# A Two Phase Probability Proportional to Size Design for Telephone Sampling 

James M. Lepkowski and Robert M. Groves, The University of Michigan

## Introduction

Two approaches are frequently used to select samples of telephone households in the U. S. The first uses a list of telephone numbers from a directory or other source to provide a frame to which a wide range of selection techniques can be applied. In most cases, the list or directory from which the sample is drawn does not provide complete coverage of the telephone household population, but investigators are attracted by the great convenience of the list frame and by the high proportion of the numbers on the list that are working residential numbers. It is also possible to use name and address information available with a list frame to send advance letters to selected units and improve response rates (Traugott, Groves, and Lepkowski, 1986). Sample selection techniques can be used which cover some telephone numbers not on the list (Sudman, 1973), but often the list is sampled directly, and its coverage deficiencies are ignored.

The second approach generates telephone numbers at random, many of which may not be residential numbers. Telephone numbers can be generated completely at random within known area code and central office code combinations, or they can be generated using a two stage procedure described by Waksberg (1978). These random digit dialing (RDD) designs provide, in principle, at least, complete coverage of the telephone population, but they suffer from inefficiencies due to generation of a large proportion of nonworking numbers.

A third alternative, dual frame list-RDD sampling, combines some of the conveniences of the list frames with the complete coverage of the RDD approach (Groves and Lepkowski, 1986). Advance letters can be sent to list frame selections to increase response rates. Although the dual frame approach incorporates the more attractive features of both list and RDD sampling into a single design, it also retains some less desirable properties, such as the high proportion of nonworking numbers in the RDD portion of the design. The dual frame design also requires the computation and use of weights to compensate for unequal probabilities of selection across the frames. A preferred strategy would be one which includes the attractive features of the dual frame design, such as complete coverage of the telephone population and advance letters to list frame selections, but which at the same time avoids unequal probabilities of selection and the difficulties of multiple frame sampling.

## A Two Phase Sample Design

An alternative which possesses many of these properties involves two phases of sample selection and the use of probability proportional to size (PPS) selection in the second phase. In the first phase, the full population of telephone numbers (RDD frame) is divided into clusters of 100 consecutive numbers defined by the area code, central office (CO) code, and the first two digits of the suffix of the number. (Use of 100 clusters of consecutive numbers is discussed here as a matter of convenience. Other size clusters could be used.) We use the term "prefix area" to refer to these 100 -banks. A sample of these prefix areas is selected, perhaps using explicit strata. It may be selected by systematic sampling from a list of area and CO code combinations that is sorted by size and geographic location of the exchange in which the CO code is located. The prefix areas are selected with equal probabilities. If there are M area-CO code combinations, a sample of $m$ prefix areas is selected from the ( $\mathrm{M} \times 100$ ) prefix areas with first phase probability $\mathrm{f}_{1}=m /(\mathrm{M} \times 100)$.

The 100 telephone numbers within each prefix area are then assembled and a file of $m \times 100$ telephone numbers is prepared and sent to a commercial telephone directory firm. The commercial firm matches telephone numbers on the list of $m \times 100$ numbers to those in its telephone household file. When a match occurs, the commercial firm appends the name, address, and other information from its telephone household file listing to the sample telephone number. (There are other ways to obtain the telephone number name and address besides preparing a file of $m \times 100$ telephone numbers and matching to the large commercial files. These alternatives depend on the nature of the commercial files and the technical abilities of the commercial firm.)

From the file with matching indicators, a variety of sample designs can be implemented in the second phase. One, using the matched telephone number file to create measures of size for selection in the second phase, is described here.

A count of the number of telephone numbers with matched name and address information is made for each of the $m$ prefix areas. In most populations, a large proportion of the prefix areas will have no or few listed numbers. However, these prefix areas may still contain working residential telephone numbers.

