This paper examines a variety of sources of error for statistics calculated from a survey. The perspective taken follows error models constructed using a finite population model, in which the purpose of the survey is to estimate some fixed parameter within the finite population (Hansen, Hurwitz, and Bershad, 1961; Fellegi, 1964; O'Muircheartaigh and Payne, 1977). The estimators used for the population parameter are seen to be subject to both variable errors and biases. Variable errors are those arising from sources that could change over replications of the survey, given its basic design. The "basic design" of the survey is often termed the "essential survey conditions", those features of the design that affect the error structure on which any estimates of the errors are conditioned. Biases are errors that are seen to remain the same over replications of the survey.

Typically, survey statistics are presented with only one source of error measured, error due to sampling, resulting from the fact that survey statistics would have different values had another sample been drawn using the same design. Other variable errors are ignored, and biases are rarely mentioned. The presence of record data for this sample survey offers a rare opportunity to measure a large set of variable errors and biases that are normally assumed to be negligible in survey analysis.

**Variable Errors and Biases to Be Examined.** A later section presents in formal notation the total survey error model employed in this paper. This section describes in a less precise way the origin and meaning of various errors.

Variable errors in surveys can be measured when there exists in the design more than one unit over which the errors vary and there is a randomization step to assure that the expected values achieved by the various units are equivalent, except for differences arising from the variable errors. This general statement needs specific examples for clarification.

Sampling error is a variable error because the deviation of the sample mean from the true population mean will vary over replications, using the same sample design. The sampling variance estimates the magnitude of variation of the sample mean over repeated drawings of the sample. Another variable error arises because errors are made in response to survey questions. Some of these arise because of inaccuracies on the part of the respondent, these response errors may also vary because different interviewers are assigned to administer the questionnaire to each respondent. Our model observes that different interviewers, through their idiosyncrasies of inflection, question delivery, and recording habits will obtain different data from the same respondent (Bailey, Moore, and Ballar, 1978; Groves and Magilavy, 1983; Kish, 1962). The effects of interviewers on the recorded data can be measured under this model through use of more than one interviewer and the randomization of sample persons to the interviewers. Interviewers can also affect the survey statistics by differential nonresponse among the sample persons assigned to them. In personal interview surveys interviewers are given assignments of varying difficulty (e.g., New York City versus a small rural area) and their resulting response rates are incomparable. In centralized telephone interviewing facilities, survey researchers have been able to observe response rate differences interviewers without confounding with other characteristics of their assignments. Similarly, there exists variability in interviewers' rates of item missing data from respondents they successfully persuade to begin the interview. Since sample persons are randomly assigned to interviewers, the variable component of nonresponse error, that due to different interviewers, can be measured.

Another source of variable nonresponse error arises from the inability of the telephone facility to contact every sample person. The survey period surrounding the interview surveys interviewers are given to call, out of town, not at home when the interviewer calls, or for some other reason not able to complete an interview. To the extent that these sample persons have different values on the survey variable than the sample persons who do respond, the respondent mean will differ from the true sample mean. The amount of noninterview error will vary over replications of the sample, dependent on the number of noninterview cases that fall into the sample. In contrast to the response error component above, therefore, this component is not viewed as a function of the interviewers assigned to work on the project. In centralized interviewing, calling on numbers is not at the discretion of individual interviewers, but rather a function of the staffing and scheduling of the facility staff. Hence, the source of variability in this component of error is seen to be within the population sampled alone.

Corresponding to some of the variable errors above are biases arising from the same sources. Respondents, independent of the interviewer, can make errors in their reports on survey questions; in addition, interviewer effects may be such that all interviewers tend to generate answers that are wrong in a similar direction. These errors when aggregated over all respondents produce differences between survey statistics and the population parameters they estimate. This kind of bias is often termed "response bias." Similarly, to the extent that persons who refuse the interview tend to have different characteristics than respondents, the mean estimated from respondents will differ from that of the entire sample, and "refusal bias" is evident. Additionally, the sample persons who cannot be contacted during the survey period, can generate a "noninterview bias."

