A STUDY OF THE RELATIONSHIP BETWEEN THE NUMBER OF SAMPLE PSUs AND BETWEEN-PSU VARIANCE

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I. INTRODUCTION

The relationship between the number of sample primary sampling units (PSUs) and between-PSU variance is an important relationship to understand when determining the optimal number of sample PSUs for a survey. (For the surveys considered here, PSUs are counties or contiguous groups of counties, except that minor civil divisions form PSUs in some northeastern states.) In this paper we begin with the reasons behind our research and its applicability. We describe the methodology used, present both qualitative and quantitative results, and discuss some of the theoretical issues behind the research.

II. BACKGROUND

In the course of our redesign research for the National Crime Survey (NCS), we surmised that there should be some optimal number of sample PSUs at which the opposing costs of field inefficiencies and between-PSU variance are in balance. To determine this optimal number of sample PSUs, we first needed to be able to predict the between-PSU variance for designs other than the current design.

We were not interested in predicting the between-PSU variance for any possible design. Surveys like NCS use stratified multi-stage designs with constraints on stratum sizes. The strata are constrained to be roughly equal in size. In such designs, there is a strong link between the number of sample PSUs and the size of the nonself-representing (NSR) population. As the number of sample PSUs decreases, fewer PSUs can be self-representing (SR) which causes the population in NSR areas to increase. Since between-PSU variance is sensitive not only to the number of sample PSUs but also to the size of the NSR population, we theorized that, within a suitably restricted class of stratifications, the between-PSU variance, for the "best" design for a given number of PSUs, should be approximately predicted by some function $f(N,m)$, where N is the size of the NSR population and m is the number of NSR strata. (This assumes one sample PSU per NSR stratum.)

We tried to derive the correct function theoretically, but realized that this could be done only by making dubious simplifying assumptions about the distribution of the characteristics of interest in the population. The actual relationship in a given situation depends strongly on the characteristics of the PSUs being stratified and the method of stratification used to select the "best" design. Therefore, we performed an empirical investigation. At first, the study was directed almost exclusively at NCS, but we later expanded it to cover a broader class of stratification problems.

The results presented in this paper relate to problems involving unbiased estimators of the prevalence of demographic characteristics based on stratified multi-stage designs where 1) the stratifications are formed with a multivariate method that does not depend on the number of NSR strata or the NSR population (a detailed discussion of the method follows in Section III), 2) there are 3 to 50 total PSUs per NSR stratum, 3) one PSU is selected per stratum with probability proportionate to size, and 4) the SR PSUs were no smaller than 75 percent of the average NSR stratum size. If a problem does not meet all of these conditions, we would not expect the relationship to be the same as presented here. We are, in fact, stretching our results to say that they apply to the entire class described above. Nevertheless, it was our aim to cover the entire class and provide general guidance on estimating between-PSU variances. The details of the problems that we selected within this class are described in the following section.

III. METHODOLOGY

Our analysis was based on nine different national sample PSU designs. They varied primarily in their number of sample PSUs. One of the designs, the 356 sample PSU design, was chosen because we wanted a design with about the same total number of sample PSUs as the current NCS design. The range of the designs was quite extensive. The smallest design consisted of 86 sample PSUs, while the largest consisted of 1,015 sample PSUs. The sample designs are presented in Table 1 along with the number of SR PSUs and NSR strata for each design. As Table 1 shows, more designs were chosen with 234 or more sample PSUs. There are basically two reasons for this. First, most of the more plausible sample designs are in that range of possible designs. We wanted a sufficient number of data points to be able to analyze that range accurately. Secondly, we had a prior interest in the behavior of the relationship as the function $f(N,m)$ approaches zero. So, we selected several designs with relatively large numbers of strata.

