CTSS/CUSUM APPLICATIONS TO INTERVENTION ANALYSIS*

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ABSTRACT

Most surveys used in monitoring undergo periodic changes by adding or deleting respondents, questions or calculations used in deriving estimates. The results of such "interventions" need to be considered when one analyzes time-series data generated from such surveys. This paper addresses the applicability of Cumulative Transitional State Score (CTSS), Transitional Scores (TS) and Cumulative Sums (CUSUMs) to detect the significance of interventions. It demonstrates the application of an adjustment to the mean value with CUSUMs when a significant change is detected. The data used in the analyses is the Energy Information Administration monthly estimates of stocks of motor gasoline, distillate fuel oil, residual fuel oil and crude oil 1976-1982. The above index computations are applied to series seasonally adjusted using X-11. A Box-Jenkins time-series analysis of CTSS and TS scores explores the charactistics of these indices and graphically displays the pattern in the partitioned successive evaluations.

I. FRAMEWORK FOR CTSS AND CUSUM INDICES

The Cumulative Transitional State Score (CTSS) and Cumulative Sums (CUSUMs) represent two types of scoring techniques which may be applied to monitoring changes in time-series data. CTSS was developed by Gardenier (1974, 1979); the principles underlying CUSUMs have been described in Barnard (1959), Johnson (1961), Johnson and Leone (1962) and DeBruyn (1961). CUSUMs are based on Wald's sequential probability ratio test (SPRT) in the early 1940's which Page (1961) applied to test sequential hypotheses. In depicting the short-term pattern of fluctuations in time-series data, CTSS has advantages over CUSUMs.

Both tests are oriented toward identifying whether statistically significant changes are apparent in the data prior to and post-intervention. CUSUMs also incorporate an adjustment to the long-term process mean value when a significant shift is observed.

A. Methodological Background

CUSUMs aggregate deviations from a long-term process average; thus one cumulates the sum of (X_i-m), where m corresponds to a reference value or a long-term process average. In applying CUSUMs, one uses two parameters, h and k in deciding whether or not the overall process has changed. k is usually expressed in standard deviation units and is added to or subtracted from the mean prior to cumulating deviations. That is, an interval $\bar{X} \pm k\sigma$ is established as an acceptable region beyond which deviations are cumulated. If the cumulative deviations reach h, the decision is made that the process has shifted to a new mean. The choice of k is usually done by the user, depending upon the long-term reliability of the data. The choice of h is dependent upon some of the basic principles of hypothesis testing, as outlined below:

Let us define H_0 as the hypothesis that the process is in control and a_0 as the probability of a false alarm or α. Let us also define H_1 as the probability that the new data are biased by g standard deviation units and a_1 as the probability of H_0 when H_1 is true (usually denoted as β in hypothesis testing). At successive observations calculating the ratio of the likelihood of observed values under H_0 and H_1 assists us in deciding whether to accept H_0, H_1 or whether to continue sampling.

The long-range process mean, μ is accepted if:

$$\frac{1}{\sigma}\sum_{i=1}^{n}(X_i-\mu) \leq \frac{1}{g}\ln\frac{a_1}{1-a_0}+\frac{1}{2}Ng$$

An alternate $\mu+g\sigma$ is accepted as the new mean if:

$$\frac{1}{\sigma}\sum_{i=1}^{n}(X_i-\mu) \geq \frac{1}{g}\ln\frac{1-a_1}{a_0}+\frac{1}{2}Ng$$

One continues to observe the process until one of the two decisions is reached.

In the literature, the run length of the CUSUM distribution is defined as the number of observations until H_0 is rejected. A small value for run length implies timely detection of shifts in the process mean.

Woods and Pike (1981) applied these concepts to detecting cumulative inventory differences and pointed out that reducing h for fixed values of k increases the false alarm rate. Other applications of the CUSUM technique to evaluating time-series data in materials inventory have been demonstrated by Cobb (1981), Markin and Shipley (1982) and Wincek et. al. (1979).

Robbins and Siegmund (1969, 1970) defined a threshold of:

$$T(N)= N(A^2+\log(N))^{\frac{1}{2}}$$

to test for an increase in the mean of the cumulative sum. A is a parameter controlling the false alarm rate.

