the BHJ procedure using the fractional allocations obtained by IPF as given in section 2 can be used.

The modified BHJ procedure, however, will break down if the fixed integer components of the allocation do not leave at least two elements to be sampled in every row and column. If this happens, the procedure of this paper should be used because the joint inclusions can be computed even when only one element is to be sampled in a given row or column.

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Table 1: LP Fitting of 3-Dimensional Incomplete Matrix

1986 ASA
S1-S27 is 3x3x3 incomplete data array, ITER is the step in the iterative procedure
ITER=0 is original array, ITER=28 checks to show that original array is convex sum of integer array
D is difference at step (ITER) k and P is the associated probability

OBS	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15
1	2.000	0.638	0.079	0.402	0.548	0.249	0.000	0.000	0.084	0.000	0.000	0.169	0.573	0.520	0.903
2	2.000	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000	0.000
3	2.000	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000
4	2.000	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	0.000
5	2.000	0.000	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000
6	2.000	0.000	0.000	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000
7	2.000	0.000	0.000	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	1.000	1.000
8	2.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	0.000
9	2.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000
10	2.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	1.000
11	2.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	1.000
12	2.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1.000	0.000	1.000
13	2.000	1.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
14	2.000	1.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	1.000
15	2.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	1.000
16	2.000	1.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
17	2.000	0.000	0.000	0.000	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	1.000
18	2.000	0.638	0.079	0.402	0.548	0.249	0.000	0.000	0.084	0.000	0.000	0.169	0.573	0.520	0.903

OBS	S16	S17	S18	S19	S20	S21	S22	S23	S24	S25	S26	S27	ITER	D	P
1	0.470	0.355	1.009	0.000	0.907	0.206	0.191	0.346	0.268	0.365	0.685	1.032	0	0.000	0.000
2	0.000	0.000	1.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	1.000	2.000	1	0.968	0.032
3	0.000	0.000	1.000	0.000	0.000	0.000	0.000	1.000	0.000	1.000	1.000	1.000	2	0.937	0.061
4	0.000	0.000	2.000	0.000	1.000	0.000	0.000	0.000	1.000	0.000	1.000	1.000	3	0.990	0.009
5	0.000	1.000	1.000	0.000	1.000	0.000	1.000	0.000	0.000	0.000	1.000	1.000	4	0.980	0.018
6	1.000	0.000	1.000	0.000	1.000	1.000	0.000	0.000	0.000	0.000	1.000	1.000	5	0.766	0.206
7	0.000	0.000	1.000	0.000	1.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000	6	0.797	0.137
8	1.000	0.000	1.000	0.000	1.000	0.000	0.000	0.000	1.000	0.000	1.000	1.000	7	0.896	0.056
9	0.000	0.000	1.000	0.000	1.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000	8	0.998	0.001
10	1.000	0.000	1.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	1.000	9	0.702	0.143
11	0.000	1.000	1.000	0.000	1.000	0.000	1.000	0.000	0.000	0.000	1.000	1.000	10	0.935	0.022
12	0.000	1.000	1.000	0.000	1.000	0.000	1.000	1.000	0.000	0.000	0.000	1.000	11	0.733	0.084
13	1.000	1.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	0.000	1.000	12	0.719	0.065
14	0.000	1.000	1.000	0.000	1.000	0.000	0.000	0.000	1.000	1.000	0.000	1.000	13	0.169	0.138
15	0.000	1.000	1.000	0.000	1.000	0.000	0.000	1.000	0.000	1.000	0.000	1.000	14	0.071	0.026
16	0.000	1.000	1.000	0.000	1.000	0.000	1.000	0.000	0.000	1.000	0.000	1.000	15	0.500	0.001
17	0.000	1.000	1.000	0.000	1.000	0.000	1.000	0.000	0.000	1.000	0.000	1.000	16	0.000	0.001
18	0.470	0.355	1.009	0.000	0.907	0.206	0.191	0.346	0.268	0.365	0.685	1.032	28	0.000	1.000