In order to estimate these various errors, the record data obtained from the firm will be treated as error free measures of the underlying variables. It should be noted that this
assumption has no validating procedure, and throughout the analysis we are forced to ignore errors in the records associated with clerks entering information into the wrong person's record, not updating information properly when changes are made, and entering the wrong information. Despite the potential errors in the records, however, it is likely that they fall far short of those in the interview reports.

Sampling Procedures. A sample was selected from a computerized list of several thousand active employees of a medium size manufacturing company. Prior to sample selection, six strata were defined using three age categories and two employee type categories, hourly and salary. Because the number of records with missing telephone numbers was larger than expected (ranging from 1.7% to 17.3% within the six strata), it was decided not to remove these records prior to sampling. The sample size was then increased to adjust for the estimated proportion of missing phone numbers in each stratum. A sample of 620 employees was drawn with interviewing conducted during June and July, 1983. Prior to selecting the sample, the list of employees was ordered by tenure within race within gender within each stratum.

The response rate (78.3 percent) was lower than originally expected, due in part to the large number of sampled individuals for whom a current working number was not available. Individuals with a current phone number listed as unpublished by directory assistance were classified as nonsample (86 individuals); individuals without a current working phone number were classified as noninterview and retained in the response rate calculations. The survey yielded 418 completed interviews.

Total Survey Error Model and its Estimators. If a sample of size \( n \) were taken from the \( N \) population elements,

- \( r \) elements would refuse or yield item missing data,
- \( o \) would be other noninterviews,
- \( c \) would be interviews,

such that \( r + o + c = n \).

\[
\bar{y} = \frac{\sum_{i=1}^{c} y_i}{c}, \quad \text{the full sample true mean.}
\]

\[
\hat{\bar{y}}_c = \frac{\sum_{i=1}^{c} x_i}{c}, \quad \text{the mean for responses for the interviewed cases.}
\]

\[
\hat{\bar{y}}_c = \frac{\sum_{i=1}^{c} y_i}{c}, \quad \text{the mean for true values of the interviewed cases.}
\]

\[
r = \frac{\sum_{i=1}^{r} y_i}{r}, \quad \text{the true mean for refused and item missing data cases.}
\]


Let \( \hat{\bar{y}}_o = \frac{1}{o} \sum_{i=1}^{o} y_i \), the true mean for other noninterviews.

The sample mean for interviewed cases can be expressed as

\[
\hat{\bar{y}}_c = \bar{y} + \frac{r}{n} (\bar{y}_r - \bar{y}_c) + \frac{o}{n} (\bar{y}_o - \bar{y}_c) + \frac{c}{n} (\bar{y}_c - \bar{y}_c) \quad (2.1)
\]

The mean square error of \( \hat{\bar{y}}_c \) is

\[
\text{MSE}(\hat{\bar{y}}_c) = E[(\bar{y} - \hat{\bar{y}})_c]^2 \quad \text{(sampling error)}
\]

\[
+ E[(\bar{y}_r - \hat{\bar{y}})_c]^2 \quad \text{(refusal error)}
\]

\[
+ E[(\bar{y}_o - \hat{\bar{y}})_c]^2 \quad \text{(noninterview error)}
\]

\[
+ E[(\hat{\bar{y}}_c - \bar{y}_c]^2 \quad \text{(response error)}
\]

\[
+ 2E[(\bar{y} - \hat{\bar{y}})(\bar{y}_r - \hat{\bar{y}})_c]) \quad \text{(covariance between sampling and refusal error)}
\]

\[
+ 2E[(\bar{y} - \hat{\bar{y}})(\bar{y}_o - \hat{\bar{y}})_c]) \quad \text{(covariance between sampling and noninterview error)}
\]

\[
+ 2E[(\bar{y} - \hat{\bar{y}})(\hat{\bar{y}}_c - \hat{\bar{y}})_c]) \quad \text{(covariance between sampling and response error)}
\]

\[
+ 2E[(\bar{y}_r - \hat{\bar{y}}_r)(\bar{y}_c - \hat{\bar{y}}_c)]) \quad \text{(covariance between refusal and noninterview error)}
\]

\[
+ 2E[(\bar{y}_o - \hat{\bar{y}}_o)(\hat{\bar{y}}_c - \hat{\bar{y}}_c)]) \quad \text{(covariance between refusal and response error)}
\]

\[
+ 2E[(\bar{y}_c - \hat{\bar{y}}_c)(\bar{y}_c - \hat{\bar{y}}_c)]) \quad \text{(covariance between noninterview and response error)}
\]

