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1. Introduction

Seasonal adjustment procedures are designed to eliminate regular movements in time series due to seasonal variation. Statistics Canada publishes thousands of socio-economic time series in seasonally adjusted form. The removal of seasonal components from these time series enables analysts to determine better where the economy stands relative to the business cycle. Such knowledge is useful in forecasting subsequent cyclical movements and provides the basis for decision-making to attempt to control the level of economic activity.

Most seasonally adjusted data published by Statistics Canada are produced by seasonally adjusting time series of estimates from periodic sample surveys. Unlike many unadjusted estimates, seasonally adjusted estimates are published without indications of their accuracy in the form of confidence intervals or variance estimates. Published seasonally adjusted survey data are subject to errors from two sources. First, the unadjusted data used as inputs to the seasonal adjustment process are survey estimates subject to sampling errors. Second, seasonally subject to sampling errors. Second, seasonally adjusted data produced from unadjusted data measured without error would be subject to errors associated with estimation of the seasonal components. Hausman and Watson (1985) recently examined the variability of seasonally adjusted data due to both these sources of error.

The majority of the seasonal adjustment methods officially adopted by statistical agencies belong to the category of moving average techniques such as the U.S. Bureau of Census Method II-X-11 variant developed by Shiskin, Young and Musgrave (1967) and the X-11-ARIMA procedure developed by Dagum (1980). These methods are nonparametric. In the absence of a parametric model it is not possible to analyze the importance of errors from the second source, that is the errors associated with the estimation of seasonal components. We assume that the true seasonal components of a time series are the estimates that would be produced by seasonal adjustment if the unadjusted series was measured without error and direct our efforts at the estimation of the variance of seasonally adjusted data due to sampling error.

Wolter and Monsour (1980) considered a method for estimating an approximate sampling variance for seasonally adjusted data produced by X-11 using a linear approximation of the seasonal adjustment procedure and estimates of the variance-covariance structure of the unadjusted series. In this study we attempt to estimate the variance of the nonlinear seasonal adjustment filters directly given the sampling design.

The paper contains five sections. The next section includes a discussion of time series decomposition, the X-11 and X-11-ARIMA seasonal adjustment methods and alternative variance estimation methodologies for seasonally adjusted data. The third section contains information about the design of the Labour Force Survey (LFS), variance estimation procedures for unadjusted data and the resampling scheme adopted for this study. The results of an application using provincial LFS estimates are given in section four. Both X-11 and X-11-ARIMA seasonal adjustment are considered. The final section contains suggestions for future work and concludes the paper.

2. Seasonal Adjustment

Classical modelling of economic time series involves the decomposition of the observed series { Y_t , t = 1, 2...T }, which is assumed to be measured without error, into unobserved seasonal and nonseasonal components. Seasonal adjustment procedures attempt to estimate these unobserved components. The seasonal variations { S_t , t = 1, 2...T } represent the composite effects of climatic and institutional events which repeat more or less regularly each year. The nonseasonal components are often further decomposed into trend-cycle (C_t) and irregular (I_t) components. The trend-cycle components represent fairly smooth movements in the series. The irregulars are unforseeable movements related to events of all kinds. In general they have a stable random appearance but in some cases extreme values may be present.

Although the process of seasonal adjustment implies that a time series measured without error is a series of realizations of a stochastic process, no explicit models for the various time series components are incorporated in most widely used seasonal adjustment procedures, including X-11 and X-11-ARIMA. For the current study we assume that seasonal components calculated by various seasonal adjustment procedures correspond to the true unobserved components in the case of data measured without error.

Given a time series of survey estimates { \hat{Y}_{t} ; t = 1, 2...T }, X-11 seasonal adjustment involves calculation of seasonal components { \hat{S}_{t} ; t = 1, 2...T } by applying a series of moving average filters to the observed series. Depending on the characteristics of the observed series the decomposition procedure involves either additive ($Y_{t} = C_{t} + S_{t} + I_{t}$) or multiplicative ($Y_{t} = C_{t} S_{t}I_{t}$) assumptions. In the case of additive assumptions, the seasonally adjusted value for time period t is calculated as $\hat{Y}_{t}^{2} = \hat{Y}_{t} - \hat{S}_{t}$. Multiplicative assumptions involve the obvious analogue.

