CONDITIONAL INFERENCE FOR SMALL AREA ESTIMATION

Michael A. Hidiroglou
Statistics Canada

Carl Erik Särndal
Université de Montréal

1. INTRODUCTION

In recent years, small area estimation has become an important concern for survey organizations. For example, Statistics Canada increased its emphasis on small area estimates in response to user requirements. The production of such small area statistics is being put into place by using existing administrative files and/or survey files. The production may be as simple a matter as geocoding the files, provided that their coverage is satisfactory and that their variables correspond conceptually to the required statistics, and tabulating them at the small area level. It may be as complex as using several files together, working out relationships between variables available in survey files, and applying these relationships to the larger administrative files, which are likely to contain fewer variables. This kind of modelling was implicit in the early development of the so-called synthetic estimation techniques, where it is assumed that the models developed for the larger areas hold for the small areas. If this assumption fails, synthetic estimation will be biased; to use synthetic estimation without validation or evaluation is a risky gamble.

Purely synthetic estimation methods have been proposed by Gonzalez and Hoza (1975), and Levy (1978). Several authors have noted that the bias potentially produced by purely synthetic methods can be reduced through procedures that essentially combine a synthetic component with another component. Such estimators have been suggested by Schaible (1979), Fay and Herriot (1979), Drew, Singh and Choudhry (1982), Särndal (1984), Hidiroglou and Särndal (1985), Fuller and Harter (1985), and Srinath and Hidiroglou (1985). The approach used for arriving at the weighting of the two components differs amongst the aforementioned authors.

The number of data points realized in a given domain is not controlled at the selection stage. The analyst must make the best of the random number of observations that happened to fall in the domain. Inference conditionally on a suitable statistic (such as the realized domain sample count) strongly suggests itself. The basic motivation for the research reported in this paper can be briefly stated as follows: it is required to develop an approach to conditional inference for small areas that is situated in the randomization theory context.

This point of view led us (in Section 3) to construct certain new estimators for (small) domains. In the following sections, we analyze the design-based conditional properties of these and other possible estimators. In the randomization theory mode of inference, there is no systematically developed conditional approach; some steps in this direction are taken in this paper. By contrast, in the model-based approach to survey sampling, conditional arguments in sampling have been used by Holt and Smith (1979), Royall and Cumberland (1985) and others. Their work pointed to important questions in conditional inference from surveys and motivated interest in the conditional approach. However, these papers are not in particular concerned with the domain estimation problem.

2. STANDARD ESTIMATORS FOR AN ARBITRARY SAMPLING DESIGN

Suppose that the population $U = \{1, ..., k, ..., N\}$ is divided into D non-overlapping domains $U_1, ..., U_d, ..., U_D$. An estimate is required for the total (or the mean) of the variable of interest for each domain. In practice, the number of domains, D, is often quite large: several hundred or more is not atypical. Let N_d be the known size of U_d. We further assume that the population is divided according to a second classification criterion into G non-overlapping groups $U_{.1}, ..., U_{.g}, ..., U_{.G}$. The number of groups, G, is usually modest, say, around 10; in any case, G is assumed small compared to D.

The cross-classification of domains and groups gives rise to DG population cells U_{dg}; d=1, ..., D; g=1, ..., G. Let N_{dg} be the known size of U_{dg}.

Then the population size N can be expressed as:

$$N = \sum_{d=1}^{D} N_d = \sum_{d=1}^{D} \sum_{g=1}^{G} N_{dg} \tag{2.1}$$

Let s denote a sample of size n drawn from U under a given probability sampling design, p(s), such that $P(k \varepsilon s) = \pi_k > 0$ for all k and $P(k \text{ and } \ell \varepsilon s) = \pi_{k\ell} > 0$ for all $k \neq \ell$. Any given sample s will distribute itself in a random manner across the domains U_d and the population cells U_{dg}. Denote as s_d and s_{dg} the parts of s that happen to fall, respectively, in U_d and U_{dg}. Let n_d and n_{dg}, which are random variables, be the respective sizes of s_d and s_{dg}. We then have that (2.1) holds for lower case n's as well, and that $0 \leq n_{dg} \leq \max(n, N_{dg})$.

The variable of interest, y, takes the value y_k for the k:th unit. For d=1, ..., D, we seek to estimate the domain total $t_d = \sum_{k \varepsilon U_d} y_k = \sum_{U_d} y_k$.

(If A is any set of units, let us write $\sum_A$ for $\sum_{k \varepsilon A}$.)

The expansion estimator (EXP), or Horvitz-Thompson estimator, is given by:

$$\hat{t}_{dEXP} = \sum_{s_d} y_k/\pi_k \tag{2.2}$$

The post-stratification estimator based on G-group counts (POSG/C) is defined as:

$$\hat{t}_{dPOSG/C} = \sum_{g=1}^{G} N_{dg} \tilde{y}_{s_{dg}} \tag{2.3}$$

where N_{dg} is known and

$$\tilde{y}_{s_{dg}} = (\Sigma_{s_{dg}} y_k/\pi_k) \ / \ (\Sigma_{s_{dg}} 1/\pi_k)$$

is the sample-weighted mean of the n_{dg} y-values in s_{dg}. If $n_{dg} = 0$, we define $\tilde{y}_{s_{dg}}$ to be zero.

A second type of post-stratified estimator is based on G group ratios (POSG/R):

$$\hat{t}_{dPOSG/R} = \sum_{g=1}^{G} X_{dg} \, (\tilde{y}_{s_{dg}}/\tilde{x}_{s_{dg}}) \tag{2.4}$$

where $\tilde{x}_{s_{dg}}$ is defined analogously to $\tilde{y}_{s_{dg}}$, and X_{dg} is the known population total of x for the dg:th cell. If $n_{dg} = 0$, define $\tilde{y}_{s_{dg}}/\tilde{x}_{s_{dg}}$ as zero (arbitrarily).

The EXP estimator is rather inefficient (see below); it serves here mainly as a benchmark against which the behaviour of other estimators is compared.

The synthetic estimation technique is also well known. Here too, we consider a "count version" and a "ratio version": For the count version, the implicit model is that the y-mean of each group is the same across all domains d. For the ratio version, the implied model is that the ratios y_k/x_k are constant within the given group, irrespective of the domain. If these assumptions of homogeneity across domains fail, the SYN estimator can be seriously biased.

The synthetic-count estimator (SYNG/C) is defined by:

$$\hat{t}_{dSYNG/C} = \sum_{g=1}^{G} N_{dg} \, \tilde{y}_{s_{.g}} \tag{2.5}$$

where $$\tilde{y}_{s_{.g}} = (\Sigma_{s_{.g}} y_k/\pi_k) \ / \ (\Sigma_{s_{.g}} 1/\pi_k) \tag{2.6}$$

is the sample-weighted mean of y in the set $s_{.g} = \bigcup_{d=1}^{D} s_{dg}$, which is the part of the sample s that belongs to the group $U_{.g}$.