Due to cost considerations, the study was performed only in two of the nine geographic divisions defined by the Office of Management and Budget. The two divisions are the South Atlantic Division and the West North Central Division. The South Atlantic Division includes Florida, Georgia, South Carolina, North Carolina, Virginia, West Virginia, Maryland, Delaware and the District of Columbia. The West North Central Division includes North Dakota, South Dakota, Minnesota, Nebraska, Iowa, Missouri and Kansas. In selecting these two divisions, we considered the following:

1. The number of PSUs in each division should be large enough so that the stratification problems would be fairly continuous in nature, rather than very discrete, but not so large so as to make the stratification costs of nine designs prohibitive.
2. One division should be relatively more heterogeneous with respect to demographic characteristics such as race, occupation, population size, etc., and the other should be relatively more homogeneous.

Based on the national sample design, the characteristics of each design for the South Atlantic and West North Central Divisions were computed. In particular, the NSR population size

(N) and the number of strata to be formed (m) were determined for each design in both divisions. As stated in Section II, we theorized that between-PSU variance should be some function of N and m. Our goal was to determine what transformation of the function variables N and m would result in that function possessing a linear relationship with between-PSU variance. In other words, what linear model "best" predicts the between-PSU variance? Our preliminary work on this subject, both theoretical and empirical, suggested three candidate functions of N and m; N/m, N^2/m, and $(N/m)^2$. N/m, N^2/m, and $(N/m)^2$ correspond to average NSR stratum size, average NSR stratum size times NSR population, and average NSR stratum size squared, respectively. The values for these three functions were computed for both geographic divisions. The results are given in Table 2.

As stated earlier, the actual relationship of between-PSU variance and the number of PSUs in a given situation will depend on the heterogeneity of the PSUs being stratified and the method of stratification. In our study we performed a multivariate stratification utilizing a modified Friedman-Rubin algorithm (Shoemaker (1983)). This modified algorithm was developed at the U.S. Bureau of Census as a part of the demographic surveys redesign research.

The modified version of the Friedman-Rubin algorithm minimizes the between-PSU variance for one sample PSU per stratum with probability proportionate to size (PPS).

To express the between-PSU component of variance mathematically, let us define,

m = the number of strata,

n_h = the number of PSUs in the h^{th} stratum,

N = the total number of PSUs,

P_{hi} = the population of the i^{th} PSU in the h^{th} stratum,

$P_{h.}$ = the population of the h^{th} stratum,

N = the total population,

U_{hi} = the population with a certain characteristic of the i^{th} PSU in the h^{th} stratum,

$U_{h.}$ = the population with a certain characteristic in the h^{th} stratum,

$U_{..}$ = the total population with a certain characteristic.

Then the between-PSU variance for estimated populations with a certain characteristic is given by the following formula:

$$(1)\quad Var_B = \sum_{h=1}^{m} \sum_{i=1}^{n_h} \frac{P_{hi}}{P_{h.}} \left(\frac{P_{h.}}{P_{hi}} U_{hi} - U_{h.} \right)^2.$$

Let the scaling factor for a characteristic j equal $\frac{1}{V_j}$, where V_j is the unstratified between-PSU variance for characteristic j as defined below.

$$V_j = \sum_{h=1}^{1} \sum_{i=1}^{n} \frac{P_{hi}}{N} \left(\frac{N}{P_{hi}} U_{hi} - U_{..} \right)^2$$

Formula (1) above, is in terms of one characteristic. For multivariate stratifications the criterion function that the modified Friedman-Rubin algorithm minimizes is presented below.

$$(2)\quad Betvar = \sum_{j=1}^{C} \frac{Var_{B,j}}{V_j}, \text{ where C = number of characteristics.}$$

Beginning with an initial stratification the algorithm iteratively transforms the strata to reach a local minimum of the between-PSU variance. This algorithm allows users to choose options. In particular, the following options were chosen for this study.