Sampling Error. The sample was a stratified element design selected systematically from six strata. Using the record data we estimate

\[
E(\bar{y} - \hat{\bar{y}})^2 \text{ by}
\]

\[
6 \sum \frac{1}{n_h(n_h - 1)} \sum_{g=1}^{n_h - 1} (y_{hg} - y_{hg+1})^2
\]

successive differences formula within strata, where \( n_h \) is the number of sample elements in the \( h \)-th stratum (\( h = 1, 2, \ldots, 6 \)), whether interviewed or not. In this treatment of the estimator we have ignored all unequal probabilities introduced into the sample in order to achieve desired numbers of

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sample workers in different classifications. If inferences were to be made to the population of employees of the firm, such a strategy would yield biased estimates, but in keeping with the desire to investigate error properties in a hypothetical population with the achieved demographic and occupation distribution, weights will not be used in the point estimates or their variance estimates.

Refusal Error. We assume the likelihood of refusals is affected by the interviewer assigned to the case, as well as the characteristics of the respondents themselves. Interviewers are viewed as a selection from a large pool of potential interviewers and random assignments of respondents were made to interviewers. We note that

$$E[(\tilde{\gamma}_c - \bar{\gamma}_c)^2] = E[\tilde{\varphi}_n^2] \text{Var}(\gamma_c), \quad (2.2)$$

under the assumption that the mean of respondent cases is not dependent on the number of refusal cases obtained in the survey. In turn

$$\text{Var}(\bar{\gamma}_c) = \frac{1}{c} \sum_{i=1}^{c} \text{Var}(\sum_{i=1}^{c} y_i)$$

$$+ \frac{1}{c^2} \gamma_c^2 \text{Var}(c) - 2\gamma_c \text{Cov}(\gamma_c, c)$$

using a Taylor Series estimator. Finally, to estimate Var($\sum_{i=1}^{c} y_i$) we use

$$\text{Var}(\sum_{i=1}^{c} y_i) = \sum_{i=1}^{c} \frac{1}{a_h - 1} \left[ \frac{a_h}{c} \sum_{i=1}^{c} y_{h,i}^2 - \bar{y}_h^2 \right]$$

(2.4)

where there are $a_h$ interviewers completing assignments within the $h$th stratum ($h = 1, 2, \ldots, 6$).

Similar estimators apply to Var($c$), Cov($\gamma_i, c$). A similar approach is used to estimate

$$E[\tilde{\varphi}_n \bar{\gamma}_r - \bar{\gamma}_r]^2$$

The final term, which contains a refusal bias component, we will estimate by

$$(\bar{\gamma}_c - \bar{\gamma}_r)^2 \tilde{\varphi}_n^2$$

Noninterview Error. The form of the noninterview error component is similar to that of the refusal error component, and all the derivations remain similar. The variance estimates, however, follow the assumption that the likelihood of noninterview is independent of the interviewer chosen to work on the survey. Instead the variable components of noninterview error arise only from the sample selection. Hence an analog of (2.4) for noninterview error is used, and similar estimators for other terms.

Response Error. Response error is viewed as having a variable and a bias component:

$$E[\bar{x}_c - \bar{y}_c]^2 = \left[ \frac{c}{\Sigma} \sum_{i=1}^{c} (x_i - y_i) \right]^2 + (\bar{x}_c - \bar{y}_c)^2$$

(2.5)

Let $x_i - y_i = d_i$, a response deviation for the $i$th respondent. Then, $E(x_i - y_i) = \bar{d}_c = \bar{x}_c - \bar{y}_c$, the response bias. Thus (2.5) becomes

$$E[\bar{d}_c - \bar{d}_c]^2 = E[\bar{d}_c]^2$$

An estimator of $E(\bar{d}_c - \bar{d}_c)^2$, the variance of the response deviations, will reflect the clustering of responses into interviewer assignments and the correlation of response deviations within an interviewer's assignment, so that the variance of response deviations

$$\text{Var}(\sum_{i=1}^{c} d_i) = \frac{1}{c} \left[ \text{Var}(\sum_{i=1}^{c} d_i) + (\frac{c}{c})^2 \text{Var}(c) \right]$$

and the estimators have forms similar to (2.4).