In contrast to CUSUMs which are based upon the sum of deviations from a target value m, the CTSS is based upon the transitions between successive observations. States are defined within CTSS as regions or partitions, quite akin to zones in control charts used in quality control. These are defined using the statistical characteristics of previously observed values. An illustration of the definition of states is given in Figure 1; the summation scheme based upon the transitions in successive states is shown in Figure 2.

In Figure 1 we may trace a sequence of observations over time and categorize them as to whether they are in states +2, +1, 0, -1 or -2. In the present analyses we have defined the range +2 to -2 as the upper and lower limits of stock bands for 1979-1980 as defined in the Weekly Petroleum Status Report (Energy Informa-

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Figure 1

An Example of the CTSS Approach to the Measurement of Change

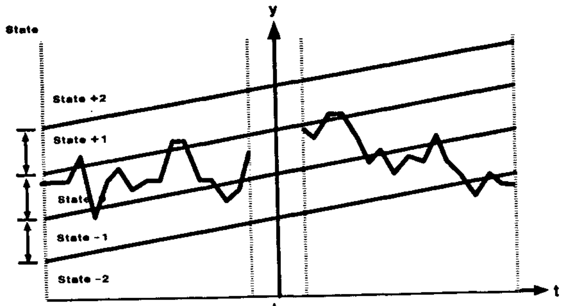

Figure 2

CTSS Transition Matrix with a 5-State System

	Final State -2	-1	0	+1	+2
Initial State -2	-1	+1	+2	+3	+4
-1	-1	0	+1	+2	+3
0	-2	-1	0	+1	+2
+1	-3	-2	-1	0	+1
+2	-4	-3	-2	-1	+1

tion Administration, 1983). Stock bands are computed using the mean and standard error of past observations, and a seasonal adjustment factor using the Census X-11 procedure (Bureau of Census, 1967). This range was then interpolated to yield the limits for +1 to -1. The horizontal line y on the diagram represents a hypothetical intervention point before and after which the status of the observations will be traced.

In Figure 2 we see a schematic diagram showing a 5x5 matrix of the possible fluctuations between one time period and the next, t_i and t_{i+1}. That is, if an observation is at state 0 and t_i, it may remain within the boundaries of the same state or may fluctuate upward or downward to any of the other possible states. The same is true for any other row of the matrix depicting initial states.

CTSS incorporates a scoring scheme to this matrix of transitions. Different evaluation scenarios may dictate different scoring schemes depending upon the pattern of past observations and their short-term transition patterns. In the present analysis, the cumulative sum was not incremented if, in their previous time period, the observations were in the -1 or +1 range and remained therein. The cumulative sum increased or decreased depending upon the number of boundary crossings in adjoining time periods. If at any time period observations were found to be outside the -2 to +2 range and remained in these states, i.e., no significant reversal toward the overall process characteristics was observed, an adjustment of the summation procedure was applied. The cumulative sum was incremented by one if the previous observation was in the -2 state.

This scoring scheme is similar to CUSUMs in that it cumulates deviations; yet it differs from CUSUMs in that it evaluates the transition matrix in successive observations. In cases where the effect of intervention is to be evaluated, the pattern of observations in the proximity of intervention time is of interest. CTSS also allows for an "uncertainty zone" in each evaluation so that outliers do not significantly affect the results.

Statistical Markovian properties of time-series data and methods for evaluating transitions have been discussed by Billingsley (1961) and Whittle (1955). A non-parametric test for randomness in multinomial observations, which may be relevant to the partitioning ranges presented in the CTSS methodology is given by Bennett (1964). Sobel et. al. (1973) used the Rao-Blackwell theorem to find the conditional expectation of CTSS up to time T for the run of 0 or 1-valued successive evaluations where 1's occur with probability p and q=1-p. The minimum unbiased estimator for the expectation of CTSS up to time T and its variance are:

$$E\{CTSS|T\} = \frac{T}{n} + \frac{T}{n}(T-1)\frac{(T-1)}{n-1} + (1-\frac{T}{n})\ T(\frac{T-1}{n-1})$$