Based on the counts of listed numbers, the sample of prefix areas is divided into two strata. The first stratum contains prefix areas with zero or few listed numbers, and is referred to as the low density stratum. Telephone households in the low density stratum will be selected by a two-stage RDD procedure (Waksberg, 1978). In the first stage, a single telephone number is drawn at random from a prefix area and called. If it is a working residential number, a fixed number of working residential numbers are selected at random from within the prefix area. This procedure requires that telephone numbers within the prefix area be called, and if they are nonworking or nonresidential, they are replaced by another number (hence, the term "rejection rule" sampling is sometimes used to refer to this design). Replacement of a nonworking or nonresidential number toward the end of a study results in fewer opportunities to call the replacement number than telephone numbers generated at the beginning of the study period.

The second stratum, referred to as the high density stratum, contains the remaining prefix areas with more than a few listed numbers. Telephone households in this stratum are selected with a probabilities proportional to counts of listed numbers. Let $\mathrm{m}_{\mathrm{T}}$ denote the number of prefix areas to be selected in the high density stratum, Mos ${ }_{a}$ denote the number of listed numbers in the ath prefix area, and $\mathrm{b}_{\mathrm{T}}$ denote the fixed number of listed numbers to be selected in each selected prefix area. Among prefix areas in the high density stratum, a sample of $\mathrm{m}_{\mathrm{T}}$ prefix areas is selected with probabilities proportional to the measures $\mathrm{Mos}_{a}$. Within selected prefix areas, telephone numbers are selected at a rate inversely proportional to the measure of size for the prefix area. The probability of selection for a telephone number (or a working residential telephone number) is

$$
\mathrm{f}_{\mathrm{T}}=\frac{m}{\mathrm{M} \times 100} \cdot \frac{\mathrm{~m}_{\mathrm{T}} \mathrm{Mos}_{a}}{\sum \operatorname{Mos}_{a}} \cdot \frac{\mathrm{~b}_{\mathrm{T}}}{\operatorname{Mos}_{a}}
$$

Operationally, a sample of telephone numbers is selected in the high density stratum in two stages. In the first stage, prefix areas with more than some minimum number of listed numbers is selected with probabilities proportional to Mos ${ }_{a}$. In
the second stage, a sample of telephone numbers (both listed and unlisted) is selected at the rate $\mathrm{b}_{\mathrm{T}} /$ Mos ${ }_{a}$ from among the
100 numbers in the prefix area.
An advanced letter can then be sent to all selected telephone numbers in the high density stratum which have a listed name and address. Both listed and unlisted sample numbers are called to obtain interviews. Three types of numbers will be encountered: numbers on the list frame, working residential numbers not on the list frame, and nonworking or nonresidential numbers. Interviews will be obtained from all working residential telephone household willing to cooperate with the survey.

There is no need to replace nonworking or nonresidential telephone numbers in the high density stratum. In addition, advance letters sent to a large proportion of the working residential numbers can increase response rates compared to a survey with no advance letters to any selected households, or compared to a dual frame survey design which sends letters to a smaller portion of the sample.

An experimental implementation of the two phase PPS design is described in the next section. Following this description, the error and cost properties of the two phase PPS and a two stage RDD sample design are compared. The paper concludes with a discussion of implications of the two phase PPS design for telephone surveys in other settings.

## An Implementation of the Two Phase PPS Design

A telephone survey of Michigan households was conducted in May and June, 1986. The survey collected information on a variety of political, social, and economic topics in a 20 to 25 minute interview administered to a randomly selected adult (18 years or older) in each sample telephone household. Samples of 250 completed interviews were needed in each of three geographic regions of the state: Detroit, the Detroit suburban area, and the remainder of the state. The data collection period was relatively brief, 17 days. A two phase PPS sample design was used to select the sample of telephone households.

There were 1,216 area and $C O$ code combinations in Michigan in January, 1986. A epsem sample of 961 prefix areas was selected from the three geographic strata. A file of 96,100 telephone numbers in the 961 prefix areas was sent to a commercial firm which maintains a file of telephone directory listings for the U. S. (i.e., the Metromail Corporation, Lincoln, Nebraska). A total of 18,751 phone numbers of the 96,100 were found in the MetroMail listed number file. A total of 439 prefix areas had no listed numbers. The remaining 522 had on average 35.9 listed numbers each. Table 1 shows the results of this first phase sample selection by geographic stratum.