Covariance Terms. The covariance terms present more complicated estimation problems and are not addressed in this analysis. It is hoped that they represent lower order magnitudes of error, and as in most past work on these response errors, they will be assumed negligible.

Estimates of Error Components. Estimates of the error components described in earlier sections were calculated for twenty statistics of interest to the study. These components are presented in Table 1. The table is divided into three major parts, the survey estimate and the true sample estimate for the population parameter, variable error components, and biases.

The survey estimate column contains unweighted estimates for means, based on unweighted observations of the respondents who supplied answers to the questions on which the statistics are based. No record data are used. The true sample value is the mean of the entire sample (respondents, refusals, other noninterviews) based on record data. The difference between the two represents the full bias due to response and nonresponse error in the sample.
are square roots of variance terms associated with error. The ratio of the estimated mean square other noninterview error, as described in earlier papers, to the square of the survey statistic will response, refusal, and noninterview errors of the different error components. This results in the true sample value in some cases. This results from slightly different cases bases for estimates of the different error components.

For variable errors, the reader will note the odd result that some of the statistics have whose values do not vary across the population. Some of these are constant for all employees (e.g., whether the employee has dental benefits, whether the employee has paid vacations); others are constant within the strata used for the sample design and thus produce a stratified estimate with no variance across replications of the sample draws (e.g., whether the employee belongs to a union, whether the employee has sick day benefits). For those statistics which are constant over the entire population there is also no refusal or noninterview bias because all the refusals and other noninterviews have the same values as the respondent cases.

Finally, the estimates for refusal bias are based on the small number of cases with complete refusal or item missing data on the question of interest. Sometimes in using the Taylor series expansion linearization of the expression, the small case base and the assumption of no covariance with the refusal and item missing data rate may produce inaccuracies and imprecision in the estimator such that the obtained value is less than zero. These estimates of refusal variance are replaced by zeroes in the table.

We will examine two different summary measures of error. The ratio of the estimated mean square error (minus covariance terms ignored in this paper) to the square of the survey statistic will be labelled, "CV^2". This may be used to compare errors across statistics. The other parameter of interest is the "bias ratio," which has the signed sum of response, refusal/missing data, and other noninterview bias in the numerator and the survey statistic in the denominator.

Table 1 illustrates statistics that suffer from different kinds of errors and Figures 1-4 are four types worth noting. Male employees are not eligible for paid maternity leave, but female employees are. As Figure 1 shows, it appears that many male respondents might have interpreted the question on maternity/paternity benefits as asking "do women employees have paid maternity benefits". The survey reports thereby grossly overestimate the proportion having benefits. The result is a survey statistic whose error is dominated by a bias term, most of it from response bias. Other error terms form only a small portion of total error.

Figure 2 illustrates a case in which biases from different sources cancel each others' effects. Salaried employees do not have formal sick day benefits (but are paid an annual salary); they disproportionately report that they do enjoy sick day benefits. The nonrespondents (both refusals and other noninterviews) tend not to have benefits, so that response bias is cancelled by nonresponse bias. Partially for that reason, the overall error structure of the estimated proportion is dominated by variable error sources. Cancellation of biases from different sources is a characteristic that researchers often hope for and sometimes assume. Table 1, however, shows that such cancelling is a rare phenomenon in these statistics.
Figure 3 may be a good illustration of weak information on the respondents' part. As soon as employees are hired they become participants in the company's pension plan. Thus, for all respondents the true value of the number of years in the pension plan is equal to the number of years an employee with the company. There are relatively few biases on reports for the number of years that the respondent has been an employee (and the statistic "mean number of years at company" has an error structure dominated by variable terms). Respondents tend to underestimate, however, the number of years they have participated in the company's pension plan. The result is a statistic (with same true value in the population) whose sample estimate has a much larger error (CV^2 greater by a factor of 100) and is dominated by response bias. Here the cause might be poor information instead of question wording.

