# IMPROUING stability of annual state waterfoll harvest estimates in HIGHLY-SKEWED DATA 

Lois M. Moyer and Paul $H$. Geissler, Patuxent Wildife Research Center

## ABSTRACT

Questionnaires returned by waterfowl hunters to the Waterfowl Harvest Survey Section of the U. S. Fish and Wildiffe Service sporadically contain records of extremely large haruests of ducks. Such reports can result in a state estimate for one year that is much larger than the estimates for other years. We approached the problem of unstable annual state estimates by winsorizing the extreme reports of duck haryest and then adjusting the estimates to remove the bias resulting from the
Winsorization. A simulation was conducted to find the method that best stabilizes the estimates as measured by the mean squared error: A computer program for calculating the estimators was developed and tested.

## BACKGROUND

The U. S. Fish and Wildlife Service's
Waterfowl Hunter Questionaire Survey is an annual survey of waterfowl hunters that estimates the numbers of waterfowl harvested within states, flyways, and the nation (Department of the Interior 1979).
Questionnaires are sent to approximately 100,000 potential hunters each year. On the average, about 70,000 questionnaires are returned. Hunters are contacted personally to verify figures they submitted if an unusually large harvest is reported. Harvest is estimated with a ratio estimate [ (mean harvest per hunter)*(number of duck stamps sold)].
Impossibly large haruests are deleted from the files and appropriate corrections are made as a result of the hunter contacts. After preliminary screening, however, there still remain some reports of large duck harvests that are apparently correct but that also may have a large effect on the sample mean and consequently on the estimated haruest.

## IDENTIFICATION OF LARGE HARVEST VALUES

The first step in stabilizing state harvest estimates is to identify the extreme values. The reported harvests are non-negative and usually small (Figures 1 and 2), with extreme values accurring only in the upper tail. The mode occurs at the minimum harvest of zero ducks. The extreme values in the upper tail appear to be accurate and a part of the underlying distribution. Rejection or Winsorization of values from the upper tail will result in a negative bias in the haruest estimates. However, these large values may have a great effect on the stability of the haryest estimates and should be investigated.

In this situation, we do not view the extreme values as outliers or discordant observations because we believe they are true values and part of the distribution. However, we use an outlier identification method to locate the observations that destabilize the harvest estimates. Instead of viewing these procedures as testing to see whether an extreme observation comes from the distribution, we view the test as simply identifying possible destabilizing obseruations. The significance level controls the number of extreme observations identified.

We require an cutlier identification procedure that identifies multiple outliers in the upper tail of a gamma-like distribution and that is computationally feasible with fairly larqe sample sizes (up to 3000 per state).

In a review of outlier detection procedures. Barnett and Lewis (1978) suggested several methods that are suitable for multiple upper outliers in a gamma distribution Unfortunately, most of the procedures are appropriate only for small or moderate sample sizes. Because our data look like a gamma distribution, we used a square root
transformation on the observations and then applied an outlier detection procedure for a normal sample with unknown mean and variance (Barnett and Lewis 1978).

The generalized extreme studentized deviate (ESD) procedure for multiple outliers (Rosner 1975. 1983) was selected for identification of extreme observations. The ESD procedure is based on the extreme studentized deviates $R_{i}, i=1,2, \ldots k$ that are computed from the
successively reduced samples of size $\mathrm{n}_{\mathrm{i}}$ n-1,....n-k+1, respectively. If the data are ordered such that $\left.x_{(i)}\right\rangle x_{(i+1)}$ then
$\left.R_{i}=\left(x_{( } \quad\right)^{-\bar{x}_{i}}\right) s_{i}$ where
$\bar{x}_{i}=\left(\sum_{j=1}^{n} x_{(j)}\right) \cdot(n-i+1)$ and
$s_{i}^{2}=\Sigma_{j}{ }_{j}^{=}\left(x_{(j, j)}-\bar{x}_{i}\right)^{2 /(n-i)}$.
We declare $x_{i}$ to be an extreme value if
$R_{i}$ > $\lambda$ where $\lambda$ is a calculated
percentage point for ( $n-i+1$ ) observations for the ESD procedure (Rosner 1983). Approximate percentage points are prouided for up to 10 outliers and a maximum sample size of 500. For values of $n$ greater than 500 , the normal distribution can be used to obtain approximate percentiles, with

$$
\begin{equation*}
\lambda=\left[z_{p}(n-i-1)\right] /\left[\left(n-i-2+z_{p}^{2}\right)(n-i)\right]^{1 / 2} \tag{1}
\end{equation*}
$$

where $i=0,1, \ldots, k-1, p=1-[\alpha /(n-i)], \alpha$ is the probability of a Type I error, and $z_{D}$ is the $0^{\text {th }}$ percentile of a normal distribution. This procedure is appropriate for normally distributed data and is an improvement ouer Rosner's earlier ESD procedure. The procedure controls the Type I error under both the hypothesis of no outliers and the alternative hrpothesis af 1,2.....k-1 outliers.