The synthetic-ratio estimator (SYNG/R) is defined by:

$$\hat{t}_{dSYNG/R} = \sum_{g=1}^{G} X_{dg} \, \tilde{R}_g \tag{2.7}$$

with $$\tilde{R}_g = (\Sigma_{s_{.g}} y_k/\pi_k) \ / \ (\Sigma_{s_{.g}} x_k/\pi_k). \tag{2.8}$$

The variance of the SYN estimators is ordinarily very small. Consequently, if in a given domain the bias also happens to be small, it is almost impossible, for any other estimation technique, to produce a better result (smaller MSE). But the possibility of a substantial bias (and large MSE) is a considerable handicap in the SYN method, which can therefore not be seriously recommended.

3. GENERALIZED REGRESSION TECHNIQUES FOR AN ARBITRARY SAMPLING DESIGN

The construction of the generalized regression estimator involves the fit of a linear regression model of y, the variable of interest, on $\underset{\sim}{x}$, a p-vector of auxiliary variables: The model, denoted ξ, postulates that $y_1, \ldots, y_N$ are independent and

$$E_\xi(y_k) = \underset{\sim}{x}_k' \underset{\sim}{\beta}; \quad V_\xi(y_k) = v_k.$$

If all N points $(y_k, \underset{\sim}{x}_k)$ were observed, the generalized least squares fit of this model would lead to estimating $\underset{\sim}{\beta}$ by

$$\underset{\sim}{B} = \underset{\sim}{Q}^{-1} \underset{\sim}{q}$$

where $$\underset{\sim}{Q} = \Sigma_U \frac{\underset{\sim}{x}_k \underset{\sim}{x}_k'}{v_k} \quad \text{and} \quad \underset{\sim}{q} = \Sigma_U \frac{\underset{\sim}{x}_k y_k}{v_k}.$$

However, in practice, y_k is observed for $k \varepsilon s$ only, and according to a sampling design with inclusion probabilities π_k. Therefore $\underset{\sim}{B}$ is in turn estimated by

$$\hat{\underset{\sim}{B}} = \left(\Sigma_s \frac{\underset{\sim}{x}_k \underset{\sim}{x}_k'}{v_k \pi_k}\right)^{-1} \Sigma_s \frac{\underset{\sim}{x}_k y_k}{v_k \pi_k}.$$

We assume that $\hat{\underset{\sim}{B}}$ is design consistent for $\underset{\sim}{B}$. For the k:th unit, let $\hat{y}_k = \underset{\sim}{x}_k' \hat{\underset{\sim}{B}}$ be the predicted value and $e_k = y_k - \hat{y}_k$ the residual. Särndal (1984) proposed the following estimator of t_d:

$$\hat{t}_{dRE} = \Sigma_{U_d} \hat{y}_k + \Sigma_{s_d} e_k/\pi_k. \tag{3.1}$$

The first term will be called the <u>synthetic term</u>:

$$\hat{t}_{dSY} = \Sigma_{U_d} \hat{y}_k = (\Sigma_{U_d} \underset{\sim}{x}_k)' \, \hat{\underset{\sim}{B}}, \tag{3.2}$$

whereas $\Sigma_{s_d} e_k/\pi_k$ will be called the <u>correction term</u>. The latter term corrects, approximately, for the bias that is generated if the synthetic term alone were used to estimate t_d. (In particular, (2.5) and (2.7) are special cases of (3.2).)

As is evident from (3.1) and (3.2), these estimators require that the domain sum $\Sigma_{U_d} \underset{\sim}{x}_k$ be known from auxiliary sources (but individual $\underset{\sim}{x}_k$ - values need not be known). Now, $\hat{t}_{dRE}$ is a consistent estimator of t_d. It is easily seen that the (unconditional) bias vanishes asymptotically:

$$E(\hat{t}_{dRE}) - t_d = - E[\hat{\underset{\sim}{B}}' (\Sigma_{s_d} \underset{\sim}{x}_k/\pi_k - \Sigma_{U_d} \underset{\sim}{x}_k)]$$

$$\doteq - \underset{\sim}{B}' \{E(\Sigma_{s_d} \underset{\sim}{x}_k/\pi_k) - \Sigma_{U_d} \underset{\sim}{x}_k\} = 0.$$

The domain estimator $\hat{t}_{dRE}$ takes a step in the right direction: Auxiliary information is judiciously used and strength is "borrowed" by fitting a model with a limited number of parameters. Hidiroglou and Särndal (1985) proposed that further improvement can be obtained by modifying $\hat{t}_{dRE}$ slightly:

$$\hat{t}_{dMRE} = \Sigma_{U_d} \hat{y}_k + N_d \frac{\Sigma_{s_d} e_k/\pi_k}{\hat{N}_d} \tag{3.3}$$

where $\hat{N}_d = \Sigma_{s_d} 1/\pi_k$ and $(\Sigma_{s_d} e_k/\pi_k)/\hat{N}_d$ is defined as zero if s_d is empty.

A comparison of $\hat{t}_{dRE}$ and $\hat{t}_{dMRE}$ suggests that the latter formula has advantages. Firstly, $\hat{t}_{dMRE}$ ordinarily has a smaller variance than $\hat{t}_{dRE}$ because of the ratio feature of the correction term. Secondly, as it will be seen in Section 5, $\hat{t}_{dMRE}$ has conditional properties which are more favourable than those of $\hat{t}_{dRE}$. Thirdly, for an SRS design, $\hat{t}_{dMRE}$ (unlike $\hat{t}_{dRE}$) is design consistent. That is, $\hat{t}_d = t_d$ when the event $s_d = U_d$ occurs: this property, however, does not hold for a general design neither for $\hat{t}_{dRE}$ nor for $\hat{t}_{dMRE}$.

In samples in which n_d is extremely small (say, five or less), the variance of the correction term (in $\hat{t}_{dRE}$ as well as in $\hat{t}_{dMRE}$) can be large. This volatility can cause "unacceptable estimates". In order to control the volatility of the correction term and reduce the risk of unacceptable estimates, we suggest to apply a "dampening factor" to the correction term of the $\hat{t}_{dMRE}$ formula, but only the below average values of $\hat{N}_d$. The result is the "dampened regression estimator":

$$\hat{t}_{dDRE} = \begin{cases} \Sigma_{U_d} \hat{y}_k + N_d \dfrac{\Sigma_{s_d} e_k/\pi_k}{\hat{N}_d} & \text{if } \hat{N}_d \geq N_d \\[2ex] \Sigma_{U_d} \hat{y}_k + \left(\dfrac{\hat{N}_d}{N_d}\right)^h N_d \dfrac{\Sigma_{s_d} e_k/\pi_k}{\hat{N}_d} & \text{if } \hat{N}_d < N_d. \end{cases} \tag{3.4}$$

The exponent h in the dampening factor $(\hat{N}_d/N_d)^h$ is a suitably chosen non-negative constant; we suggest h=2 as a general purpose value. (We have examined alternative values; see Section 7.) The correction term is defined as zero whenever s_d is empty.