Table 1 also presents the mean listed rates for prefix areas with one, ten, and twenty or more listed numbers. The mean listed rates increases in each geographic stratum as the minimum number of listed numbers increases, but because the number of listed numbers in prefix areas with fewer than 20 listed numbers is small, the increases in the mean listed rate is not large.

Up to three subsamples of size $\mathrm{b}_{\mathrm{T}}$ listed numbers were to be selected in successive surveys from the Mos ${ }_{a}$ in each prefix area. Hence the minimum size for a prefix area should be at least $3 \times \mathrm{b}_{\mathrm{T}}$. The average cluster size depends on the sampling rates within strata. Since approximately equal sample sizes were desired in each stratum, the rates will vary across strata, and for a fixed $m_{T}$ in each geographic stratum, the desired subsample size $\mathrm{b}_{\mathrm{T}}$ will also vary across strata. It was expected that $b_{T}$ should range from around 2 to 4 if 100 prefix areas are selected from each geographic stratum. Over

Table 1
Primary phase sample selection for two phase PPS sample, May - June, 1986

|  | Geographic region |  |  |  |
| :--- | ---: | ---: | ---: | ---: |
|  | Detroit | Suburbs | Outstate | State <br> Total |
|  |  |  |  |  |
| Population Data | 166 | 210 | 840 | 1216 |
| CO codes | 16600 | 21000 | 84000 | 121600 |
| Prefix areas |  |  |  |  |
|  |  |  |  |  |
| Sample Data | 274 | 258 | 429 | 961 |
| Selected prefix areas | 93 | 77 | 269 | 439 |
| No listed numbers | 181 | 181 | 160 | 522 |
| 1 or more listed | 161 | 170 | 137 | 468 |
| 10 or more listed | 119 | 158 | 136 | 413 |
| 20 or more listed |  |  |  |  |
|  | 16.4 | 26.1 | 17.2 | 22.6 |
| Mean number listed | 25.6 | 37.2 | 46.1 | 35.9 |
| 1 or more listed | 28.4 | 39.5 | 53.1 | 39.7 |
| 10 or more listed | 33.1 | 41.3 | 53.4 | 42.9 |
| 20 or more listed |  |  |  |  |

the course of three surveys of the same size from these same listings, the minimum size of a cluster should be at least 12 listed numbers. Given uncertainties about the working household rates within prefix areas with one, ten, or twenty listed numbers, and the relatively small proportion of listed numbers in prefix areas with fewer than 20 listed numbers, we decided to set the minimum size at 20 listed numbers per prefix area.

Thus, the low density stratum consisted of 548 prefix areas with fewer than 20 listed numbers each (see Table 2). These 548 numbers were called and 25 ( 4.6 percent) were working residential telephone numbers. The working rate expected if an equal probability of selection method (epsem) had been used is given in the last column under weighted results: 3.0 percent. This is much lower than the overall working number rate for Michigan typically obtained (approximately 28 percent), because prefix areas with fewer than 20 listed numbers have fewer residential telephone numbers.

In the second stage of selection in the low density stratum, the working rates within the 25 selected prefix areas ranged from 22.7 to 59.1 percent across the geographic regions. This compares with rates of approximately 60 percent typically observed at this stage in two stage RDD samples for Michigan.

Table 2

## Low density stratum sample selection for two phase PPS sample, May - June, 1986

|  | Geographic region |  |  |  |
| :--- | ---: | ---: | ---: | ---: |
|  | Detroit | Suburbs | Outstate | State <br> Total |
|  | 155 | 100 | 293 |  |
| Selected prefix areas <br> Working <br> Working rate | 18 | 4 | 3 |  |
| Secondary working <br> rate | 0.116 | 0.040 | 0.010 | 0.030 |

${ }^{1}$ Weighted to compensate for unequal sampling rates across geographic regions.

Overall, the expected working rate for telephone numbers within selected low density stratum prefix areas for an epsem design was 33.7 percent. Thus, in terms of primary and secondary stages of selection, low density stratum has very low working rates and poor efficiency compared to the two stage RDD design which we have typically used for sample selection in Michigan.