The ESD procedure appears to perform better than other procedures when outliers are all on the same side of the mean: fur thermore, it is computationally simole (Rosner 1975). Simonoff (1982) notes that the sequential ESD techniques are preferable over other methods such as Johns' adaptive estimator because the ESD procedure is consistently effective for both large and small data sets. He demonstrates that the ESD technique is the preferred method over robust estimators for asymmetric outliers.

Rosner"s ESD test statistics for the ESD procedures are not prone to the effect of masking, which is the inability of a testing procedure to identify a sirigle outlier in the presence of several suspected values. Also; his procedure requires only the knowledge of $k$, the maximum number of outliers present (Beckman and Cook 1983). Rosner (1975) asserts that the greatest increase in overall power occurs when the outliers are on the same side of the mean (as in our data) and are approximately of the same magnitude. This is also the situation where the masking problem is most serious.

## mINSORIZATION

After identifying the extreme observations, we wanted to reduce the effect of these values and stabilize the waterfowl harvest estimates. Early accommodation procedures included a suggestion by Riders (1933) that an observation which differs widely from the rest should be retained tut assigned a smaller weight or be replaced with a value closer to the mean. A robust estimator of the mean can be obtained by Winsorizing, or replacing the extreme
observations with the nearest retained neighbors and taking an unweighted average from the modified sample. Dixon (1960) showed the efficiency of Winsorized estimators. Tukey (1960) favored Winsorized means on the grounds that long-tailed distributions are more common than short. We used a form of Winsorization called semi-winsorization or the S-rule (Gut tman and Smith 1969)
If $\left.\left.R_{i}=\left(x_{(i)}\right)_{i} \bar{x}_{i}\right) s_{i}\right\rangle \lambda$ and $R_{i+1}<\lambda$
for 1 亿 $i \leqslant k$, we replace
$x_{(1)} \ldots . x_{(i)}$ with $x_{i+1}+x_{i+1}$.
In other words, we replace any observation greater than $\bar{x}_{i}+\lambda s_{i}$ with that
yalue rather than its nearest neighbor. The estimates are then based on the modified sample.

## BIAS ADJUSTMENT

Winsorizing extreme values from the upper tail but not from the lower tall of the distribution biases the estimated annual state harvests to be too low. National estimates are not as seriously affected by the extreme yalues. Therefore, we adjusted Winsorized state estimates by a proportion so that they sum to the un-Winsorized national estimate as in:

$$
\begin{equation*}
A_{i}=W_{i}\left(\Sigma U_{j} / \Sigma W_{j}\right) \tag{2}
\end{equation*}
$$

where $A_{i}$ is the adjusted estimate for state $i$. $U_{j}$ is the un-winsorized state estimate over all states and $W_{j}$ is the Winsorized state estimate over all states. This adjustment takes the extreme haruest peaks that were remoued by Winsorization and redistributes the peak harvest among the states, raising the lower values and decreasing the extreme values.

## SIMULATION STUDY

A simulation was performed to compare three methads for estimating state haruects. These were the unadjusted estimates, the semiWinsorized estimates and the semi-winsorized estimates with a bias adjustment. From these three methods, we determined which method best statilizes the estimates as measured by the mean squared error (MSE).

Fifty states in the nation were classified into 15 high success states and 35 low success states. Twa empirical frequency distributions were constructed to represent these high and low success states using Waterfowl Harvest Survey data for the years 1978 through 1980. For each simulated year, a sample of 100 hunters was selected from the empirical distribution for each of the high success etates and a sample of 50 hunters was selected for each of the low success states. Each year we obtained unadjusted estimates, semi-Winsorized estimates and hias-adjusted semi-winsorized estimates. Becauce we only wanted to Winsorize the most extreme values in the tail. we set $\alpha=0.001$. Mean biases and mean squared errors were calculated separately for high and low succese
states each year. Bias and MSE estimates were standardized by dividing by the true yalue and the true value squared, respectively, One hundred simulated years constitute the independent replicates. Estimates of the mean bias and mean squared error and their standard errors were calculated from the annual means.

Because many extreme values were already removed before the data were keypunched, we also contaminated some of the samples with the most extreme observations in the two true distributions. We substituted the largest true yalue for one other value in one high and one low success state each year ireplicate).

## SIMIULATION RESULTS AND DISCUSSION

We found that our procedure only minimally reduced the MSE in the uncontaminated data (P)0.05: Table 1). With these data, extreme values do not occur frequently enough to be detected in our simulation: On the other hand, use of our procedure when it is not needed, does not increase the MSE.

As expected, Winsorization does result in a negative bias. The unadjusted estimates had the smallest bias, the bias-adjusted Winsorized values were next, and the Winsorized values had the largest bias. The adjustment removes the bias for all states combined (P)0.05); however, the high success states had a positive bias whereas the low success states had a neqative bias (P(0.01). This bias of about $1 \%$ is not considered to be serious. It occurs because proportionally more harvest is removed by Winsorization in the low success states as compared to the high success states.

