ESTIMATING INTERVIEWER VARIANCE FOR DICHOTOMOUS ITEMS USING A LATENT VARIABLE MODEL

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1. INTRODUCTION

There are at least two important reasons for attempting to measure the correlated component of response variance due to interviewers, or interviewer variance, in sample surveys. First, a large correlation may indicate a problem with interviewer training on an item or with the design of the question itself. If such items can be identified, they might be improved.

Second, the presence of the correlation inflates the variance of the sample mean $\hat{\mu}$, which is commonly used as the estimator of the population mean of the item responses. Let d_{ij} be the response of the j^{th} unit of interviewer i's assignment, $i = 1,..., k$; $j = 1, ..., n_i$. Writing $N = \Sigma n_i$, $\overline{n-1} = \Sigma n_i(n_i - 1)/N$, we have

$$V(\hat{\mu}) = V(\Sigma\Sigma\, d_{ij} / N)$$
$$= [\, V(d_{ij}) + (\overline{n-1})\, Cov(d_{ij}, d_{ij'})]/ N$$
$$= V(d_{ij})\, [1 + (\overline{n-1})\, \rho]/ N,$$

where $\rho = Cov\,(d_{ij}, d_{ij'})/V(d_{ij})$. (1)

If $(\overline{n-1})$ is moderately large, even a very small ρ can dramatically increase the variance of $\hat{\mu}$. Further, the usual estimate of variance of the sample mean, $v(\hat{\mu}) = s^2/N$, underestimates $V(\hat{\mu})$, leading to dangerously optimistic impressions of the precision of $\hat{\mu}$. If good estimates of ρ were available, improved estimates of precision could be produced.

Traditionally, random and mixed analysis of variance models have been used to describe interviewer variability and ρ is then just a ratio of variance components. Recently, an interesting approach has been suggested by Anderson and Aitkin (1985) for describing interviewer variability for dichotomous items. Their model includes a parameter which is a measure of interviewer variability, but which is different from ρ as defined in (1). In this paper, we show the relationship between their parameter and ρ, and then show how an estimate of ρ can be obtained using their procedure.

2. THE MODEL

The usual approach to modeling interviewer errors is to think of the interviewer bias as a random effect in an analysis of variance model. Then the response d_{ij} would be expressed as

$$d_{ij} = \mu + \beta_i + \epsilon_{ij},$$

where $\beta_i \sim (0, \sigma_\beta^2)$, $\epsilon_{ij} \sim (0, \sigma_\epsilon^2)$, and β_i and ϵ_{ij} uncorrelated. (The $\sim (\mu, \sigma^2)$ means the random variable has a distribution with mean μ and variance σ^2.) If the respondents are chosen from populations having different means, then this complication is accomodated in the model by including fixed effects. This model leads to the desired covariance structure; i.e.,

$$Cov(d_{ij}, d_{i'j'}) = \sigma_\beta^2 + \sigma_\epsilon^2 \quad \text{if } i = i',\ j = j'$$
$$= \sigma_\beta^2 \quad \text{if } i = i',\ j \neq j'$$
$$= 0 \quad \text{otherwise.}$$

Then the intra-interviewer correlation defined in (1) is

$$\rho = \sigma_\beta^2 / (\sigma_\beta^2 + \sigma_\epsilon^2).$$

Estimates of the variance components σ_β^2 and σ_ϵ^2 and of ρ can be obtained using standard methods of variance component estimation for random or mixed models. Some of these methods yield estimators that are optimal in some sense under the assumption that the random effects are normally distributed. In addition, standard errors for these estimators are available (Searle 1971), under the same assumptions.

Most questionnaire items for which measures of interviewer variability are needed are categorical, however. This means that the assumptions of normality are not met so that the most commonly used estimators of the variance components may not be good and further, expressions for their variance are not known. The usual tests for significance of fixed effects are not appropriate either. For these reasons, Anderson and Aitkin (1985) proposed a method for the estimation of interviewer variability for dichotomous items.

Their method hypothesizes the presence of an unobservable continuous random variable which determines the outcome of the observed variable. Again letting d_{ij} denote the response of the j^{th} unit in interviewer i's assignment, Y_{ij} the corresponding unobservable variable, and t_{ij} the "threshold" value, they assume

$$p_{ij} = Pr[d_{ij} = 1 |\, i] = Pr[Y_{ij} > t_{ij} |\, i]. \quad (2)$$

t_{ij} is assumed to be a random variable as well, but since its mean and variance can be absorbed into those of Y, we may, with no loss of generality, write $p_{ij} = Pr\,[Y_{ij} > 0 \mid i]$. Y may be treated as having any distribution, but assuming a normal or logistic gives rise to a familiar probit or logit model for D. Any recognized effects on Y may be included as fixed or random effects in an ANOVA model for Y; i.e., we may write