1. Size constraints: The user could assign the minimum and/or maximum size of the stratum. Initially, we utilized this option by constraining the stratum size within the South Atlantic Division. According to this constraint a stratum was not permitted to vary more than 20 percent from the average NSR stratum size. An early examination of between-PSU variances indicated that the size constraints had negligible effect on our study. Therefore, the size constraints were dropped in later analyses to reduce costs.
2. Constraints on the iterative procedure: One could limit the number of iterations performed to reduce the between-PSU variance. For our study, a maximum limit of 10 iterations was used. Convergence was almost always obtained.
3. Random clustering: According to this option, the PSUs are placed randomly into strata.
4. Scaling of chacteristics: The program permitted "preference factors" which would give greater importance to some characteristics. In this research all characteristics were considered equally important.

The set of characteristics whose Betvar is being minimized is referred to as the "stratifiers". The stratifiers for the nine designs were chosen from the list of 1980 redesign stratifiers for the NCS and the Current Population Survey (CPS). Stratifications were performed using CPS stratifiers in addition to NCS stratifiers to enable us to generalize our results to the larger class of stratifications described in Section II.

Small sets of items that were considered to be correlated with the items to be estimated by NCS or CPS were used to evaluate the stratifications. These sets of items are referred to as the "evaluators". The stratifiers could not be used as evaluators because these items were used by the Friedman-Rubin algorithm to form the strata; the estimated variances on these items would be seriously negatively biased. Some evaluators are 1970 decennial census sample estimates while the others are estimates from the Uniform Crime Reports (UCR), which are not subject to sampling error. Thus, formula (1) is biased for the 1970 decennial census evaluators because they were calculated using sample estimates at the PSU level, but not for the UCR evaluators. Based on a small study, the relative bias on these estimates is not believed to have a significant effect on our results. The results of this small study of the relative bias are included in the Appendix of the full paper.

In the South Atlantic Division two sets of stratifications were produced; one based on the NCS stratifiers and the other based on the CPS stratifiers. In this division, the evaluators for the NCS stratification were the UCR items. The evaluators for the CPS stratification were the 1970 decennial census sample estimates and some of the UCR items. In the West North Central Division, only one set of stratifications was produced; it was based on NCS stratifiers. The stratifications were evaluated using the same items as used with the CPS stratifiers in the South Atlantic Division.

Within each set of stratifications, three stratifications were performed for each of the nine designs, each with a different random start. Then the between-PSU variances for the corresponding set of evaluators for each stratification were calculated using equation (2). Note, three replications were performed for each design because we did not have any prior knowledge as to the variability of between-PSU variance of the evaluators.

IV. RESULTS

In practice, one would like to select the stratification with the minimum value of Betvar. Hence, if we could have taken sufficient random starts for each design, using the minimum value of the between-PSU variance would have been the appropriate approach. However, since cost restrictions required a small number of random starts, we would have had to deal with the distribution of the minimum observation, which would have complicated the analysis. Also, it was observed (earlier in the analysis) that the graph of minimum value of the between-PSU variances had the same form as the average value of the three stratifications. Therefore, we decided to use the average between-PSU variance in our analysis.

A. Graphs

Our initial analysis of the data was to graph the average of the between-PSU variances of both the initial random stratifications and the locally optimal unconstrained stratifications for one of the evaluation items. As stated in the previous section, the constrained stratification was negligibly different from the unconstrained stratification, so the constrained stratification was dropped from the analysis. The item chosen for display was the UCR - Number of Reported Crimes (except assault and burglary). The graphs for this item were typical of what we observed with all of the items. This item was chosen because it is correlated with NCS characteristics of interest. The purpose of examining these graphs was to visualize the relationship between the functions and between-PSU variances. Our goal was to determine which single function, if any, was the "best" linear predictor of between-PSU variance.

Let us first examine Figures 1a-c. The graphs are based on between-PSU variances for the UCR item from the South Atlantic Division stratifications using NCS stratifiers. The X-axes for the graphs are: 1a) Average NSR Stratum Size, 1b) Average NSR Stratum Size Times NSR Population, and 1c) Average NSR Stratum Size Squared. The Y-axis of the graphs is based on between-PSU variances. The between-PSU variances have been standardized in order to make the graphs easier to interpret. The standardization was accomplished by dividing each variance by the between-PSU variance of the random stratification for the 356 sample PSU design. This design was chosen because of its close relationship with the current NCS design. This standardization method does not affect the look of the graphs, only the units of the Y-axis. Also, for each of the nine designs, the average value of the between-PSU variances of the three stratifications was used in the graphs. As expected, between-PSU variance increases as the value for the functions f(N,m) increases. The difference between the random stratification line and the unconstrained stratification line is the reduction in between-PSU variance from the modified Friedman-Rubin algorithm, compared to a random stratification.