$$= \frac{T}{n} + \frac{T(T-1)}{n(n-1)}\ T-1+n-T = \frac{T^2}{n}$$

$$\sigma^2(T^2/n) = \frac{E\ T^4-(ET^2)^2}{n^2}$$

$$= \frac{pq}{n}\ [1+6(n-1)p+2(n-1)(2n-3)p^2]$$

B. Adjustment to the Methods in the Present Application

Initial exploratory application of the CUSUM technique to the four data series showed that H_o was being rejected very often due to the seasonality exhibited in the data series. Thus an adjustment to the CUSUM computations was made, correcting for seasonality in the data using the X-11 procedure. The CUSUM index in the present analyses cumulated $\bar{X} + S \pm k$, where S corresponded to the seasonal factor for each month. $\bar{X}$ was calculated for the years 1975-1978; k was chosen as ½ standard deviations of the variation in the series during the same time period; the rejection level h corresponded to 4k.

When H_o was rejected at time T for a particular series, an adjustment to the long-term process average was made using the following formula:

$$\bar{X}_N = \bar{X} \pm ((N\ k + (CUSUM\ /\ N))$$

where $\bar{X}_N$ corresponds to the new process average and N refers to tne number of successive observations where positive deviations were found to occur. The process for monitoring CUSUMs were reinitiated after each such modification of the process mean.

In using CTSS, the transitional state score (TS) was recorded as well as the value of CTSS for each month 1976-1982. CTSS was tested for significance using 6-month and yearly time intervals prior to and post-intervention. The cutoff points for significance at α=.05 and α=.01 were established by simulating the possible values of CTSS for T=6 and T=12. In addition, contingency tables were formed for each data series tabulating, by year, the number of observations where TS was observed as 0, $\geq$1 or $<$1. Reclassifying TS values which were greater than or less than 1 as +1 was done in order to minimize the number of zero entries in the contingency tables. A Chi-square test was applied to each such table to test for time-related effects.

II. APPLICATION OF CTSS AND CUSUM TO THE DATA SERIES

Table 1 shows a summary presentation of the results relating to CTSS at yearly and 6-month intervals. For motor gasoline, a significant increase is observed during 1977 and 1980, prior to the intervention point of January, 1981. The same trend exists for distillate--a significant increase during 1977 and 1980. Distillate stocks show a decrease in CTSS after mid-1981 and continue to decrease throughout 1982. CTSS differences for residual fuel oil start increasing late 1978 and show a significant increase until the intervention point, early 1981. A decrease is observed thereafter which becomes statistically significant by late 1980, and continue to increase thereafter.

Significance test results from the Chi-square application to the total matrix, to the pre- versus post-intervention time periods, and each consecutive pair of two years is shown in Table 2. The number of columns in the Chi-square analyses have been consistently taken as three, collapsing and TS score more extreme than ±1 into ±1 in order to avoid difficulties in applying the analysis to a sparse matrix. A cut-off p-values of .10 has been used.

In summary, differences in the pattern of TS values can be traced in all the four data series during the 1976-1982 time interval. In the two years prior to the intervention, 1979-1981 significant differences are also observed in all the four data series. For motor gasoline and distillate fuel oil and crude oil the transition matrices seem to have been stabilized by the end of 1981, about a year after the intervention.

Table 1

CTSS Comparisons at Yearly and Six-Month Intervals

Time	Motor Gasoline			Distillate			Residual			Crude Oil		
	CTSS	p_m	p_a	CTSS	p_m	p_a	CTSS	p_m	p_a	CTSS	p_m	p_a
June 1976	-1	ns		+1	ns		-1	ns		-4	ns	
Dec 1976	0	ns	ns	-2	ns	ns	-2	ns	ns	-2	ns	ns
June 1977	+5	.05		+1	ns		+1	ns		-2	ns	
Dec 1977	+6	.01	.01	+6	.01	.05	+4	ns	ns	+5	.05	ns
June 1978	-3	ns		-1	ns		-2	ns		+2	ns	
Dec 1978	-2	ns	ns	0	ns	ns	+2	ns	ns	-1	ns	ns
June 1979	+1	ns		-6	.01		+3	ns		0	ns	
Dec 1979	-1	ns	ns	+2	ns	ns	+6	.01	.01	+1	ns	ns
June 1980	+6	.01		+6	.01		+6	.01		+6	.01	
Dec 1980	+6	.01	.01	+1	ns	.05	+6	.01	.01	+6	.01	.01
June 1981	+4	ns		+3	ns		-1	ns		+6	.01	
Dec 1981	0	ns	ns	-5	.05	ns	-1	ns	ns	+6	.01	.01
June 1982	-5	.05		-2	ns		-6	.01		+4	ns	
Dec 1982	+2	ns	ns	-6	.01	.05	-6	.01	.01	+6	.01	.01

Note: CTSS entries are differences between cumulative values of intervals being compared. Annual CTSS differences, upon which significance tests are based, are obtained by the sum of six-month interval differences.