4. CONDITIONING IN THE CONTEXT OF RANDOMIZATION THEORY

In the context of randomization theory, expected value and variance are interpreted with reference to repeated draws of samples s under the fixed probability sampling design p(s). Let ζ be the set of all possible different sets s. Conditioning means that attention is focused on samples s having some specific property (for example, samples s that contain exactly 10 observations from the d:th domain). The collection of samples s that display the specific property form a subset ζ_c of ζ, where the subscript c (here and in other symbols used below) indicates "conditional". Thus, conditional expectation (c-expectation), conditional bias (c-bias) and conditional variance (c-variance) derive their interpretation from repeated draws of samples s according to the design p(s), but such that s obeys $s \varepsilon \zeta_c$. The conditional probability of drawing s is

$$p_c(s) = p(s) \Big/ \sum_{s \varepsilon \zeta_c} p(s).$$

The c-expectation of an estimator $\hat{t}_d = \hat{t}_d(s)$ is:

$$E_c(\hat{t}_d) = E(\hat{t}_d | s \varepsilon \zeta_c) = \sum_{s \varepsilon \zeta_c} \hat{t}_d(s)\, p_c(s), \tag{4.1}$$

the c-bias is $B_c(\hat{t}_d) = E_c(\hat{t}_d) - t_d$, and the c-variance is:

$$V_c(\hat{t}_d) = E(\hat{t}_d^2 | s \varepsilon \zeta_c) - [E_c(\hat{t}_d)]^2. \tag{4.2}$$

By "<u>the conditional approach</u>" to the construction of a confidence interval for t_d, we mean: an approximately c-unbiased estimator $\hat{t}_d$ is considered (that is, $E_c(\hat{t}_d) \doteq t_d$). For the c-variance, $V_c(\hat{t}_d)$, we assume that a c-consistent estimator, $\hat{V}_c(\hat{t}_d)$, can be found. An approximately $100(1-\alpha)\%$ conditional confidence interval for $\hat{t}_d$ is then constructed as:

$$\hat{t}_d \pm z_{1-\alpha/2} \sqrt{\hat{V}_c(\hat{t}_d)} \tag{4.3}$$

where the constant $z_{1-\alpha/2}$ is exceeded with probability $\alpha/2$ by the unit normal variate. (Here we assume that the distribution of $\hat{t}_d$, given $s \varepsilon \zeta_c$, is approximately normal.) In repeated draws of samples, roughly $100(1-\alpha)\%$ of all samples s obeying $s \varepsilon \zeta_c$ will contain the true total t_d.

It follows that since the approach gives a c-coverage rate of roughly $1-\alpha$ for <u>any</u> specific set ζ_c, the <u>un</u>conditional coverage is automatically about $1-\alpha$. That is, unconditionally speaking, nothing has been lost by conditioning; the confidence statement is valid conditionally as well as unconditionally. By contrast, in <u>the unconditional approach</u> one would find the unconditional variance, $V_u(\hat{t}_d)$, then find a consistent estimator thereof, $\hat{V}_u(\hat{t}_d)$, and use it to construct the unconditional confidence interval:

$$\hat{t}_d \pm z_{1-\alpha/2} \sqrt{\hat{V}_u(\hat{t}_d)} \tag{4.4}$$

In this procedure, the coverage rate in repeated samples will be (roughly) the desired $1-\alpha$

unconditionally, that is, over all possible samples $s \epsilon \zeta$. However, c-conditionally on s in some $\zeta_c \subset \zeta$, the confidence level will ordinarily differ from the desired $1-\alpha$, even if $\hat{t}_d$ is c-unbiased.)

5. CONDITIONAL AND UNCONDITIONAL PROPERTIES UNDER SIMPLE RANDOM SAMPLING

In the domain estimation problem, on what set ζ_c should one condition? When the π_k's are arbitrary, it is reasonable to consider the conditioning set ζ_c composed of all s in which the estimated domain size $\hat{N}_d$ is constant (or near-constant, since $\hat{N}_d$ is generally not integer). To systematically carry out an analysis of the conditional behaviour of estimators may not be easy in the case of arbitrary inclusion probabilities. In one important case it is, however, rather simple, namely, when the design p(s) is simple random sampling without replacement (SRS). Consider SRS with n units drawn from N, so that $\pi_k = n/N$ for all k and $\pi_{k\ell} = n(n-1)/N(N-1)$ for all $k \neq \ell$. We shall let ζ_c be subset of all s for which $\hat{N}_d = Nn_d/n$ is a fixed positive constant. That is, the statistic conditioned on is the realized sample count, n_d, in the domain. For SRS, the "tilde means" of the type used in (2.4) become straight arithmetic means, which will be denoted by overbar: $\bar{y}_s = \Sigma_s y_k/n$; $\bar{x}_s = \Sigma_s x_k/n$, etc.

Let ζ_c be the subset of ζ containing the samples s for which n_d is fixed. Let us study the bias, variance and mean square error (conditionally as well as unconditionally) of some of the proposed estimators.

In the absence of auxiliary information, the straight expansion estimator (EXP) given by (3.2) would have been used. Although unconditionally unbiased, this estimator has, for $n_d \geq 1$, a conditional bias given by

$$B_c(\hat{t}_{dEXP}) = (\frac{N}{n} n_d - N_d)\, \bar{y}_{U_d}$$

This bias is near zero only if the realized domain sample count n_d is near its expected value $n\,N_d/N$. Attaching a conditional confidence interval to $\hat{t}_{EXP}$ is thus in general not meaningful; nevertheless, it is instructive to observe that a c-unbiased estimator of the c-variance for $\hat{t}_{dEXP}$ is given (if $n_d \geq 2$) by:

$$\hat{V}_c(\hat{t}_{dEXP}) = (\frac{Nn_d}{n})^2\, (\frac{1}{n_d} - \frac{1}{N_d})\, S^2_{s_d} \tag{5.1}$$

where $$S^2_{s_d} = \frac{1}{n_d-1} \Sigma_{s_d} (y_k - \bar{y}_{s_d})^2$$

and $$\bar{y}_{s_d} = \Sigma_{s_d} y_k/n_d,$$

and that an unbiased estimator of the unconditional variance for $\hat{t}_{dEXP}$ is given by:

$$\hat{V}_u(\hat{t}_{dEXP}) = N^2(\frac{1}{n} - \frac{1}{N})\, \frac{n_d-1}{n-1}\, \{S^2_{s_d} + \frac{n_d}{n_d-1}\,(1 - \frac{n_d}{n})\, \bar{y}^2_{s_d}\} \tag{5.2}$$

As is easily shown, the c-variance of which (5.1) is an unbiased estimate is an increasing function of n_d for $0 \leq n_d/N_d \leq 0.5$ (which is ordinarily the most important range of n_d - values). Thus, a weakness of the EXP estimator (in addition to the conditional bias) is that its conditional variance ordinarily increases (rather than decreases, as seems reasonable) when the data base in the domain expands. As for the unconditional variance estimator (5.2), one can show that it is c-biased.