It can be seen that the shape of the unconstrained stratification line for Figures 1a and 1b are generally the same, with graph 1b being slightly more linear. Given a linear model, linearity is directly related to predictability. The more linear the graph of a function is, the better the function is as a linear predictor of between-PSU variance. The unconstrained stratification line for Figure 1c is clearly not linear. While a regression line for each of the functions seems to possess a y-intercept around zero, the y-intercept of the average NSR stratum size times NSR population functions appears to be the closest to zero. This is important because in theory the origin should lie on the graph. The random stratification lines indicate the same general results, with graph 1b appearing the most linear. However, they are subject to more variability than the unconstrained stratifications and should be referred to with this in mind. Based on Figures 1a-c both the average NSR stratum size and the average NSR stratum size times NSR population are close to being linearly related to between-PSU variance. Hence, they are good linear predictors of between-PSU variance.

The shape of the graphs for each of the functions for the other various combinations of evaluators, survey stratifiers, and geographic divisions are generally very similar to Figures 1a-c. Thus, it seems reasonable to conclude that the relationship between the number of sample PSUs and between-PSU variance does apply to the class of stratifications as described in Section II. Our general conclusion is that both the average NSR stratum size and average NSR stratum size times NSR population are reasonable linear predictors of between-PSU variance, with the average NSR stratum size times NSR population appearing to be a slightly better predictor. As a linear predictor, the average NSR stratum size squared did not seem to be an appropriate function for predicting between-PSU variances. In the following paragraphs we present a more quantitative approach to analyzing the data.

B. Quantitative Analysis

After examining the graphs, we wanted to perform a more quantitative analysis of the shape of the curves. Our approach was to perform regressions using a least-squares method.

One of the assumptions when performing a least-squares regression is that the variance of the dependent variable is the same for any value of the independent variables. This assumption of homoscedasticity seemed to be violated upon examination of the data, so a weighted least-squares regression procedure seemed appropriate. In addition, we believed that two sources of error needed to be considered for our experimental design. The model is given below:

$$Y_{ij} = A + BX_i + \alpha_i + \varepsilon_{ij},$$

where $\alpha_i \sim N(0,\tau_i^2)$, $\varepsilon_{ij} \sim N(0,\sigma_i^2)$, and the $\{\alpha_i\}$ and $\{\varepsilon_{ij}\}$ are both mutually independent.

The sum $A + BX_i$ represents the theoretical line corresponding to $f(N,m)$. The ϵ_{ij} represents the variation in the observations due to choosing a random start for the stratification. However, not all of the variation of the observations from the theoretical line can be explained by the ϵ_{ij}. Consider the particular group of PSUs being stratified for a design as one of infinitely many unique groups that could have been stratified. In other words, the particular group of PSUs are part of some "superpopulation". The α_i represents the deviation from the theoretical line at the i^{th} design for the group of PSUs that was actually chosen from the "superpopulation". However, we have only one observation of α_i for each design. With only one observation it was impossible to estimate τ_i^2, the variance of α_i, directly. So, we developed an ad hoc procedure, namely an iterative weighted least-squares procedure, to estimate the "best" regression line.

The procedure first involves performing an unweighted least-squares regression on the observed averages. From this regression line, the average of the squared residuals was computed for each design. The inverses of these values were the weights to be applied to the data. A weighted least-squares regression using these weights was performed to compute a new regression line. Based on this new regression line a new set of weights was computed. This process was continued until the regression coefficients converged to within certain limits.