In the high density stratum (prefix areas with 20 or more listed numbers), a systematic PPS selection of 100 prefix areas was selected in each geographic region. Nine of the 119 prefix areas in Detroit had enough listed numbers that they were selected twice; two subsamples of telephone numbers were selected without replacement in each.

Within the selected prefix areas in the high density stratum, the proportion of telephone numbers that were working residential numbers is shown in Table 3. Overall telephone numbers, the rates range from 60.5 to 68.7 percent across geographic regions, with an expected 65.9 percent rate overall for an epsem design. Combining these working rates with those obtained in the low density stratum, the overall working rate within prefix areas across all sample prefix areas is 62.3 percent, which compares favorably with the usual 60 percent expected secondary working rate for two stage RDD samples in Michigan.

Table 3 also provides the working residential rates for listed and other numbers called within the high density stratum prefix areas. A high percentage of the listed numbers (88.5) were working residential numbers, while only 46.7 percent of the remaining sample numbers were residential. The improvements in the dialing efficiency apparent in the high density stratum can be attributed to the high working rates among listed telephone numbers.

## Table 3

High density stratum working residential rates (and standard errors) for two phase PPS sample, May June, 1986

|  | Geographic region |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Detroit | Suburbs | Outstate | State <br> Total |
|  |  |  |  |  |
| High density stratum | 60.5 | 63.0 | 68.7 | 65.9 |
|  | $(1.7)$ | $(2.6)$ | $(2.5)$ | $(1.6)$ |
| Listed numbers | 82.1 | 90.9 | 82.6 | 88.5 |
|  | $(2.6)$ | $(1.9)$ | $(1.6)$ | $(1.2)$ |
| Other numbers | 49.9 | 44.7 | 46.7 | 46.7 |
|  | $(2.3)$ | $(3.0)$ | $(3.9)$ | $(2.2)$ |
|  |  |  |  |  |
| Low and high density |  |  |  |  |
| strata combined | 50.7 | 59.1 | 68.3 | 62.3 |
|  | $(2.9)$ | $(4.0)$ | $(2.6)$ | $(1.9)$ |

${ }^{1}$ Weighted to compensate for unequal sampling rates across geographic regions.

Response rates varied greatly across the low density stratum, the high density stratum listed numbers, and the high density stratum other numbers (see Table 4). The high density stratum listed numbers had the highest response rates (72.4 percent for an epsem design), and the low density stratum numbers had the lowest ( 35.1 percent for an epsem design). The extremely low rates obtained in the low density stratum are likely the consequence of the replacement scheme for nonworking numbers in the second stage of the RDD sampling procedure. Many of the second stage telephone

## Table 4

Response rates (and standard errors) in experimental implementation by type of sample number, May - June, 1986

|  | Geographic region |  |  | State <br> Total ${ }^{1}$ |
| :---: | :---: | :---: | :---: | :---: |
|  | Detroit | Suburbs | Outstate |  |
| Total | $\begin{gathered} 55.4 \\ (2.2) \end{gathered}$ | $\begin{array}{r} 64.4 \\ (2.4) \end{array}$ | $\begin{gathered} 67.6 \\ (2.1) \end{gathered}$ | $\begin{aligned} & 64.9 \\ & (1.4) \end{aligned}$ |
| Low density stratum | $\begin{aligned} & 36.0 \\ & (5.0) \end{aligned}$ | $\begin{array}{r} 40.0 \\ (9.4) \end{array}$ | $\begin{array}{r} 30.8 \\ (11.5) \end{array}$ | $\begin{array}{r} 35.1 \\ (4.5) \end{array}$ |
| High density stratum | $\begin{aligned} & 60.3 \\ & (2.4) \end{aligned}$ | $\begin{gathered} 65.4 \\ (2.4) \end{gathered}$ | $\begin{gathered} 68.9 \\ (2.1) \end{gathered}$ | $\begin{aligned} & 66.8 \\ & (1.5) \end{aligned}$ |
| Listed numbers | $\begin{gathered} 66.6 \\ (4.0) \end{gathered}$ | $\begin{aligned} & 74.7 \\ & (3.0) \end{aligned}$ | $\begin{aligned} & 72.4 \\ & (2.8) \end{aligned}$ | $\begin{gathered} 72.4 \\ (2.0) \end{gathered}$ |
| Other numbers | $\begin{aligned} & 55.2 \\ & (3.5) \end{aligned}$ | $\begin{aligned} & 53.0 \\ & (3.7) \end{aligned}$ | $\begin{aligned} & 61.7 \\ & (3.7) \end{aligned}$ | $\begin{aligned} & 57.8 \\ & (2.3) \end{aligned}$ |