Thus for two reasons (c-bias of the estimator, and increasing tendency of the conditional variance) $\hat{t}_{dEXP}$ is unsuitable for the conditional approach.

Let us turn to the post-stratified estimators POSG/C or POSG/R given, respectively, by expressions (2.3) and (2.4). They are (nearly) unbiased, both conditionally and unconditionally. Here, one can easily take the conditioning argument one step further and condition not simply on the domain count n_d, but on the individual cell counts n_{dg}, g=1, ..., G. Then the conditional estimator of variance for $\hat{t}_{dPOSG/C}$ (derived from the c-variance) is, if all $n_{dg} \geq 2$,

$$\hat{V}_c(\hat{t}_{dPOSG/C}) = \sum_{g=1}^{G} N^2_{dg}(\frac{1}{n_{dg}} - \frac{1}{N_{dg}})\, S^2_{s_{dg}}, \tag{5.3}$$

while an estimator of variance suggested by the unconditional variance (via an analogy involving the standard ratio estimator) is

$$\hat{V}_u(\hat{t}_{dPOSG/C}) = N^2(\frac{1}{n} - \frac{1}{N})\, \sum_{g=1}^{G} \frac{n_{dg}-1}{n-1}\, S^2_{s_{dg}}, \tag{5.4}$$

where $S^2_{s_{dg}}$ is defined analogously to $S^2_{s_d}$. Formula (5.3) will give approximately valid conditional confidence intervals, whereas (5.4) will not. This fact is illustrated empirically by Table 1, Section 7.

6. CONDITIONAL ANALYSIS FOR ESTIMATORS BASED ON REGRESSION

To facilitate a conditional analysis, under SRS, of $\hat{t}_{dDRE}$, $\hat{t}_{dMRE}$ and $\hat{t}_{dSY}$, let us express each of them in a suitable form. Given $s \epsilon \zeta_c$, we shall assume that there is a constant vector value, $\underset{\sim}{B}_c$, for which $\hat{\underset{\sim}{B}}$ is c-consistent, namely,

$$\underset{\sim}{B}_c = \{E[(\Sigma_s \frac{\underset{\sim}{x}_k \underset{\sim}{x}'_k}{v_k}) \mid s\epsilon\zeta_c]\}^{-1}\, \{E[(\Sigma_s \frac{\underset{\sim}{x}_k y_k}{v_k}) \mid s\epsilon\zeta_c]\}$$
$$= \underset{\sim}{Q}_c^{-1}\, \underset{\sim}{q}_c, \tag{6.1}$$

say. (c-consistency should in a more formal analysis be placed in a context where n_d increases, with proportional increases in n and N.) Define a "conditional residual" for the k:th unit as

$$E_{ck} = y_k - \underset{\sim}{x}_k' \underset{\sim}{B}_c \quad (6.2)$$

The conditional behaviour of $\hat{t}_{dSY}$ can be analysed through the following identity for the deviation of the estimator from the true value t_d:

$$\hat{t}_{dSY} - t_d = - \Sigma_{U_d} E_{ck} + \underset{\sim}{t}_{xd}'(\hat{\underset{\sim}{B}} - \underset{\sim}{B}_c) \quad (6.3)$$

where $\underset{\sim}{t}_{xd} = \Sigma_{U_d} \underset{\sim}{x}_k$.

Correspondingly, for the estimator $\hat{t}_{dMRE}$ we have

$$\hat{t}_{dMRE} - t_d = - \Sigma_{U_d} E_{ck} + N_d \frac{\Sigma_{s_d} E_{ck}}{n_d} - \Delta_d \quad (6.4)$$

with $\Delta_d = (N_d \frac{\Sigma_{s_d} \underset{\sim}{x}_k}{n_d} - \underset{\sim}{t}_{xd})' \ (\hat{\underset{\sim}{B}} - \underset{\sim}{B}_c)$,

while the identity for $\hat{t}_{dDRE}$ is:

$$\hat{t}_{dDRE} - t_d = - \Sigma_{U_d} E_{ck} + (\frac{\hat{N}_d}{N_d})^h N_d \frac{\Sigma_{s_d} E_{ck}}{n_d}$$

$$+ (1 - (\frac{\hat{N}_d}{N_d})^h) \underset{\sim}{t}_{xd}'(\hat{\underset{\sim}{B}} - \underset{\sim}{B}_c) - (\frac{\hat{N}_d}{N_d})^h \Delta_d \quad (6.5)$$

if $\hat{N}_d < N_d$, whereas for $\hat{N}_d \geq N_d$, $\hat{t}_{dDRE} - t_d$ is given by the right hand side of (6.4). In these expressions, $\Sigma_{U_d} E_{ck}$ is a constant. Looking at (6.4), the random variable Δ_d is of lower order of importance than $N_d \Sigma_{s_d} E_{ck}/n_d$, since $(N_d \Sigma_{s_d} \underset{\sim}{x}_k/n_d - \underset{\sim}{t}_{xd})$, a vector with c-expected value zero, has a c-variance of the same order as that of $N_d \Sigma_{s_d} E_{ck}/n_d$, and since $\hat{\underset{\sim}{B}} - \underset{\sim}{B}_c$ converges conditionally to the vector $\underset{\sim}{0}$.

Thus, for $s \varepsilon \zeta_c$,

$$\hat{t}_{dMRE} - t_d \doteq - \Sigma_{U_d} E_{ck} + N_d \Sigma_{s_d} E_{ck}/n_d, \quad (6.6)$$

where $\doteq$ expresses "of the same order in probability". Similarly, in (6.5), Δ_d is of lower order than the terms that precede, so that:

$$\hat{t}_{dDRE} - t_d \doteq - \Sigma_{U_d} E_{ck} + (\frac{\hat{N}_d}{N_d})^h N_d \frac{\Sigma_{s_d} E_{ck}}{n_d}$$

$$+ (1 - (\frac{\hat{N}_d}{N_d})^h) \underset{\sim}{t}'_{xd} (\hat{\underset{\sim}{B}} - \underset{\sim}{B}_c) \quad (6.7)$$

when $\hat{N}_d < N_d$, and for $\hat{N}_d \geq N_d$, $\hat{t}_{dDRE} - t_d$ is given by (6.6).