For our analysis, the dependent variable was the average of the between-PSU variances of the unconstrained stratifications for each design and the independent variables were the average NSR stratum size, average NSR stratum size times NSR population, and average NSR stratum size squared. Let us examine the results of the regressions under several models for UCR item from the South Atlantic Division stratifications using NCS stratifiers. Figures 1a-c correspond to this case. The regression results are given in Column 1 of Table 3 under Two Parameter Models. For compactness of presentation, we present only the proportion of the total variance (r^2) explained by each model.

The r^2 for all three models was high. The model with the average NSR stratum size times NSR population (N^2/m) gave the highest value of r^2. These results seem to agree with our general observations of Figures 1a-c. However, these are two parameter models, with a constant and one independent variable. In keeping with our objectives we really needed to determine which one parameter model is "best", that is, gives the highest r^2 value. Note, the regression line from a one parameter model is forced to pass through the origin. The results are presented in Column 1 of Table 3 under One Parameter Models. As before, the highest value of r^2 belonged to the model with the average NSR stratum size times NSR population (N^2/m) as the independent variable. Notice the very small reduction in r^2 when the constant term was dropped from the N^2/m model, while the reductions in r^2 for the other models were relatively much larger. Clearly, the loss in assuming the "best" regression line passes through the origin is minimized with the N^2/m model.

The results from the stratifications for the other combinations of geographic divisions and survey stratifiers are given in Columns 2-5 of Table 3. The results were generally the same. The value of r^2 for the N^2/m two parameter model was the highest for each combination. In addition, r^2 for the N^2/m one parameter model was still the highest, along with having the smallest reduction in r^2 due to dropping the constant term. Consequently, we believe that the average NSR stratum size times NSR population to be the "best" of these three linear predictors of between-PSU variance.

One of the reviewers suggested another possible model which involved a log transformation. The model had several advantages over the one we used. Unfortuneately, we did not have did not have time to pursue a more detailed study of this model.

V. DISSCUSSION: SOME SIMPLE THEORETICAL EXAMPLES

In Figure 1 it seems that the between-PSU variance increases linearly with either the average NSR stratum size or the average NSR stratum size times NSR population. To try to understand the implications of this relationship, we considered some simple idealized models for the population to see what functions $f(N,m)$ are found for models corresponding to totally ineffective stratifications and to very effective stratifications.

In these examples, the universe is assumed to contain J NSR PSUs all having the same population. The PSUs actual numbers (U_j) with a certain characteristic of interest are uniformily spaced over an interval by defining $U_j = j$, $j = 1,2,\ldots,J$. These PSUs will be divided into H strata, of which H-L have K PSUs and L have K+1 PSUs, so that $J = (H-L)K + L(K+1) = HK + L$. The examples will differ regarding how the strata are formed. Note that here H takes the place of m, and N is proportional to J. First expressions for the between-PSU variance under the two models will be derived.

Example 1 (Ineffective Stratification)

In this example, the strata are formed by selecting a simple random sample of either K or K+1 PSUs without replacement. (After the first stratum is selected, the second is selected from the remaining PSUs, etc.)

Conditional on the particular division into strata, the between-PSU variance, corresponding to (1), is

$$(3)\quad Var_B = \sum_{h=1}^{H-L} \sum_{i=1}^{K} \frac{1}{K} (KU_{hi} - U_{h.})^2 + \sum_{h=H-L+1}^{H} \sum_{i=1}^{K+1} \frac{1}{(K+1)} [(K+1)U_{hi} - U_{h.}]^2,$$

where U_{hi} is the number with the characteristic of interest in the i^{th} PSU in the h^{th} stratum, and $U_{h.}$ is the total for stratum h. Taking the expectation over all possible divisions into strata,

$$(4)\quad E\ Var_B = \left[K + \frac{L}{H} - K\left(\frac{H-L}{J} + \frac{L}{J-L}\right)\right] \frac{J^2-1}{12} \left(\frac{J}{J-1}\right) \cdot J,$$

using the fact that $E \frac{1}{K-1} \sum_{i=1}^{K} \left(U_{hi} - \frac{U_h}{K}\right)^2$ is equal to the variance of a random value U_j

selected at random without replacement from $(1,\ldots,J)$, namely $\frac{J^2-1}{12}\left(\frac{J}{J-1}\right)$ and that $J = HK+L$.