$$Y = Xm + \Sigma_r Z_r B_r + e, \qquad (3)$$

where Y is the vector of y_{ij}'s, m, B_r, and e are the vectors of fixed, random, and error components, respectively, X and Z_r are the associated design matrices, $E(B_r) = E(e) = 0$, $V(e) = \sigma_e^2 I$, and $V(B_r) = \sigma_{br}^2 I$. Then as a simple illustration, suppose there were only one random effect (r = 1) and that due to interviewers, and we write $V(B_1) = \sigma_b^2 I$. Then the intra-interviewer correlation in Y is

$$\rho_Y = \frac{Cov(y_{ij}, y_{ij'})}{V(y_{ij})} = \sigma_b^2 / (\sigma_b^2 + \sigma_e^2).$$

Anderson and Aitkin explain how the maximum likelihood estimators of the variance components σ_b^2 and σ_e^2 can be computed. They can then be used to obtain an estimate of ρ_Y.

The parameter ρ_Y can be helpful in identifying problem items on a questionnaire, since it is a measure of interviewer variability. However, it cannot be used directly for adjusting the variance of D, as suggested by (1), since $\rho_Y \neq \rho$, the intra-interviewer correlation of D. However, a relationship exists between these two parameters which can be used to obtain an estimate of ρ.

Suppose we assume a normal distribution for all the random components of Y in (3) and thus for Y itself. Then the intra-interviewer correlation as defined in (1) is

$$\rho = \frac{Cov(d_{ij}, d_{ij'})}{V(d_{ij})} \qquad (4)$$

To illustrate, we assume only one random effect $B_1' = (b_1, ..., b_k)$ and assume that all sample units have the same mean; i.e., $E(Y_{ij}) = m$. Then from (2) and (3), we have

$$Cov(d_{ij}, d_{ij'}) = E[\, Cov(d_{ij}, d_{ij'}) | i] + Cov\,[\, E(d_{ij} | i), E(d_{ij} | i)] = V[\, \Phi((-m-b_i)/\sigma_e)] \qquad (5)$$

and

$$V(d_{ij}) = E\,[\Phi((-m-b_i)/\sigma_e)] - E^2[\, \Phi((-m-b_i)/\sigma_e)] \qquad (6)$$

where Φ is the standard normal distribution function. (5) and (6) can be evaluated numerically for specific values of m, σ_b, and σ_e, yielding the relationship between ρ and ρ_Y through (4).

Alternatively, using the delta method we obtain the approximations

$$V\,[\Phi((-m - b_i)/\sigma_e)] = (\sigma_b/\sigma_e)^2\, \phi^2(m/\sigma_e)[1 - (1/4)(m/\sigma_e)^2(\sigma_b/\sigma_e)^2] \qquad (7)$$

and

$$E[\Phi((-m-b_i)/\sigma_e)] = \Phi(-m/\sigma_e) + (1/2)(\sigma_b/\sigma_e)^2(m/\sigma_e)\, \phi(m/\sigma_e). \qquad (8)$$

(7) and (8) were used to obtain Figure 1. It shows the ratio ρ/ρ_Y as a function of m/σ_e for several values of ρ_Y. We can see that for ρ_Y between .01 and .10, which includes the typical values for intra-interviewer correlation, the corresponding values for ρ will be at most 70% as large as that of ρ_Y.

Anderson and Aitkin make the observation that the estimate of ρ_Y using their method is larger than the ANOVA estimate of ρ. Our discussion and Figure 1 should make it clear that this relationship must hold for the parameters themselves.

3. CONCLUSION

In Section 1, we identified two uses for an estimate of interviewer variance. The first use was that of identifying problem items. We note that ρ_Y is just as useful as ρ for that purpose, since one will be large only when the other is. The second use was to provide a way to obtain more realistic assessments of the precision of the sample mean. For this purpose, ρ_Y alone is not adequate, since it is not a measure of the proportion of variation in d_{ij} attributable to the interviewer. However, using (7) and (8) and estimates of m/σ_e and σ_b^2/σ_e^2, we can obtain an estimate of ρ from Anderson and Aitkin's model.

Since their approach is maximum likelihood, it also offers methods for evaluating the (aymptotic) standard errors of their estimators. From them, approximate standard errors for $\hat{\rho}$ can be obtained. And finally, they discuss methods for determining the significance of the effects in their model. This method can be used directly for help in modeling interviewer errors; i.e., for determining which fixed effects must be included in the model.

REFERENCES

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Figure 1. RELATIONSHIP BETWEEN ρ AND ρ_y