For the difference $(\hat{\underset{\sim}{B}} - \underset{\sim}{B}_c)$ appearing in (6.3), (6.5) and (6.7), we have, for $s \varepsilon \zeta_c$,

$$\hat{\underset{\sim}{B}} - \underset{\sim}{B}_c = (\Sigma_s \frac{\underset{\sim}{x}_k \underset{\sim}{x}_k'}{v_k})^{-1} \Sigma_s \frac{\underset{\sim}{x}_k E_{ck}}{v_k} \doteq \underset{\sim}{Q}_c^{-1} \Sigma_s \frac{\underset{\sim}{x}_k E_{ck}}{v_k} \quad (6.8)$$

Here we have used that $\Sigma_s \frac{\underset{\sim}{x}_k \underset{\sim}{x}_k'}{v_k}$ is c-consistent for $\underset{\sim}{Q}_c$ (rather than for $\underset{\sim}{Q} = \Sigma_U \frac{\underset{\sim}{x}_k \underset{\sim}{x}_k'}{v_k}$). Now, it is easily derived that:

$$E_c(\hat{\underset{\sim}{B}}) - \underset{\sim}{B}_c \doteq \underset{\sim}{Q}_c^{-1} (\underset{\sim}{q}_c - \underset{\sim}{Q}_c \underset{\sim}{B}_c) = \underset{\sim}{0}. \quad (6.9)$$

The c-bias for the three estimators follows from (6.3), (6.6), (6.7) and (6.9):

$$B_c(\hat{t}_{dSY}) \doteq - \Sigma_{U_d} E_{ck} \quad (6.10)$$

$$B_c(\hat{t}_{dMRE}) \doteq 0 \quad (6.11)$$

$$B_c(\hat{t}_{dDRE}) \doteq \begin{cases} 0 & \text{if } \hat{N}_d \geq N_d \\ - (1 - (\frac{\hat{N}_d}{N_d})^h) \ \Sigma_{U_d} E_{ck} & \text{if } \hat{N}_d < N_d \end{cases} \quad (6.12)$$

Here, (6.11) shows that $\hat{t}_{dMRE}$ is approximately c-unbiased whatever the value of n_d. The expressions (6.10) and (6.12) contain the residual sum $\Sigma_{U_d} E_{ck}$. Although in principle dependent on n_d, this sum is in many practical settings roughly constant as n_d varies. In fact, as shown below,

$$\Sigma_{U_d} E_{ck} \approx \Sigma_{U_d} E_k \quad (6.13)$$

where $E_k = y_k - \underset{\sim}{x}_k' \underset{\sim}{B}$ is the unconditional residual. In view of (6.13), we conclude from (6.10) that the c-bias of $\hat{t}_{dSY}$, seen as a function of n_d, is essentially constant. This is confirmed by our empirical evidence (see Graph 1). Moreover, from (6.12) we see that $\hat{t}_{dDRE}$ is approximately c-unbiased when $\hat{N}_d \geq N_d$. Otherwise, $\hat{t}_{dDRE}$ has a c-bias which becomes increasingly large as $\hat{N}_d$ decreases away from N_d, and as $\hat{N}_d$ approaches zero, the c-bias of $\hat{t}_{dDRE}$ obliques towards the constant c-bias level of $\hat{t}_{dSY}$. This behavior, too, is confirmed by our empirical work (see Graph 1).

Turning now to c-variances and their estimation, we get from (6.6), for $n_d \geq 1$,

$$V_c(\hat{t}_{dMRE}) \doteq N_d^2\left(\frac{1}{n_d} - \frac{1}{N_d}\right) \frac{\Sigma_{U_d}(E_{ck} - \bar{E}_{cU_d})^2}{N_d - 1} \quad (6.14)$$

with $\bar{E}_{cU_d} = \Sigma_{U_d} E_{ck}/N_d$. The expresssion has this form since, given n_d, the sample s_d realized in the domain behaves as a SRS selection of n_d from N_d.

Consequently, a c-consistent variance estimator is, for $n_d \geq 2$,:

$$\hat{V}_c(\hat{t}_{dMRE}) = N_d^2\left(\frac{1}{n_d} - \frac{1}{N_d}\right) \frac{\Sigma_{s_d}(e_k - \bar{e}_{s_d})^2}{n_d - 1} \quad (6.15)$$

where $\bar{e}_{s_d}$ is the mean of the residuals e_k for $k \epsilon s_d$. In deriving this expression we have replaced the theoretical residual $E_{ck} = y_k - \underset{\sim}{x}'_k \underset{\sim}{B}_c$ in (6.15) by the sample-based residual $e_k = y_k - \underset{\sim}{x}'_k \hat{\underset{\sim}{B}}$, which for any k is c-consistent for E_{ck}.

Remark. An unconditional variance estimator is given by:

$$\hat{V}_u(\hat{t}_{dMRE}) = N^2 \left(\frac{1}{n} - \frac{1}{N}\right)$$

$$\left\{\frac{\Sigma_{s_d}(e_k - \bar{e}_{s_d})^2 + n_d\,(1-n_d/n)\,\bar{e}_{s_d}^2}{n-1}\right\} \quad (6.16)$$

Given our conditional outlook, we favour the use of the c-variance estimator (6.15) for the construction of confidence intervals with $\hat{t}_{dMRE}$. Formula (6.16) gives incorrect conditional (but correct unconditional) confidence levels.

Remark. We shall also use (6.16) when forming confidence intervals with the estimator $\hat{t}_{dDRE}$. This will tend to overstate the c-variance for $\hat{N}_d < N_d$. However, it turns out that the overstatement helps to maintain a correct conditional coverage rate for small n_d - values. (The normality assumption is not adequate for small n_d; a constant greater than 1.96 = $z_{0.975}$ (for α = 5%) would be needed to give roughly 95% conditional coverage rate.)

Remark. The correction term in $\hat{t}_{dRE}$ given by (3.1) involves a direct expansion estimator in the residuals. Therefore $\hat{t}_{dRE}$ will suffer from drawbacks similar to those observed earlier for $\hat{t}_{dEXP}$: The estimator $\hat{t}_{dRE}$ is c-biased, thus unsuitable for conditional confidence statements.