The contamination assured that some extreme values occured in each replication. Both the Winsorized and adjusted winsarized estimates in the contaminated data reduced the $M S E$ ( $P<0.01$ ) but no difference could be found between these procedures $\{P 30.05$, Duncan's multiple range test). In the contaminated samples, as in the uncontaminated samples, the tias adjustment corrected any significant bias resulting from the semi-winsorization.

In conclusion, our methods to stabilize the annual waterfonl state harvest estimates appear to decrease the MSE whenever there are extreme values in the data, and do not increase it when extreme values are not present. The adjusted Winsorized estimates are recommended because they reduce the bias that is introduced by the Winsorization. The procedure daes not intraduce any serious bias into the estimates.

The authors would like to thank Christine M. Eunck and Dr. B. Kenneth williame for their reuiew of yarious drafts of the paper and many helpful suggestions.

## REFERENCES

EECKMAN, R.J. and R.D. Cook (1983). "Outlier....ss," Technometrics, 25, 119-149.

BARNETT, U, and T. Lewis (1978), Outliers in Statistical Data, John Wiley and Sons.

DIXON, W. J. (1960), "Simplified Estimation from Censored Normal Samples," Anmals of of Pathematical statistice, 31, 385-391.

GUTTMAN, I, and D.E. Smith (1969), "Investigation of Rules for Dealing with Outliers in Small Samples from the Normal Distribution. I: Estimator of the Mean," Technometrics, 11, 527-550.

HUEER, P.J. (1972), "Robust Statistics: A Feuiew (The 1972 Wald Lecture)." Annals of Mathematical Statistics, 43, 1041-1067.

RIDER, P.R, (1933), "Criteria for Rejection of Observations." Washington University Studies--New Series, Science and Tectinology, 8, 3-23.

ROSNER, B. (1975), "On the Detection of Many Outliers," Technometrics, 17, 221-227.

ROSNER. B, (1983), "Percentage Pointe for a Generalized ESD Many-0utiler Procedure," Technometrics, 25, 165-172.

SIMONOFF, J.S. (1982), "A Comparison of Robust Methods and Detection of

Qutliers Techniques When Estimating a Location Parameter," Froceedings of the SAS Users Group International Conference, 278-281.

TUKEY, J.W. (1960), "A Survey of Sampling from Contaminated Distributions," in Olkin (1960).

UNITED STATES DEPARTMENT OF THE INTERIOR (1982), Fish and Wildife Special Scientific Report, Wildiffe No, 246, Waterfowl Status Report 1979, Washington, D. C.

## RELATIVE MSE AND BIAS ESTIMATES FOR WATERFOWL HARVEST SURVEY

UNCONTAMINATED SURVEY DATA

| HIGH/LOW | METHOD | MEAN BIAS | MEAN MSE | SE BIAS | SE MSE |
| :---: | :--- | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
| H | NONE | .0082 | .0266 | .0039 | .0011 |
| H | WIN | .0016 | .0304 | .0039 | .0011 |
| H | ADJ | .0137 | .0263 | .0039 | .0011 |
| L | NONE | .0844 | .0815 | .0847 | .0021 |
| L | WIN | -.0264 | .0770 | .0046 | .0020 |
| L | ADJ | -.0147 | .0784 | .0047 | .0020 |
| COMBINED | NONE | .0063 | .0541 | .0031 | .0023 |
| COMBINED | WIN | -.0124 | .0537 | .0032 | .0020 |
| COMBINED | ADJ | -.0005 | .0523 | .0032 | .0022 |

CONTAMINATED SURVEY DATA

| H | NONE | . 0278 | . 0332 | . 8040 | . 0014 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| H | WIN | . 8101 | . 0329 | . 0840 | . 0011 |
| H | ADJ | . 8348 | . 0301 | . 8041 | . 0812 |
| L | NONE | . 8283 | . 1036 | . 0049 | . 0027 |
| L | WIN | -. 8162 | . 0820 | . 8049 | . 0023 |
| L | ADJ | . 0071 | . 0857 | . 0050 | . 0025 |
| COMBINED | NONE | . 0281 | . 0684 | . 0032 | . 0029 |
| COMBINED | WIN | -. 0031 | . 8575 | . 8033 | . 9822 |
| COMBINED | ADJ | . 0205 | . 8579 | . 0034 | . 0024 |

## EMPIRICAL PDF FOR WATERFOWL HARVEST SURVEY HIGH SUCCESS STATES



## CDF FOR WATERFOWL HARVEST SURVEY HIGH SUCCESS STATES



## EMPIRICAL PDF FOR WATERFOWL HARVEST SURVEY LOW SUCCESS STATES



CDF FOR WATERFOWL HARVEST SURVEY LOW SUCCESS STATES