${ }^{1}$ Weighted to compensate for unequal sampling rates across geographic regions.
numbers did not have a final status assigned by the end of data collection because they were ringing but unanswered at each call. Lacking any other evidence about the status of these numbers, we have classified them as working residential numbers without interviews, which decreases the response rates in the low density stratum.

Over both the low density and high strata, the response rate is 64.9 percent. The effect of the low response rate in the low density stratum on the overall response rate is small since only a small portion of the total population of residential telephone households is in the low density stratum.

The higher response rates among listed numbers in the high density stratum are likely due to the advance letter, which all working residential numbers in this group received, and to the nature of the population. Previous experimental studies have demonstrated increases in response rates when listed telephone households received letters (Traugott, Groves, and Lepkowski, 1986; Groves and Lepkowski, 1986). In addition, the listed telephone household population appeared to have higher response rates than the unlisted households.

Table 5 presents the percentage of various types of telephone numbers that were listed in the high density stratum. Only 46.0 percent of all telephone numbers selected were listed numbers. But 61.7 percent of the residential sample numbers were listed. Further, among all completed interviews, 66.9 percent were listed numbers. Thus, to the extent that improvements in response rates for listed numbers can be achieved through the use of an advance letter, the two phase design will improve response rates for 61.7 percent of the sample households.

Groves and Lepkowski (1986) report a 61.6 percent response rate for two stage RDD sampling and a 72.6 percent response rate for list frame cases receiving an advance letter in a telephone survey in Michigan similar to the one reported here. Given the 72.4 percent response rate for list frame cases receiving a letter in this survey, a 61.8 percent response rate might be expected for a comparable two stage RDD survey. Thus the complete two phase survey (with a 64.9 percent response rate) has a higher rate than might be expected for a two stage RDD survey.

Table 5
Ratio of total calls to interviews (and standard errors) by type of sample number, May - June, 1986

|  | Type of sample number |  |  | State <br> Total ${ }^{1}$ |
| :---: | :---: | :---: | :---: | :---: |
|  | Nonres. | Nonint. | Int. |  |
| Total | 2.04 | 4.44 | 3.77 | 10.25 |
|  | (0.20) | (0.30) | (0.14) | (0.43) |
| Low density stratum | 9.55 | 15.41 | 4.28 | 29.24 |
|  | (3.09) | (3.36) | (0.37) | (5.99) |
| High density stratum | 1.79 | 4.07 | 3.75 | 9.61 |
|  | (0.18) | (0.29) | (0.14) | (0.41) |
| Listed numbers | 0.31 | 3.00 | 3.49 | 6.80 |
|  | (0.05) | (0.32) | (0.17) | (0.37) |
| Other numbers | 4.78 | 6.23 | 4.28 | 15.29 |
|  | (0.57) | (0.64) | (0.22) | (1.07) |

${ }^{1}$ Weighted to compensate for unequal sampling rates across geographic regions.
As a further investigation of the operational efficiency of the two phase design, the ratio of the number of calls of various types to the total number of completed interviews was computed (see Table 6). For an epsem design, a total of 10.25 calls would have been made for each completed interview. These calls would have consisted of 2.04 calls to nonresidential numbers, 4.44 to residential numbers which were noninterviews, and 3.77 calls to residential numbers which eventually resulted in an interview.