Figure 4 shows two error pie charts that illustrate the effect of transformations of data on the error structure of resulting statistics. The top chart presents the error structure for the estimated mean 1981 earnings; the bottom, that for the mean logarithm for 1981 earnings. The raw data produce an estimated mean in which 48 percent of the mean square error is associated with bias; there is a tendency to underestimate 1981 earnings. When a logarithmic transformation is made (a common transformation with economic data) the estimated mean suffers from few effects of bias. There is a small tendency to overestimate earnings, the opposite of that with the raw data! These results reflect a tendency for those with high earnings to underreport them and those with low earnings to overreport them. The logarithmic transformation has disproportionately affected those with high earnings, and the mean of the logged variable achieves a cancelling of biases because the nonrespondents have lower earnings than the overall sample. Note also that the logarithmic transformation has reduced the overall CV^2, reflecting a reduction in the relative mean square error from the transformation. This finding both threatens common assumptions in response error models of independence between the magnitude of the response error and the true value of the measure and notes that post-survey recoding of data can have dramatic impact on the magnitudes of mean square errors.

Summary. Table 2 presents some summary data from the statistics appearing in Table 1. The proportion of the total CV^2 from different sources is displayed separately for the full set of twenty statistics and the twelve statistics that are subject to sampling variance in the design. The table shows that the bias terms form the largest single source of error. Of the biases, response bias is often very large in this survey. Of the variable errors, that associated with refusals and missing data is smallest, and the others have values quite similar to one another. These data offer a unique opportunity to compare values of different errors, but the inference to other survey error structures must be qualified by the rather low levels of refusals and missing data that the survey exhibits and the unusually low variability in true values within strata of the sample design. Hence, the relative values of sampling variance and refusal errors might be much higher in other surveys.

Of particular interest in the results presented is the effect of data transformation on the relative values of error sources and the failure of many of the biases to cancel one another. These findings both offer challenges to existing survey error models and direction for future work.

Footnotes

1. Response rate is defined as the ratio of all complete and partial interviews divided by the total number of individuals sampled (number of interviews + number of refusals + number of individuals for whom no new working number could be found). Ten interviewers were used on this study. Five were experienced Survey Research Center telephone interviewers and five had little or no previous experience. Replicate random samples of respondent phone numbers within the two design groups were assigned to interviewers to permit an analysis of interviewer variability.

References


FIGURE 1
Proportion With Paid Maternity Leave, CV² = 3.9 E-1

SQUARED BIAS 97.1%
NONINTERVIEW VARIANCE 0.3%
RESPONSE VARIANCE 2.0%
SAMPLING VARIANCE 0.1%

FIGURE 2
Proportion With Paid Sick Day Benefits, CV² = 1.6 E-2

SQUARED BIAS 96.4%
NONINTERVIEW VARIANCE 5.5%
RESPONSE VARIANCE 0.6%
SAMPLING VARIANCE 0.5%

FIGURE 3
Number of Years at Company, CV² = 8.1 E-4

SQUARED BIAS 97.4%
NONINTERVIEW VARIANCE 0.3%
RESPONSE VARIANCE 2.0%
SAMPLING VARIANCE 0.1%

FIGURE 4
Number of Years in Pension Plan, CV² = 1.9 E-2

SQUARED BIAS 96.1%
NONINTERVIEW VARIANCE 0.4%
RESPONSE VARIANCE 1.4%
SAMPLING VARIANCE 0.5%

TABLE 2
MEAN PROPORTION OF TOTAL [MSE/(ESTIMATE²)] BY ERROR SOURCE

<table>
<thead>
<tr>
<th>ERROR SOURCE</th>
<th>ALL TWENTY STATISTICS</th>
<th>TWELVE STATISTICS WITH SAMPLING VARIANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAMPLING VARIANCE</td>
<td>.105</td>
<td>.176</td>
</tr>
<tr>
<td>RESPONSE VARIANCE</td>
<td>.152</td>
<td>.129</td>
</tr>
<tr>
<td>REFUSAL/MD VARIANCE</td>
<td>.029</td>
<td>.040</td>
</tr>
<tr>
<td>NONINTERVIEW VARIANCE</td>
<td>.116</td>
<td>.156</td>
</tr>
<tr>
<td>SQUARED BIAS</td>
<td>.598</td>
<td>.499</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>