7. RESULTS FROM THE EMPIRICAL STUDY

In order to confirm and illustrate the conditional and other results discussed in the preceding sections, we carried out a simulation study involving repeated draws of simple random samples. This study can be summarized as follows. The province of Nova Scotia was chosen as our population with N = 1678 sampling units (unincorporated tax filers). The variable of interest, y, is Wages and Salaries, and the auxiliary variable, x, is Gross Business Income (Income, for short). It is assumed that $x_1, ..., x_N$ are known. Domains of the population were formed by a cross-classification of four industry types (i=1, ..., 4) with eighteen areas (a=1, ..., 18). The industry types were Retail (515 units), Construction (496 units), Accommodation (114 units) and Others (553 units). The areas were the 18 census divisions of the province. This produced 70 non-empty domains (out of the 72 possible domains, two had no units). Thus, 70 domain totals t_{ai}, are to be estimated every time a sample is drawn. The domain index d used in earlier sections is expressed in our empirical study, by the double index ai. Consequently, earlier notation such as U_d, s_d, n_d, s_{dg}, n_{dg} ... now becomes U_{ai}, s_{ai}, n_{ai}, s_{aig}, n_{aig} ... The overall correlation coefficient between x and y was 0.42 for Retail, 0.64 for Construction, 0.78 for Accommodation and 0.61 for Others. The average domain size was 28.6 for Retail, 27.5 for Construction, 7.1 for Accommodation and 30.7 for Other. The overall average domain size was 24.0. The smallest domain has 1 unit; the largest 130 units.

For the Monte Carlo simulation, 500 simple random samples, s, each of size n=419, were selected from the population of N=1678 units, and for each sample, a number of estimators were calculated. (The sampling fraction is thus 419/1678 = 25%.) The selected sample units, within each sample, were classified by domain (that is, by industrial type and census division), as well as by Income Group, indexed by g=1, ..., G. Two income groupings were used: (i) G=3 groups with income classes given by \$25K-\$50K, \$50-\$150K, and \$150K-\$500K; (2) G=1, which means that no income grouping was attempted. The average behaviour of the estimators is summarized below through (i) conditional performance measures; (ii) overall unconditional performance measures. These are defined in detail below.

The simulation included the "count version" DRE estimator:

$$\hat{t}_{aiDREG/C} = \sum_{g=1}^{G} \{N_{aig}\, \bar{y}_{s_{.ig}} + F_{ai}\, \hat{N}_{aig}\, (\bar{y}_{s_{aig}} - \bar{y}_{s_{.ig}})\} \quad (7.1)$$

where $F_{ai} = (N_{ai}/\hat{N}_{ai})$ if $\hat{N}_{ai} \geq N_{ai}$ and $F_{ai} = (\hat{N}_{ai}/N_{ai})^{h-1}$ if $\hat{N}_{ai} < N_{ai}$. Here $\hat{N}_{aig} = Nn_{aig}/n$, $\hat{N}_{ai} = Nn_{ai}/n$, and $\bar{y}_{s_{.ig}}$ and $\bar{y}_{s_{aig}}$ are straight means. The "ratio version" DRE estimator is

$$\hat{t}_{aiDREG/R} = \sum_{g=1}^{G} \{X_{ai}\, \hat{R}_{ig} + F_{ai}\, \hat{N}_{aig}\, (\bar{y}_{s_{aig}} - \hat{R}_{ig}\, \bar{x}_{s_{aig}})\} \quad (7.2)$$

with $\hat{R}_{ig} = \bar{y}_{s_{.ig}}/\bar{x}_{s_{.ig}}$. The estimator (7.2) is generated by the general formula (3.3) if the underlying model is taken as:

$$E(y_k) = \beta_{ig}\, x_k;\; V(y_k) = \sigma^2_{ig}\, x_k, \qquad (7.3)$$

for units k in the i:th industrial type and g:th income group; i=1, ..., 4; g=1, ..., G. In the case of G=3 groups (defined as mentioned above), this implies that each simple random sample of size 419 is used to calculate 12 slope estimates, $B_{ig} = \bar{y}_{s_{.ig}} / \bar{x}_{s_{.ig}}$, where $\bar{y}_{s_{.ig}}$ and $\bar{x}_{s_{.ig}}$ are straight means.

We also used the model (7.3) with G=1, meaning that a single slope was estimated for each industry type.

The count version (7.1) of the DRE estimator is generated by the model (7.3) with $x_k \equiv 1$. With G=3, we then have 12 parameter estimates, $\hat{B}_{ig} = \bar{y}_{s_{.ig}}$. With G=1, the count version of the DRE is less interesting, since industry type by itself will pick up only a modest amount of the total variation in y.

The simulation study also included the EXP, POSG/C, POSG/R, SYNG/C and SYNG/R estimators given by formulas (2.2)-(2.7), with $\pi_k = n/N = 0.25$ for all k (since simple random sampling was used). For these estimators, too, we considered the cases of G=3 and G=1 income groups.

A. CONDITIONAL PERFORMANCE MEASURES

For each domain, the 500 repeated samples were distributed over the different realized domain sample count values n_{ai}. For a fixed value of n_{ai}, and for domain ai, the conditional performance measures were computed over that subset of the 500 samples for which the domain sample count was exactly n_{ai}.

The following conditional performance measures were calculated: (a) Relative Conditional Bias (RCB); (b) Root Conditional MSE (RCMSE); (c) Conditional Standard Error (CSE); (d) Conditional Coverage Rate (CCR).

Because of space constraint, we limit ourselves here to a graphical illustration of these results, involving one selected domain; namely, Retail, Region 8, with $N_{ai} = 23$; $E(n_{ai}) = 5.75$. Since the synthetic estimator is considerably c-biased, for this domain it is of interest to observe how alternative estimators behave. The graphical comparison involves the EXP estimator and several estimators based on G=1 group: POS1/C, POS1/R, SYN1/R, and DRE1/R (with h=2). The main conclusions from Graphs 1 to 4 are as follows:

(a) **Relative Conditional Bias (RCB).**

If $\hat{t}_{ai}$ is one of the estimators studied, the RCB was calculated as

$$RCB(\hat{t}_{ai}) = \frac{1}{R}\sum_{r=1}^{R} (\hat{t}_{ai,r} - t_{ai})/t_{ai}$$

where $\hat{t}_{ai,r}$ is the value of the estimator $\hat{t}_{ai}$ in the r:th of the R, say, samples (out of the 500) for which the sample count in the ai:th domain equals the fixed number n_{ai}. As seen in Graph 1, the RCB curve of the SYN1/R estimator is situated at an essentially constant, clearly non-zero level over the entire n_{ai}-range. Confirming the theory in Section 6, the DRE1/R estimator is seen to be essentially c-unbiased when n_{ai} is greater than expected; below the expected value point, the RCB of DRE1/R increases as n_{ai} approaches zero, at which point it tends to join the RCB curve of SYN1/R. The POS1/R estimator displays a RCB curve near the zero level, while EXP is heavily c-biased, except in the immediate vicinity of the expected value point.