Table 6
Percentage (and standard error) of telephone numbers, residential numbers, and completed interviews that are listed numbers in the high density stratum, May - June, 1986

|  | Geographic region |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | State |  |  |  |
|  | Detroit | Suburbs | Outstate | Total $^{1}$ |
|  |  |  |  |  |
| Telephone numbers | 33.0 | 39.6 | 52.5 | 46.0 |
|  | $(1.8)$ | $(2.3)$ | $(2.7)$ | $(1.6)$ |
| Residential numbers | 44.8 | 57.1 | 67.7 | 61.7 |
|  | $(2.4)$ | $(2.7)$ | $(2.8)$ | $(1.8)$ |
| Completed interviews | 43.0 | 63.7 | 71.1 | 66.9 |
|  | $(2.9)$ | $(2.8)$ | $(3.1)$ | $(2.1)$ |

${ }^{1}$ Weighted to compensate for unequal sampling rates across geographic regions.

Among the different components of the sample design, the listed numbers in the high density stratum had the best efficiency with 6.80 calls per completed interview. The worst efficiency occurred, not unexpectedly, among the low density stratum numbers with 29.24 calls per completed interview. While for the listed numbers only 0.31 calls to nonresidential numbers were made per completed interview, the ratio is 9.55 among the low density stratum numbers. In addition, the the low density stratum numbers also required a large number of
calls to households that eventually became noninterviews. We have no explanation for this finding, except to note that few completed interviews were obtained for this stratum in general, while a large number of calls were made to numbers for which the result was a ring with no answer.

This survey is only an experimental implementation of the two phase methodology, and not an experimental comparison of the two phase method to a survey based on the two stage RDD sample design. As such, direct comparison of the relative efficiencies and errors of the two designs cannot be made. As noted previously, the two phase design appears to have higher working rates within prefix areas in the high density stratum, as well as over all prefix areas than typically observed in two stage RDD sampling in similar Michigan telephone surveys. Response rates also appear to be higher for the two phase design. Comparison of the ratio of calls to completed interviews is more difficult because of differences between surveys such as length of interview, interviewing periods, and survey topics between surveys.

An indirect comparison between two phase and two stage RDD sampling can be made once suitable cost and error models for the two designs are constructed. In the next section cost and error models for each survey design are described. They are used to compare the relative efficiencies of the two sample designs.

## Cost Models for the Two Stage and Two Phase Sample Designs

The cost components of the two phase telephone survey design can be estimated from the implementation of the design presented previously. Those for the RDD survey design can be derived from similar types of results from telephone surveys conducted using the two stage RDD design. Values for the error terms are more difficult to determine because bias is typically unknown for most survey designs. However, given the cost characteristics of the two survey designs, the relative error characteristics can be simulated under various assumptions about the relative performance of the data collection strategies, and then the designs compared for a fixed overall budget.

In this comparison, the total survey budget will be fixed, and the number of completed interviews achieved under the two designs will be determined. The relative error properties of the two designs will then be compared.

Let $m_{R}$ denote the desired number of working primary numbers in the RDD design. Suppose that the number of completed interviews per prefix area is fixed at $b=4$ for the low and high density strata of the two phase design and for the two stage RDD design.

The costs for implementing the two phase design can be expressed in terms of two basic components: (1) fixed costs which do not vary across samples of different size, (2) variable costs which are a function of the sample size. The variable costs in turn can be disaggregated into distinct components: (a) selection of the first phase sample of prefix areas, (b) purchase of listed numbers in first phase prefix areas, (c) mailing advance letters, and (d) data collection.

Two phase fixed costs. The fixed costs are limited to those that concern selection of the sample. The components include a programmer to select the sample of prefix areas and to process the listed numbers in preparation for advance mailing and data collection, costs of the sampling frame, administrative processing of the purchase listed numbers in selected prefix areas, and the charges incurred for passing the numbers in prefix areas by the complete file of listed numbers (a processing charge from the list frame processing firm). We assume that these fixed costs are amortized over 3 studies.

For a. study similar to the two phase sample design implementation described previously, the amortized fixed costs are estimated to be $\$ 560$.

Selection of first phase sample prefix areas. These costs are estimated as

$$
\left(m_{T} / t\right) \cdot C_{g} \cdot(1 / 3)
$$

where $t$ denotes the proportion of prefix areas with 20 or more listed numbers and $\mathrm{C}_{\mathrm{g}}$ denotes the cost to generate a telephone number at random. These costs are amortized over 3 uses of the sample. From the experimental implementation, $t=0.384$, and from other considerations $\mathrm{C}_{\mathrm{g}}=\$ 0.035$.