Example 2 (Effective Stratification)
In this example, the strata are formed by taking consecutive strings of K or K+1 PSUs in increasing order. Thus, the first stratum would consist of the PSUs with U_j equal to $1,\ldots,K$, the second with U_j equal to $K+1,\ldots,2K$, and so forth for the other strata. Here the strata are non-random, so $E\ Var_B = Var_B$. Using (3),

$$(5)\ Var_B = \frac{J}{12}[(K + \frac{L}{H})^3 - (3K^2(\frac{L}{H}) + 3K(\frac{L}{H})^2 + (\frac{L}{H})^3) + (\frac{L}{J})K(3K^2 + 6K + 2) - (\frac{H}{J})K^2].$$

A closer inspection of the expressions (4) and (5) shows that the ineffective stratification is roughly a linear function of the average stratum size $\frac{J}{H}$, while the effective stratification example is convex, varying basically as $(\frac{J}{H})^3$. Note that for the case of one PSU in each stratum ($K=1$, $H=J$, $L=0$), both (4) and (5) are equal to zero. For one stratum consisting of J PSUs ($K=J$, $H=1$, $L=0$), both (4) and (5) pass through the point $J^2\left(\frac{J^2-1}{12}\right)$. Thus, if the two functions (4) and (5) are graphed against $\frac{J}{H} = K + \frac{L}{H}$, the graphs will have the same beginning and end points. Between these points, the two functions have a very different form. Let J be held fixed at a very large value. As $\frac{J}{H}$ increases, H and L become negligible compared to J. Consequently, for Example 1,

$E\ Var_B \doteq \left[\frac{J}{H}\right]\frac{J^2-1}{12}\left(\frac{J}{J-1}\right)\cdot J$, in other words, $E\ Var_B$ increases roughly linearly with $\frac{J}{H}$ once $\frac{J}{H}$ becomes large enough. For Example 2, $Var_B = \frac{J}{12}\left(\frac{J}{H}\right)^3$, which increases roughly proportional to $\left(\frac{J}{H}\right)^3$, ignoring terms containing $\frac{L}{J}$ or $\frac{H}{J}$ and leaving out lower-order terms in K. (These lower order terms cause the function to oscillate about the basic cubic equation, as L goes from zero to K-1 and back to zero. The amplitude of the oscillation decreases as $\frac{J}{H}$ increases.)

Based on these simple models, an ineffective stratification would produce roughly a straight line when plotting Var_B vs the average NSR stratum size while an effective stratification should generally follow a convex (cubic) curve.

One important difference between our simple models and the real situation shown in Figure 1 is that the NSR population is fixed in our examples while in the actual study it increases as 1/H increases. One way to correct for this would be by plotting Var_B/(NSR Population) against the average NSR stratum size, or alternatively by plotting Var_B vs (average NSR stratum size)(NSR population). Thus, the curve in Figure 1b would seem to be the most comparable to the simple examples considered here.

The straight lines for the random stratifications in Figure 1b are therefore what would be expected for an ineffective stratification. The Freidman-Rubin stratifications produce roughly linear relationships, indicating that the stratification is fairly ineffective over the range of values shown on the X-axis. However, this stratification cannot be totally ineffective because the curve lies well below the curve for the random stratification. In fact, extending the graph indicates that the graph would begin to curve upwards toward the line for the random stratification; for H=1 the two curves meet.

The above discussion leads to the conclusion that the stratifiers are useful for splitting the population into a few large groups, but after that the stratification does not do much better than a random splitting.