(b) **Root Conditional Mean Squared Error (RCMSE).**

This measure was calculated for a given domain and n_{ai}-value as

$$RCMSE(\hat{t}_{ai}) = \{\frac{1}{R}\sum_{r=1}^{R} (\hat{t}_{ai,r} - t_{ai})^2\}^{\frac{1}{2}}.$$

For the two domains in question, we see from Graph 2 that the DRE1/R estimator behave best in terms of RCMSE. It is followed by the SYN1/R and POS1/R estimators, while the EXP estimator falls way behind the others, due in large part to a considerable c-bias.

(c) **Conditional Standard Error (CSE).**

This measure was calculated as the average of $\{\hat{V}_c(t_{ai})\}^{\frac{1}{2}}$ over those R samples that yielded a given n_{ai}-value in a given domain ai. That is, the CSE is proportional to the average length of the conditional confidence interval calculated by (4.3). (As it makes little sense in this comparison to include estimators for which there is no valid design-based confidence interval procedure, we could not consider the SYN1/R estimator in Graph 3. Graph 3 shows decreasing CSE curves for $\hat{t}_{aiDRE1/R}$ and $\hat{t}_{aiPOS1/C}$ (which is intuitively sound) while the CSE curve for $\hat{t}_{aiEXP}$ increases with n_{ai} (which underscores the less satisfactory performance of this estimator). Formula (6.15) was used for the DRE1/R estimator; (5.3) with G=1 was used for POS1/C.

(d) **Conditional Coverage Rate (CCR).**

This performance measure was computed for a given ai and n_{ai} as

$$CCR(\hat{t}_{ai}) = \frac{1}{R}\sum_{r=1}^{R} I_{c,r}(\hat{t}_{ai}),$$

where $I_{c,r}(\hat{t}_{ai}) = 1$ if the r-th conditional confidence interval based on $\hat{t}_{ai}$ contains the true total t_{ai}, and zero otherwise. The nominal rate 95% was used in the simulation. The intervals were computed using formula (4.3), inserting the appropriate conditional variances. Graph 4 shows that the CCR curve for $\hat{t}_{aiDRE1/R}$ is roughly constant at (but sometimes a bit short of) the nominal 95% rate. This satisfactory performance is also observed when n_{ai} is less than expected, despite a certain conditional bias in the estimator for this range of n_{ai}-values. For $\hat{t}_{aiPOS1/C}$, the CCR is also reasonably well maintained near the nominal 95% rate, except for n_{ai} values near zero. (The normal approximation is then inadequate.) The CCR curve for $\hat{t}_{dEXP}$ is close to 95% only if n_{ai} is near its expected value, since it is only in this neighbourhood that the c-bias of $\hat{t}_{dEXP}$ is small.

Table 1 illustrates (for the domain Retail, Region 8) the differences that may arise between the conditional and unconditional approaches, for the POS1/C and DRE1/R estimators. CSE and CCR denote conditional standard error and conditional coverage rate. Viewed as functions of n_{ai}, both concepts are well-behaved for the two estimators: CSE decreases as n_{ai} increases, and the CCR is roughly constant throughout the range of n_{ai}-values. (DRE1/R performs better in the latter respect). On the whole, both CCR's are, however, on the short side of the nominal 95% rate, suggesting that the CSE formula underestimates.

As illustrated by the last four columns of Table 1, the unconditional approach is unsuitable when valid inferences are required for a fixed domain sample count. In the case of both estimators, the unconditional standard error (USE; given by (5.4) with G=1, and (6.16)) is increasing with n_{ai}, contrary to what is reasonable. Consequently, the unconditional coverage rate, UCR (while near 95% on the average over all n_{ai}-values) is 100% for large n_{ai}-values, but drops toward zero for small n_{ai}-values.

B. OVERALL UNCONDITIONAL PERFORMANCE MEASURES

These serve to measure the bias and the MSE of the various estimators over all 500 repeated samples and over all A domains of a given industry type (A = 18 except for Accommodation, where A = 16). Since collapsed over all n_{ai}-values, these measures are unconditional in nature. As before, $\hat{t}_{ai,r}$ denotes the estimate obtained by a certain estimator $\hat{t}_{ai}$ in the r-th repeated sample.

The Overall Absolute Relative Bias (OARB) was calculated as

$$\overline{ARB}(\hat{t}_{ai}) = \frac{1}{500A} \sum_{a=1}^{A} \left| \sum_{r=1}^{500} (\hat{t}_{ai,r}/t_{ai} - 1) \right|$$

The Overall Relative Efficiency (OREFF) was calculated as

$$EFF(\hat{t}_{ai}) = \{\overline{MSE}(\hat{t}_{ai}) / \overline{MSE}(\hat{t}_{aiEXP})\}^{\frac{1}{2}}$$

where

$$\overline{MSE}(\hat{t}_{ai}) = \frac{1}{500A} \sum_{a=1}^{A} \sum_{r=1}^{500} (\hat{t}_{ai,r} - t_{ai})^2$$

For this level of comparison, we examined the expansion estimator, EXP, the post-stratified estimators POS1/C and POS1/R, the synthetic estimators SYN1/C, SYN3/C, SYN1/R, SYN3/R, the dampened regression estimators DRE1/C, DRE3/C, DRE1/R and DRE3/R, each with h = 0.5, 1.0, 2.0, 8.0. It is not possible here to show detailed results from the simulation for each of these estimators. We limit ourselves to a brief summary.

The results on OARB were as follows for the EXP estimator and for the regression-based estimators of ratio type with G=1:

Industrial Type	Estimator			
	EXP	POS1/R	SYN1/R	DRE1/R (h=2)
Retail	0.02	0.11	0.32	0.09
Construction	0.02	0.05	0.16	0.05
Accommodation	0.04	0.27	0.41	0.24
Other	0.02	0.05	0.26	0.08

In terms of OARB, the estimators thus rank as follows from most favourable (low OARB) to least favourable (high OARB) : 1. EXP; 2. POS1/R and DRE1/R (essentially tied); 3. SYN1/R. Given what is known from theory, there are no surprises in this ranking. One notes that all estimators except EXP are more heavily biased in Accommodation, where domain sizes are very small and zero domain sample counts frequently occur. One consequence is that the DRE1/R estimator will often equal the SYN1/R estimator, with increased bias as a result.