Purchase of listed numbers in first phase prefix areas. These costs are estimated as

$$
100 \cdot\left(\mathrm{~m}_{\mathrm{T}} / \mathrm{t}\right) \cdot\left(\pi \cdot \pi_{l}\right) \cdot \mathrm{C}_{l} \cdot(1 / 3)
$$

where $\pi$ denotes the proportion of all telephone numbers that are residential, $\pi_{l}$ denotes the proportion of all residential numbers that are listed, and $C_{l}$ denotes the cost to purchase listed number information for each selected telephone number. These costs are amortized over 3 uses of the sample. From the experimental implementation of the two phase sample, $\pi=0.283, \pi_{l}=0.622$, and $C_{l}=\$ 0.055$.

Mailing advance letters to selected listed numbers in the high density stratum. These costs are estimated as

$$
\mathrm{m}_{\mathrm{T}} \cdot\left(\mathrm{~b} / \mathrm{R}_{\mathrm{T}}\right) \cdot \mathrm{P}_{i} \cdot \mathrm{C}_{\mathrm{m}}
$$

where $\mathrm{R}_{\mathrm{T}}$ is the response rate in the high density stratum prefix areas, $\mathrm{P}_{l}$ is the proportion of residential numbers in prefix areas in the high density stratum that are listed, and $\mathrm{C}_{\mathrm{m}}$ is the cost to mail an advance letter. For this comparison, $R_{T}=0.700$ was fixed arbitrarily, and response rates in the low density stratum and the RDD design determined proportionately from this rate. From the experimental implementation of the two phase sample, $P_{l}=0.617$ and $C_{m}=\$ 0.52$.

Data collection. The data collection costs for the two phase sample are considered separately for the low and high density strata. The high density stratum data collection costs are estimated as

$$
\mathrm{m}_{\mathrm{T}} \cdot \mathrm{~b} \cdot \mathrm{C}_{\mathrm{T}}
$$

where $\mathrm{C}_{\mathrm{T}}$ is the cost per completed interview in the high density stratum prefix areas. From the experimental implementation of the two phase design with a 22 minute interview, $\mathrm{C}_{\mathrm{T}}=\$ 22.86$.

The low density stratum data collection costs are estimated as

$$
\begin{gathered}
((1-t) / t) \cdot m_{\mathrm{T}} \cdot\left(\left(\mathrm{C}_{\mathrm{s}} / 3\right)\right. \\
\left.\left.+\pi_{\mathrm{TR}} \cdot \mathrm{~b} \cdot\left\{\left(\mathrm{C}_{\mathrm{g}} / \mathrm{R}_{\mathrm{TR}} \cdot \mathrm{P}_{\mathrm{TR}}\right)+\mathrm{C}_{\mathrm{TR}}\right)\right\}\right)
\end{gathered}
$$

where $C_{s}$ is the cost to determine the residential status of a primary number, $\pi_{T R}$ is the proportion of all telephone numbers in the low density stratum that are residential, $\mathrm{R}_{\mathrm{TR}}$
is the low density stratum response rate, $\mathrm{P}_{\mathrm{TR}}$ is the proportion of telephone numbers in selected low density stratum prefix areas that are residential, and $\mathrm{C}_{\mathrm{TR}}$ is the low density stratum cost per completed interview. From the experimental implementation of the two phase design, $C_{s}=\$ 2.08$, $\pi_{\mathrm{TR}}=0.030, \mathrm{P}_{\mathrm{TR}}=0.337$, and $\mathrm{C}_{\mathrm{TR}}=\$ 48.72$. In addition, $R_{T R}=0.368$ based on results from the experimental implementation and on the fixed value $R_{T}=0.700$.