This conclusion must be regarded as tentative without further corroboration. Our simple models leave out the effect of variation in PSU size. (This can be very important when size constraints are used; however, these constraints were not a major factor in this study and were omitted.) The models also assume a very uniform distribution of PSU means throughout the universe, with no natural clusterings. The reality is probably very different. The conclusion could be corroborated by using the stratifiers to divide the population into a few large groupings of PSUs and then forming strata at random within these groups. The discussion in this section suggests the conjecture that the resulting Var_B will be similar to that from the full Friedman-Rubin stratification using the same variables, for the same values of N and m.

VI. CONCLUSION

Our general conclusion, based on our qualitative, quantitative, and theoretical research is that the best linear predictor of between-PSU variance is the average NSR stratum size times NSR population (N^2/m). The graphs of the functions indicated that (N^2/m) as the X-axis resulted in the most linear graphs. The value of r^2 was the greatest when N^2/m was the independent variable for both the least-squares regression and the iterative weighted least-squares regression. The results, in Section IV, were the same for all the various combinations of evaluation items, survey stratifiers, and geographic divisions that we examined. These empirical results were further substantiated by some theoretical examples in Section V.

These results could be corroborated by examining stratifications in some other geographic divisions based on different stratifiers with different evaluators. In addition, alternative models could be used to analyze the data, such as, the log transformation model mentioned in Section IV.

VII. ACKNOWLEDGMENTS

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REFERENCES

Draper, N.R., and Smith, H. (1966). Applied Regression Analysis. John Wiley & Sons, Inc., New York.

Jewett, R.S. (Unpublished paper). "A Method of Stratification When Size Constraints are Imposed." U.S. Bureau of the Census Report.

Kleinbaum, D.G., and Kupper, L.L. (1978). Applied Regression Analysis and Other Multivariate Methods. Duxbury Press, North Scituate, Massachusetts.

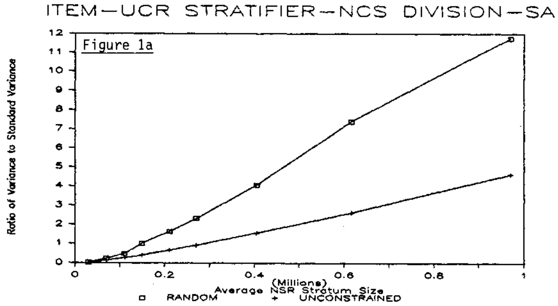
Shoemaker, H. (Unpublished Memorandum for Documentation). CPS Redesign -Stratification of PSUs-Documentation of CPS Stratification Program (ID #A1-37)." U.S. Bureau of the Census, April 13, 1983.

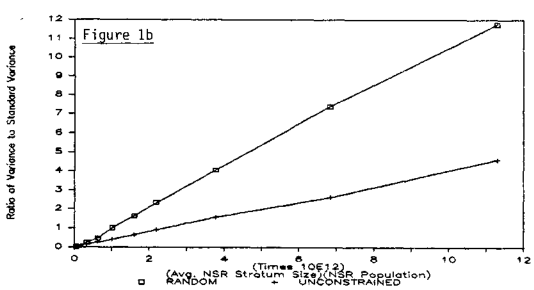
TABLE 1

NUMBER OF SAMPLE PSUS FOR VARIOUS NATIONAL DESIGNS

Total # of PSUs	Number of SR PSUs	Number of NSR Strata
1,015	618	397
601	248	353
440	169	271
356	133	223
280	98	182
234	84	150
176	60	116
129	40	89
86	23	63