As for the other estimators included in the study, the following observations were made:

1. SYN estimators (SYN1/C, SYN3/C, SYN1/R, SYN3/R): The ratio versions had smaller OARB than the count versions, except for Retail, where the correlation between x and y is weakest.
2. DRE estimators (DRE1/C, DRE3/C, DRE1/R, DRE3/R, each with h=0.5, 1, 2 and 8): The OARB increases rather modestly with the value of the exponent h. For a fixed value of h, there was no clear indication that one of the four DRE versions would have a decidedly smaller OARB. There was no clear evidence that grouping (G=3), as opposed to no grouping (G=1), would necessarily reduce the OARB.

Turning to the OREFF, we have the following results for ratio version estimators with G=1:

Industrial Type	Estimator		
	POS1/R	SYN1/R	DRE1/R (h=2)
Retail	1.24	2.27	1.80
Construction	1.86	1.93	2.10
Accommodation	1.86	3.27	2.38
Other	1.64	2.04	1.78

Overall, the SYN estimator will outperform all its competitors if its bias is small enough in all domains. Under this condition, alternative estimators cannot overcome the variance advantage of the SYN estimator. Here we see that SYN is better than the alternatives in the Retail and Accommodation types.

The ranking (from best to worst) in terms of OREFF of the other two estimators is: 1. DRE1/R and 2. POS1/R. The highest efficiency gains (relative to EXP) are realized in Accommodation and Construction, the industry types with the highest correlation between x and y.

For the other estimators included in the simulation we observed the following:

1. DRE estimators. The OREFF increases with the value of h (despite some increase in bias), but not markedly beyond h=2.

2. POS, SYN and DRE estimators: not surprisingly, for the count version, G=3 groups gives considerably higher OREFF than G=1 group. However, for the ratio versions, such an increase was not always observed. (DRE3/R was, however, more efficient than DRE1/R for all four industry types and all h-values.) The comparison between G=3/C and G=1/R (which should achieve roughly the same purpose), was not conclusive. Sometimes the former is more efficient, sometimes the latter.

8. CONCLUSIONS

The use of conditional inference has permitted the development of new estimators which have desirable conditional properties. It has also shown that although some of the estimators are unconditionally unbiased, they can be conditionally biased (EXP, RE). The construction of confidence intervals based on conditional variances is more likely to achieve the target nominal rates as opposed to those based on unconditional variances.

The dampened regression estimator (DRE) has several advantages over the other estimators (EXP, SYN, and POS). It is more efficient than either the EXP or the POS estimators. Although the SYN estimator may under some circumstances (when it is conditionally unbiased) be more efficient than DRE, it is biased (conditionally and unconditionally). The DRE estimator formula is straightforward to apply, and the associated conditional confidence interval procedure is not complicated.

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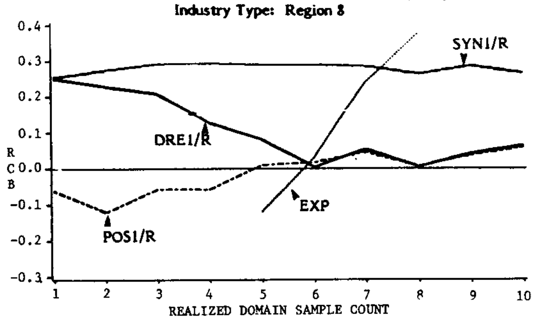
Table 1. Standard error and coverage rate in conditional and unconditional approaches. Retail, Region 8.

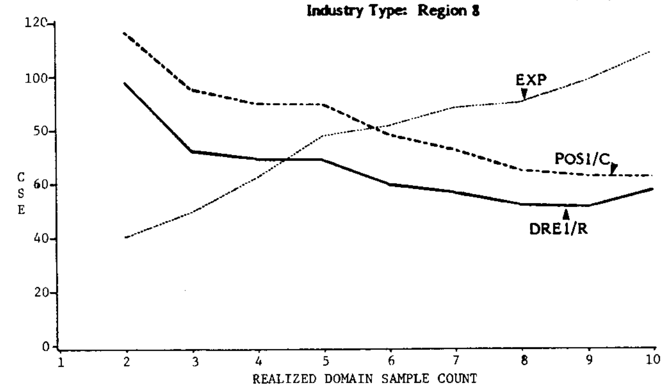
DOMAIN SAMPLE COUNT	FREQUENCY OF SAMPLES	CONDITIONAL APPROACH POS1/C CSE	CONDITIONAL APPROACH POS1/C CCR	CONDITIONAL APPROACH DRE1/R CSE	CONDITIONAL APPROACH DRE1/R CCR	UNCONDITIONAL APPROACH POS1/C USE	UNCONDITIONAL APPROACH POS1/C UCR	UNCONDITIONAL APPROACH DRE1/R USE	UNCONDITIONAL APPROACH DRE1/R UCR
1	7	-	-	-	-	0.0	0.00	23.3	0.00
2	25	116.6	0.56	97.8	0.84	26.2	0.28	37.6	0.12
3	45	95.2	0.67	72.4	0.95	37.9	0.58	40.7	0.40
4	59	89.8	0.80	69.4	0.98	51.8	0.64	54.5	0.86
5	106	89.6	0.85	68.8	0.95	68.5	0.82	64.8	0.98
6	94	78.3	0.87	59.8	0.84	75.4	0.86	68.0	0.98
7	77	72.4	0.88	56.6	0.83	85.1	0.90	75.4	0.97
8	42	64.8	0.93	52.0	0.83	90.8	0.98	80.9	1.00
9	31	62.9	0.87	51.3	0.87	103.4	1.00	91.9	1.00
10	10	62.7	0.90	57.8	1.00	119.5	1.00	114.1	1.00

@ — confidence interval not defined.

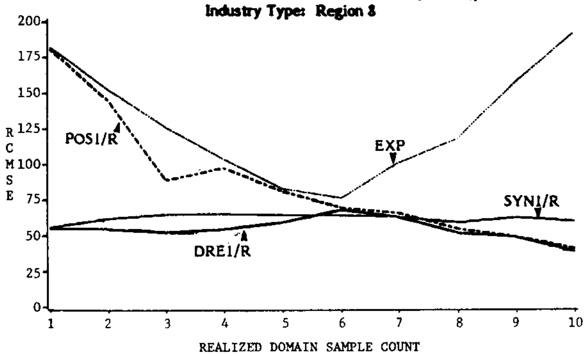
GRAPH 1: RELATIVE CONDITIONAL BIAS (RCB)
Industry Type: Region 8



GRAPH 3: CONDITIONAL STANDARD ERROR (CSE)
Industry Type: Region 8



GRAPH 2: ROOT CONDITIONAL M.S.E. (RCMSE)
Industry Type: Region 8



GRAPH 4: CONDITIONAL COVERAGE RATE (CCR)
Industry Type: Region 8