Two stage RDD cost model. The cost model for a two stage RDD design also has fixed and variable costs. The fixed costs are somewhat lower than those for the two phase design, $\$ 186$ (amortized over three uses of the primary stage sampling units). The variable costs are estimated as

$$
\begin{gathered}
\left(m_{R} / \pi\right) \cdot\left(C_{g}+C_{s}\right) \\
+m_{R} \cdot b \cdot\left\{\left(C_{g} / R_{R} \cdot P_{R}\right)+C_{R}\right\}
\end{gathered}
$$

where $R_{R}$ is the two stage $R D D$ response rate, $P_{R}$ is the proportion of telephone numbers in working two stage prefix areas that are residential, and $C_{R}$ is the cost per completed interview in the two stage RDD design. In this case, $R_{R}=0.721, P_{R}=0.593$, and $C_{R}=\$ 23.15$.

## Cost and Error Comparison

Given the two phase and two stage RDD design cost models, consider a survey with a fixed total budget of $\$ 25,000$. The two phase design, including the use of advance letters to the high density stratum listed numbers, would be able to obtain 976 completed interviews. The two stage RDD design would obtain 1,036 completed interviews, 6.2 percent more than the two phase design. Some of this relative improvement can be attributed to the increased cost in the two phase design of mailing advance letters.

To the extent that the two designs have similar variance properties, and only variance is considered, the choice between the two designs clearly favors the two stage RDD design. However, if the error considerations are extended to include bias, the two phase design may be preferred because higher response rates are possible with the use of advance letters to a large proportion of the sample. To the extent that higher response rates lead to lower bias due to nonresponse, the two phase design may be preferred to the two stage RDD design.

Consider simple error models for the two designs comprising only sampling variance and bias. For the two phase design,

$$
\mathrm{MSE}_{\mathrm{T}}=\sigma^{2} \delta_{\mathrm{T}} / 976+\mathrm{B}_{\mathrm{T}}^{2}
$$

and for the two stage RDD design,

$$
\operatorname{MSE}_{R}=\sigma^{2} \delta_{R} / 1036+\mathrm{B}_{\mathrm{R}}^{2}
$$

where $\mathrm{MSE}_{\mathrm{T}}$ and $\mathrm{MSE}_{\mathrm{R}}$ denote the mean squared error, $\delta_{\mathrm{T}}$ and $\delta_{R}$ denote the design effects, and $B_{T}$ and $B_{R}$ denote the bias of the two phase and the two stage RDD surveys, respectively. $\sigma^{2}$ denotes the variance of elements in the population.

Suppose that $\delta_{T} \cong \delta_{R}=1.2$, and for an estimated proportion $p=0.5, \sigma^{2}=(0.5)(1-0.5)=0.25$. Then for almost any level of bias, the two phase design has a lower
mean squared error than the two stage design. For example, Table 7 presents the ratio $\mathrm{MSE}_{\mathrm{R}} / \mathrm{MSE}_{\mathrm{T}}$ for several levels of relative bias, the ratio of the bias to the estimate p. For a 1 percent relative bias under the two phase design and a 2 percent relative bias under the two stage RDD design, the

Table 7
Ratio of two stage RDD to two phase design mean squared error for selected levels of relative bias under each design

| Relative bias |  |  |
| :---: | :---: | :---: |
| $\mathrm{B}_{\mathrm{T}} / \mathrm{p}$ | $\mathrm{B}_{\mathrm{R}} / \mathrm{p}$ | $\mathrm{MSE}_{\mathrm{R}} / \mathrm{MSE}_{\mathrm{T}}$ |
| 0.01 | 0.02 | 1.17 |
| 0.05 | 0.06 | 1.28 |
| 0.05 | 0.07 | 1.62 |
| 0.10 | 0.11 | 1.18 |
| 0.10 | 0.12 | 1.39 |
| 0.10 | 0.13 | 1.61 |

mean squared error for the two stage RDD design is 17 percent larger than for the two phase design. The mean squared errors for the two phase design are smaller than that for the two stage RDD design for all levels of relative bias shown in Table 7. Thus, even though the two stage RDD design achieves a larger sample size than the two phase design for the same fixed survey budget, the improvements in precision are likely to be overwhelmed by even small differences in bias that are attributed to improved response rates for the two phase design.

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