ITEM—UCR STRATIFIER—NCS DIVISION—SA





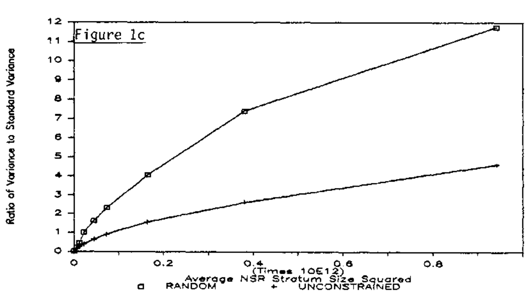


TABLE 2

CHARACTERISTICS OF VARIOUS DESIGNS

Number of SR PSUs	SR Population	Number of NSR PSUs	NSR Population	Number of NSR Strata	Average NSR Stratum Size	Average NSR Stratum Size Times NSR Population	Average NSR Stratum Size Squared
SOUTH ATLANTIC DIVISION							
140	14,572,878	120	1,656,704	55	30,122	49,903,057,157	907,328,312
53	11,500,009	207	4,729,573	68	69,553	328,953,834,740	4,837,556,393
37	10,581,307	223	5,648,275	51	110,750	625,549,225,012	12,265,671,079
26	9,489,280	234	6,740,302	45	149,784	1,009,592,690,027	22,435,393,112
20	8,641,680	240	7,587,902	36	210,775	1,599,340,465,600	44,426,124,044
17	8,131,109	243	8,098,473	30	269,949	2,186,175,497,724	72,872,516,591
12	6,911,155	249	9,318,427	23	405,149	3,775,351,380,623	164,145,712,201
7	5,141,813	253	11,087,769	18	615,987	6,829,923,410,965	379,440,189,498
6	4,584,503	254	11,645,079	12	970,423	11,300,655,409,687	941,721,284,141
WEST NORTH CENTRAL DIVISION							
50	4,720,479	333	2,623,279	87	30,153	79,098,766,803	909,181,228
15	3,653,579	368	3,690,179	53	69,626	256,932,472,680	4,847,782,503
8	3,193,111	375	4,150,647	38	109,228	453,365,013,648	11,930,658,254
6	2,995,555	377	4,348,203	29	149,938	651,961,011,352	22,481,414,185
4	2,701,164	379	4,642,594	22	211,027	979,712,684,038	44,532,394,729
4	2,701,164	379	4,642,594	17	273,094	1,267,863,473,461	74,580,204,321
3	2,459,665	380	4,884,093	12	407,008	1,987,863,702,721	165,655,308,560
3	2,459,665	380	4,884,093	8	610,512	2,981,795,554,081	372,724,444,260
2	1,866,856	381	5,476,902	6	912,817	4,999,409,252,934	833,234,875,489

TABLE 3

ITERATIVE WEIGHTED-LEAST-SQUARES REGRESSION RESULTS
(R Squared Values for Two Parameter and One Parameter Models)

Y = Average Unconstrained Between-PSU Variance
(N/m) = Average NSR Stratum Size
(N^2/m) = Average NSR Stratum Size Times NSR Population
$(N/m)^2$ = Average NSR Stratum Size Squared

UCR = Uniform Crime Reports
CLF = Civilian Labor Force

		WEIGHTED R SQUARED 1	2	3	4	5
	Item	UCR	UCR	UCR	CLF	CLF
	Sratifier	NCS	CPS	NCS	CPS	NCS
TWO PARAMETER MODELS	Division	SA	SA	WNC	SA	WNC
$Y_{ij} = A + B_1(N/m) + \alpha_i + E_{ij}$		0.99394500	0.99961585	0.99872166	0.99501664	0.99725010
$Y_{ij} = A + B_2(N^2/m) + \alpha_i + E_{ij}$		0.99926992	0.99987629	0.99876508	0.99634572	0.99821866
$Y_{ij} = A + B_3(N/m)^2 + \alpha_i + E_{ij}$		0.98296834	0.99294561	0.99323666	0.99170572	0.99044798
ONE PARAMETER MODELS						
$Y_{ij} = B_1(N/m) + \alpha_i + E_{ij}$		0.96251432	0.99032307	0.97361580	0.96821249	0.97928984
$Y_{ij} = B_2(N^2/m) + \alpha_i + E_{ij}$		0.99884749	0.99951011	0.99431721	0.99340149	0.99475746
$Y_{ij} = B_3(N/m)^2 + \alpha_i + E_{ij}$		0.95540127	0.91865705	0.99320172	0.99259272	0.96472913