The X-11 procedure involves some nonlinearities due to i) the identification and modification of outliers and ii) use of multiplicative assumptions. When multiplicative assumptions are used, moving average filters are applied to the logarithms of the original series. The seasonal adjustment procedure is not linear in logarithms since it involves equating annual sums of seasonally adjusted and unadjusted data rather than annual products.

If procedures for outlier identification and modification are ignored and the additive model is used, observations more than k periods from either end of the input series are seasonally adjusted using the same moving average filter, called a central-term filter. Seasonally adjusted values in the central portion of the series can be written as

$$\hat{\mathbf{Y}}_{t}^{a} = \sum_{s=-k}^{k} \hat{\mathbf{h}}_{s} \hat{\mathbf{Y}}_{s} .$$
(1)

The values of k and the filter coefficients depend on the seasonal adjustment options used. The default options involve k = 72 for monthly data. Coefficients corresponding to time periods more than three years from t are relatively small in all cases. Refer to Young (1968). Each observation less than k time periods from one end of the input series is seasonally adjusted using a different assymetric or end-term filter. Seasonally adjusted data for recent years (m < k) can be written as

$$\hat{Y}_{T-m}^{a} = \sum_{i=-k}^{m} h_{m,i} \quad \hat{Y}_{T-m+i}.$$
(2)

The seasonal adjustment procedure used by Statistics Canada for most time series is the X-11-ARIMA method. This method is a modified version of X-11 that basically consists of

- i) Modelling the input series using an ARIMA model of the Box and Jenkins (1970) type,
- ii) Using the fitted model to generate one year of forecasts and one year of backcasts, and
- Seasonally adjusting the extended series using X-11.

When the additive version of X-11 is used and there is no modification of extreme values, seasonally adjusted data from X-11-ARIMA for time periods $% \left(\frac{1}{2} \right) = 0$ more than k periods for either end of the input series are calculated using X-11 central-term moving average filters. Observations fewer than k periods from the ends of the input series are seasonally adjusted using a convolution of an X-11 end-term filter and the ARIMA extrapolation filter. If the ARIMA model is estimated in levels or differences of levels and the coefficients of the extrapolation filter are considered fixed, seasonally adjusted data for these time points can be written as a moving average. Estimation of the parameters of the ARIMA model and the fitting of the model to the logarithms of the unadjusted series introduce additional sources of nonlinearity in seasonally adjusted data for these observations.

One approach to estimation of the variance of seasonally adjusted data involves approximation of the seasonal adjustment process by a moving average filter. If each seasonally adjusted observation can be written as a moving average,

$$\hat{\mathbf{Y}}_{t}^{a} = \sum_{s=1}^{T} \mathbf{w}_{ts} \, \hat{\mathbf{Y}}_{s}, \qquad (3)$$

and an estimate of the intertemporal variance-covariance matrix of the unadjusted data, say $\hat{V}(\hat{Y})$ where $\hat{Y} = (\hat{Y}_1, \hat{Y}_2, ..., \hat{Y}_T)$ is available from the survey design, an estimate of the variance-covariance matrix of the vector of unadjusted data can be calculated as

$$V(\underline{Y}^{a}) = W'V(\underline{\hat{Y}})W.$$
(4)

This methodology was used by Wolter and Monsour to estimate the sampling variance of data seasonally adjusted using X-11.

In the current study we attempt to estimate the variability of seasonally adjusted data directly using the sampling design. For observations less than k periods from either end of the input series data seasonally adjusted using X-11-ARIMA, which depend on extrapolated values from an ARIMA filter and consequently on the estimated values of the ARIMA parameters, are nonlinear functions of all unadjusted estimates used as inputs to the X-11-ARIMA procedure,

$$Y_{s}^{a} = f_{T,s}(\hat{Y}) \quad s < k, \quad s > T - k.$$
 (5)

Seasonally adjusted values for observations more than ${\sf k}$ periods from either end of the input series are

calculated using X-11 filters. For these time periods the seasonally adjusted value is a nonlinear function of 2k + 1 unadjusted estimates

$$\hat{Y}_{s}^{a} = g_{s} (\hat{Y}_{s-k}, \hat{Y}_{s-k+1}, \hat{Y}_{s+k}) \quad k < s < T-k.$$
 (6)