Table 2

Results of Statistical Comparisons

	Motor Gasoline Stocks		Distillate Fuel Oil Stocks		Residual Fuel Oil Stocks		Crude Oil Stocks	
	Chi-Square	p	Chi-Square	p	Chi-Square	p	Chi-Square	p
Full Matrix: 1976-1982	36.15	<.005	29.67	<.005	56.11	<.001	77.05	<.001
Pre/post intervention	1.45	ns	7.10	<.05	9.61	<.01	31.48	<.001
1976/1977	12.88	<.005	3.72	ns	4.13	ns	13.77	<.005
1977/1978	12.57	<.005	4.47	ns	1.50	ns	3.82	ns
1978/1979	3.28	ns	3.60	ns	5.47	<.10	2.81	ns
1979/1980	8.34	<.02	6.60	<.05	*	*	17.14	<.001
1980/1981	4.76	<.10	4.92	<.10	12.00	<.005	*	*
1981/1982	3.14	ns	2.40	ns	8.00	<.02	*	*

* Not evaluable due to sparse matrix

Wide fluctuations in the data are observed for residual fuel oil at a few points several years before the intervention combined with an upward trend. The 1981 intervention seems to have elicited a downward trend.

A parallel analysis using seasonally adjusted CUSUMs was applied to the four data series. Wherever H_0 was rejected the long-term mean was adjusted upward or downward and the CUSUM computations were reinitiated. Table 3 summarizes the results of this analysis. It shows a tracking of the mean of each data series over time showing when it was updated as well as the adjustments made to the mean. A summary comparison of times to detecting change prior and post-intervention is shown in Table 4. An initial scan of the results based upon CUSUMs shows no significant shift in the pattern of observations after the intervention point. The one point of departure is in distillate stocks data where the only adjustment of the series was made a year after the intervention date.

In summary CTSS and TS scores may prove helpful in maintaining data series to (a) detect changes over time and (b) to compare the magnitude of changes over short time intervals as compared to those that may be due to external intervention.

Table 3

Switching Points Observed Prior to and Post-Intervention and Adjustments to the Mean Value Using CUSUMs

Series	$\pm$	New $\bar{X}$	Date
Motor Gasoline	-20	219	4/1976
	+16	231	11/1976
h=32	+16	247	2/1977
k=$\pm$8	+18	265	8/1977
Initial $\bar{X}$ = 239	-16	249	4/1978
	-20	229	7/1978
	+29	258	3/1980
	=====	=====	=====
	-15	243	3/1982
	-17	226	5/1982
	+15	241	11/1982
Distillate	-47	143	1/1982
h=80			
k=$\pm$20			
Initial $\bar{X}$ = 190			
Residual	+ 9	83	10/1977
	-14	69	3/1978
h=16	+ 7	76	10/1978
k=$\pm$4	+10	86	7/1979
Initial $\bar{X}$ = 74	+10	96	3/1980
	-12	84	10/1980
	=====	=====	=====
	-10	74	8/1981
	- 9	65	4/1982
	- 8	58	10/1982
Crude Oil	-12	288	7/1976
	+33	321	6/1977
h=48	+21	342	12/1977
k=$\pm$12	-19	323	8/1978
Initial $\bar{X}$ = 300	+26	349	1/1980
	+23	372	6/1980
	=====	=====	=====
	-29	343	1/1982

Table 4

Time to Detect Change Using CUSUMs (Months)

	Prior to Intervention	Post Intervention	
	Mean	Mean	to First
Motor Gasoline	7.8	7.3	14
Distillate	---	---	12
Residual	7.2	7.3	8
Crude	9.6	---	12

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