3. The LFS Design and Variance Estimation

The methodology was applied to the Canadian Labour Force Survey, which is a monthly survey used to estimate labour force size and charateristics. The LFS involves two separate sampling designs. In large urban or self-representing (SR) areas the random group method of selection by Rao-Hartley-Cochran procedure (1962) is employed whereby strata (subunits) containing contiguous blocks and block faces (clusters) are delineated into six random groups by the RHC procedure. Within each group one cluster is selected with probability proportional to size without replacement and within the selected cluster, a systematic sample of dwellings is selected at a subsampling rate such that the overall sampling fraction at the subunit level is fixed. There are variations in the sampling fraction by province in the 1971 Census based design (old design) and by economic regions within province in the 1981 Census based design (new design). In the rest of the country, called non-selfrepresenting (NSR) areas, the small urban and rural portions of each economic region are delineated into one to about five strata and within each stratum two to four PSU's are selected with PPS. Sub-sampling within the urban and rural portion of each selected PSU is taken independently in two or three more stages so that the overall selection probability is constant within each stratum with sampling fractions varying between provinces in the old design and between economic regions in the new design. There are some differences in the sampling procedures between the old and new designs that need not concern us here.

The estimates of totals of many LFS characteristics such as employed and unemployed are derived as Horvitz-Thompson (1952) sample-weighted estimates and adjusted for non-response. In the final stage of estimation, ratio estimates using post-stratified age-sex cells within each province are obtained. Since June 1981 a raking ratio estimation procedure has been used with the age-sex population as one dimension and metropolitan or non-metropolitan/economic region population as the other dimension. Two iterations of this raking ratio are computed.

For the purposes of variance estimation for unadjusted LFS estimates the standard assumption that first stage units are selected with replacement is made. Variance estimation for unadjusted LFS estimates is based on Taylor linearization of the final stage raking ratio procedure. Taylor linearization was not a feasible variance estimation method for seasonally adjusted data. The derivatives of the seasonal adjustment filters are not analytically tractable. Numerical differentiation was attempted without practical success.

To compute variance estimates for seasonally adjusted data a resampling methodology was adopted. In SR areas two pseudo-PSU's were formed in each stratum by combining random groups. Strata with two selected first stage units in the resulting set-up were resampled using partially balanced repeated replication (PBRR). Other strata were resampled using the bootstrap. For a stratum with n_h (> 2) selected first stage units in the survey design, a with

replacement sample of size n_h-1 was drawn for each replicate. Design weights for these strata were adjusted by the factor $(n_h-1)/n_h$. For strata resampled using PBRR, design weights were increased by a factor of two.

The LFS involves a partially rotating sample with one-sixth of the dwellings replaced every month. The dwellings are selected by a 2 or 3 stage sample within each stratum independently and the first stage units, i.e., primary sampling units within NSR strata and clusters within SR subunits, remain in the sample for several years to be replaced by another PSU or cluster by a rotation procedure. Thus, each subsample of each previous survey may be derived in a manner such that the subsample consists of identical or "replaced" PSU's and clusters as in the current survey. The "replaced" PSU or cluster refers to the PSU or cluster that rotated out between the earlier and current survey. This procedure is quite straight-forward as long as the same design is in effect.

Each decennial census, the LFS is completely redesigned with new strata and subunits with new PSU's and clusters in turn and finally, new dwellings. The time period used for the application reported in section four, January 1979 to March 1986, included an LFS redesign. There was an overlap period between the old and new designs to mitigate potential abrupt changes in the statistics resulting from independent as opposed to partially rotating samples. The new design was phased in gradually over a six-month period. The subsamples for surveys before October 1985, based on the 1971 census, were somewhat arbitrarily set up to correspond to the subsamples of the new design. There was considerable flexibility in this operation since the old and new samples were, to all practical purposes, completely independent. There was a small potential dependence in the case of dwellings being reselected in the new design after the old one has been discarded. To reduce response burden, these dwellings were deleted from the current sample and the sample weight adjusted.

An attempt was made to duplicate as much of the LFS estimation procedure as possible for each replicate. The final stage raking ratio adjustments (and the simple ratio adjustments used before June 1981) were recomputed in each case. Weight adjustments for non-response were also recalculated in SR areas. It is impossible to replicate precisely by means of resampling procedures all of the events that lead to the sampling and non-sampling errors present in the estimates based on the full sample, since the original LFS sample design is not set up to ensure pure replication. The imprecise replication of the events arises from interviewer assignments crossing over replicates, weight adjustments for non-response that cross over replicates and finally, selection of PSUs and clusters without replacement rather than with replacement. Consequently, the variance estimates based on resampling procedures, with appropriate adjustments for sample size, will tend to over-estimate the sampling variance between PSUs in NSR strata, over-estimate the sampling variance between clusters in SR sub-units and to under-estimate the correlated non-sampling variances because the non-sampling covariance (likely positive) between dwellings in different PSUs or clusters is omitted from the variance estimate.

4. Application to Provincial LFS Estimates

In order to determine the practical feasibility of the use of resampling methods to estimate the

variance of seasonally adjusted data and examine the empirical properties of resampling variance estimates, a pilot study was conducted using data from Nova Scotia and Manitoba. For each province four characteristics-total employed, men employed, women employed and total unemployed - were considered. Time series of estimates for each replicate and characteristic were constructed over the period January 1979 to March 1986.

Variance estimates for both unadjusted and seasonally adjusted estimates for Nova Scotia were computed using 44 replicates partially balanced over strata with two selected first stage units and for Manitoba using a partially balanced set of 48 replicates. The partially balanced designs were of order two. That is, the strata with two selected first stage units were divided into two groups and the same balanced design applied to both groups. The variance estimators used has a form analogous to the estimator \dot{V}_{BRR-H} defined in the literature. For unadjusted data, the variance estimate for a particular month and characteristic was calculated by

$$\hat{V}_{PBRR-B}$$
 $(\hat{Y}_{t}) = \sum_{r=1}^{R} (\hat{Y}_{tr} - \hat{Y}_{t})^{2}/R$ (7)

where R is the number of replicates, \hat{Y}_t is the full sample unadjusted estimate of the characteristic for month t and \hat{Y}_{tr} is the PBRR-bootstrap unadjusted estimate for replicate r. Variance estimates for seasonally adjusted data were defined analogously.

Before the resampling methodology was applied to seasonally adjusted data, PBRR-bootstrap variance estimates for unadjusted data were compared to estimates computed using Taylor linearization of the LFS raking ratio estimator over the period April 1984 to March 1986. The Taylor linearization variance estimation progam uses the formulae for the variance of a raking ratio estimator given in Brackstone and Rao (1979). Six random groups are used in each SR strata and it is asumed that first stage units are selected with replacement. The resampling variance estimates were somewhat more volatile than the Taylor linearization estimates. Refer to Table 1 for an example. This volatility difference is expected in view of the results of recent Monte Carlo experiments such as the study by Kovar (1985).

Averages of estimated coefficients of variation for unadjusted LFS estimates over the period April 1984 to March 1986 are reported in Table 2. These differences are not unreasonably large when compared to available evidence concerning the bias in the Taylor linearization variance estimator. Refer to Choudhry and Lee (1986). The PBRR-bootstrap and Taylor linearization variance estimators involve slightly different methodological assumptions. Unlike PBRRbootstrap variance estimates, the Taylor linearization estimates do not incorporate the effects of nonresponse adjustments in SR areas or ratio adjustments for numbers of urban and rural dwellings applied in The Taylor linearization variance NSR areas. estimator uses LFS final weights resulting from two raking ratio iterations in the variance formula for a single iteration. In addition, characteristic estimates computed using replicates were rounded before being used to compute PBRR-bootstrap variance estimates. Rounding was done to ensure that the PBRR-bootstrap variance estimates for unadjusted data would be directly comparable to the variance estimates for seasonally adjusted data (LFS data are rounded before being seasonally adjusted). These methodological differences could produce systematic differences between the variance estimates. Even in the absence of methodological differences, the PBRR-boostrap and Taylor linearization variance estimators would have different small sample biases, although this difference might be negligible for a survey as large as the LFS.

The comparisons were intended as a method of verifying the PBRR-bootstrap variance estimation methodology before it was applied to seasonally adjusted data. There is nothing here that would suggest that experimental use of the PBRR-bootstrap methodology for seasonally adjusted data is inappropriate.

To compute variance estimates 'for seasonally adjusted data the time series corresponding to each replicate were seasonally adjusted twice, one using X-11-ARIMA with the options used to seasonally adjust Statistic's Canada's published data and once using X-11 without the ARIMA extrapolation filter. Averages of estimated coefficients of variation for unadjusted data, data seasonally adjusted using X-11 and data seasonally adjusted using X-11-ARIMA are reported in Table 3 for a twelve month period in the middle of the period used for seasonal adjustment. Since these observations are more than 36 months from both ends of the input unadjusted series they are effectively seasonally adjusted using X-11 (compare the second and third columns). The averages of coefficient of variation estimates for seasonally adjusted data are lower than the corresponding averages for unadjusted data in all cases. One should note, however, that standard errors for the differences between these two statistics are not available.

Averages of estimated coefficients of variation for the last twelve months of the period used for seasonal adjustment are reported in Table 4. For observations in this period, X-11 and X-11-ARIMA do not produce the same seasonally adjusted values. Comparing Tables 3 and 4 one notes that the averages of estimated coefficients of variation for data seasonally adjusted using X-11 are generally larger, relative to the corresponding averages for unadjusted data, near the ends of the series. In the case of Manitoba total employed, for example, the average of CV estimates for seasonally adjusted data is 4% lower than the corresponding average for unadjusted data in Table 3. In Table 4 the reduction is only 2.6%. Reductions in average estimated CV for X-11-ARIMA seasonal adjustment are considerably larger than those for X-11 in Table 4. For Manitoba total employed, X-11-ARIMA seasonal adjustment leads to a 10.3% decrease in average estimated CV. Similar, although smaller, differences in the reductions of averages of CV estimates using X-11-ARIMA as opposed to X-11 seasonal adjustment were observed for earlier time periods.

The results for X-11 reported here are generally similar to those of related studies. Wolter and Monsour report the results of work involving a linear approximation to X-11. Six monthly American series were considered. In each case it was assumed that the relative variance of the unadjusted series was constant. Relative variance is the variance of the logged series and is approximately equal to the square of the coefficient of variation. Relative variances for seasonally adjusted series were smaller than relative variances for unadjusted data. The differences were greatest in the central portions of the series where relative variances for seasonally adjusted data were between, approximately, 3% and 17% lower than the corresponding variances for unadjusted data.

Hausman and Watson (1985) studied the effects of

sampling error on the variance of seasonally adjusted estimates using a linear approximation to the X-11 filter under the assumption that the true trend-cycle components of two American unemployment rate time series were generated by certain parametric models. The use of models for the trend-cycle components allowed them to measure the mean squared error of the trend-cycle estimates. The increase in the mean squared error of the trend-cycle estimates due to the sampling error was less than the variance of the sampling error for both time series.

Theoretical evidence also suggests that variances of seasonally adjusted data should be lower than variances for unadjusted data. When the additive decomposition model is used and nonlinearities due to treatment of extreme values are ignored, each seasonally adjusted estimate produced using X-11 can be exactly represented as a moving average of unadjusted estimates. The sum of the moving average coefficients is one. In this case, if the variances of the unadjusted estimates are constant over time, one can show that the variances of seasonally adjusted estimates will be smaller than the variance of the unadjusted estimates.

The differences between the results obtained for X-11 and X-11 ARIMA are apparently due to seasonal terms in the ARIMA extrapolation filters. When X-11-ARIMA is used, all the series considered in this study are seasonally adjusted after extrapolation using an ARIMA model with seasonal differencing and a seasonal moving average coefficient. Some sampling error is relatively highly correlated with sampling error in the original unadjusted data one year earlier.

Refer to Table 5 and note that the estimated correlation for s=12 is considerably larger when averaged over the period April 1985-March 1986 than when averaged over April 1984-March 1985. This large seasonal correlation leads to an increase in the quantity of sampling error removed by the X-11 filters applied in the third step of X-11-ARIMA seasonal adjustment.

5. Conclusion

In this paper we have investigated the sampling variability of data from the Labour Force Survey seasonally adjusted using the X-11 and X-11-ARIMA A resampling methodology combining methods. partially balanced repeated replication and the bootstrap was used. Variance estimates for unadjusted data obtained using this methodology were reasonably using Taylor estimates computed close to linearization. For data seasonally adjusted using X-11 the results reported here were similar to results obtained by Wolter and Monsour (1980) using a methodology involving linear approximation of the seasonal adjustment filters. Averages of estimated coefficients of variation for seasonally adjusted data were generally lower than averages for unadjusted data in the middle of the time series. The reductions in averages of estimated coefficients of variation were somewhat smaller near the ends of the series when X-11 seasonal adjustment was used. Reductions in averages of CV estimates for X-11-ARIMA near the ends of the series were considerably larger than reductions for X-11.

In view of the instability of the resampling variance estimates, further work is required. This may involve investigation of approaches using Taylor linearization to estimate $\hat{V}(\hat{Y})$ and linear approximations to the seasonal adjustment filters, use of the

jackknife and investigation of schemes for the minimization of the inefficiency of PBRR variance estimates relative to BRR estimates (Rust, 1984).

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| Table 1 | | | |
|---|--|--|--|
| Estimated Coefficients of Variation - Unadjusted Data | | | |
| Manitoba Total Employed | | | |

| Month 1985 | Taylor | PBRR-B |
|------------------------------|--------------|--------------|
| January February March | 0.87 | 0.89 0.96 |
| April May | 1.05 | 0.96 0.93 |
| July August | 1.0 | 1.13 |
| October November | 0.88 0.91 | 0.80 |

Table 2Average Values of Taylor and PBRR-BootstrapCoefficients of Variation Estimates-Unadjusted DataApril 1984 - March 1986

| Characteristic | Average of C | V Estimates |
|--|----------------------------------|----------------------------------|
| | PBRR-B | Taylor |
| Nova Scotia | | |
| Men Employed Women Employed Total Employed | 1.152 1.966 | 1.172 1.923 |
| Total Unemployed | 4.611 | 4.790 |
| Manitoba | | |
| Men Employed Women Employed Total Employed Total Unemployed | 0.977 1.472 0.893 5.239 | 1.003 1.461 0.899 5.352 |

Table 3Averages of Estimated Coefficients of Variation-LevelsFebruary 1982 - January 1983

| | Unadjusted Data | Seaso X-11 | onally Adjust X | ed Data -11-ARIMA |
|-------------------|--------------------|---------------|--------------------|----------------------|
| Nova Scotia | | | | |
| Men Employed | 1.39 | 1.36 | (-2.2)* | 1.36 |
| Women Employed | 1.84 | 1.73 | (-6.0) | 1.72 |
| Total Employed | 1.17 | 1.16 | (-0.9) | 1.16 |
| Total Unemployed | 4.54 | 4.50 | (-0.9) | 4.50 |
| Manitoba | | | | |
| Men Employed | 1.04 | 1.01 | (-2.9) | 1.01 |
| Women Employed | 1.97 | 1.85 | (-6.1) | 1.86 |
| Total Employed | 1.01 | 0.97 | (-4.0) | 0.97 |
| Total Linemployed | 5.14 | 4.94 | (-3.9) | 4.95 |

Table 4Averages of Estimated Coefficients of Variation-LevelsApril 1985 - March 1986

| | Unadjusted | | Seasonally A | djusted | Data |
|------------------|------------|------|--------------|---------|---------|
| | Data | X | -11 | X-11 | -ARIMA |
| Nova Scotia | | | | | |
| Men Employed | 1.18 | 1.16 | (-1.7)* | 1.11 | (-4.3)* |
| Women Employed | 1.93 | 1.94 | (0.5) | 1.83 | (-5.7) |
| Total Employed | 1.08 | 1.03 | (-4.6) | 0.96 | (-11.1) |
| Total Unemployed | 4.66 | 4.73 | (1.5) | 4.33 | (-7.1) |
| Manitoba | | | | | |
| Men Employed | 1.06 | 1.07 | (0.9) | 0.99 | (-6.6) |
| Women Employed | 1.61 | 1.59 | (-1.2) | 1.46 | (-9.3) |
| Total Employed | 0.97 | 0.95 | (-2.6) | 0.87 | (-10.3) |
| Total Unemployed | 5.80 | 5.77 | (-0.5) | 5.60 | (-3.4) |

* Percent difference relative to average estimated CV for unadjusted data

| Table 5 |
|--|
| PBRR-Bootstrap Estimates of Autocorrelation Function of Sampling Error |
| Manitoba Total Employed - Unadjusted Data Augmented by ARIMA Forecasts |

| | Estimated Correlation Between $\hat{\boldsymbol{\gamma}}_t$ and $\hat{\boldsymbol{\gamma}}_{t+s}$ | | | | |
|---|---|---|--|--|--|
| S | Average for t from April 1984 to March 1985 | Average for t from April 1985 to March 1986 | | | |
| 1 2 3 4 5 6 7 8 9 10 11 | 0.656 0.472 0.364 0.272 0.199 0.130 0.077 0.070 0.076 0.068 0.084 | 0.787 0.643 0.553 0.474 0.418 0.386 0.407 0.432 0.466 0.507 0.553 | | | |