SAMPLE ALLOCATION BY GEOMETRIC PROGRAMMING

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INTRODUCTION

When planning a sample, allocating it to strata is a central problem. One tries to achieve a sample with least possible cost that provides estimates with sampling errors no larger than specified goals. When only one mean or proportion is estimated, with specified sampling error, the problem is a classical one [Cochran (1953), p. 75]. When several upper bounds on sampling errors are specified, the problem is more complex, but it has been solved by non-linear mathematical programming [Kokan (1963), Jagannathan (1965), Schwartz (1978)]. An interesting and powerful method of this kind is geometric programming [Duffin, Peterson and Zener (1967), Beightler and Phillips (1976), Ecker (1980)]. In this paper, it is applied to the allocation of stratified samples when several constraints on sampling errors and sample sizes are imposed. An example from the allocation of integrated samples [Schwartz, (1978)] is used to illustrate the method. Allocations with complex variance constraints and constraints requiring equal workloads over time are also shown. They were prepared for use by the Redetermination Review System for quality control in the Supplemental Security Income program of the Social Security Administration of the United States of America.

STRATIFIED SAMPLE ALLOCATION

Optimum allocation in stratified random sampling is discussed by Cochran (1953). In his notation (p. 66), a population of N items is divided into L strata, indexed by h. The population sizes N_h are known. Also known, or estimated externally, are variances S_h^2 and costs per sampled unit C_h in each stratum. A sampling allocation consists of choices of n_h, the sample size in each stratum. Clearly, $0 \le n_h \le N_h$, since the sample cannot be larger than the population.

The cost of the entire sample is $C = a + \sum_{h=1}^{L} C_h n_h$, where a is an overhead cost. The variance of the estimate of the mean is

$$V(\bar{y}_{st}) = \frac{1}{N^2} \sum_{h=1}^{L} N_h (N_h - n_h) \frac{S_h^2}{n_h}$$

An optimal sample allocation is found by minimizing the variance V with respect to the sample sizes n_h, subject to fixed cost Co. Using a Lagrange multiplier, one minimizes $V + \lambda(C - C_o)$. An alternative is to minimize the cost C with respect to the n_h, subject to a fixed variance V_o. Then one minimizes $C + \lambda'(V - V_o)$, an equivalent problem. Actually, the constraint on the variance then specified could be found at the same minimal cost. If several variance constraints are imposed, the cost of sampling is minimized. Several constraints imply several terms with Lagrange multipliers in the quantity to be minimized. Some constraints, however, do not actively constrain the problem, even though they are satisfied. Then their Lagrange multipliers are zero. Lagrange multipliers for active constraints are like weights expressing the importance of the constraints. They are also the variables in the dual problem of geometric programming, by means of which we allocate samples.

GEOMETRIC PROGRAMMING

Developed in an engineering context by Duffin, Peterson and Zener (1967), geometric programming is a technique for minimizing a function called a "posynomial" subject to several constraints consisting of "posynomials" being less than or equal to 1. A "posynomial" is a polynomial in several variables with positive coefficients in all terms. The powers to which the variables are raised can be any real numbers. Both the cost function and the variance constraint functions are "posynomials", so geometric programming is applicable to these allocation problems.

Geometric programming transforms the primal problem of minimizing a "posynomial" subject to "posynomial" constraints to a dual problem of maximizing a function of the weights on each constraint. Usually, there are fewer constraints than strata, so the transformation simplifies. Exhibit 1, copied from Duffin, Peterson and Zener (1967, pp. 78-81) defines geometric programming concisely. Ecker (1980) reviews the extensive literature.

Application of geometric programming to allocation is best described in an example. First, allocation of sampling units to strata in an integrated sample is stated as a problem. Second, the problem is interpreted as a primal problem in geometric programming. Third, the dual problem is found by transforming from the primal problem. Fourth, the dual problem is solved, partly analytically and partly by an iterative numerical calculation.

ALLOCATION IN INTEGRATED SAMPLING

Schwartz (1978) designed an integrated sample combining quality control samples from three welfare programs of the U.S. Federal Government: Aid to Families with Dependent Children (AFDC), Food Stamps (FS) and Medicaid (Md). Seven strata were identified, including families with all possible combinations of assistance. Three variance constraints were established, to achieve acceptable estimates of the proportions of errors in each program. Population sizes, costs and variance constraint coefficients were:

Stratum	Population Size, N_h	Cost hours	Variance constraint coeff. 1 AFDC	2 FS	3 Md	Cost times variance coeff. 1 AFDC	2 FS	3 Md
1 AFDC	9,000	9.5	67.01	0	0	636.27	0	0
2 AFDC,FS	18,000	11.5	294.55	119.83	0	3387.27	1378.01	0
3 AFDC,FS,Md	2,000	13.5	2.97	2.15	0.27	40.15	29.00	3.59
4 AFDC,Md	1,000	11.5	0.66	0	0.10	7.56	0	1.10
5 FS	23,000	9.5	0	312.20	0	0	2965.87	0
6 FS,Md	7,000	11.5	0	23.65	3.38	0	271.96	38.87
7 Md	45,000	9.5	0	0	193.10	0	0	1834.47

PRIMAL PROBLEM

Minimize $g_0(t) = 9.5t_1 + 11.5t_2 + 13.5t_3 + 11.5t_4 + 9.5t_5 + 11.5t_6 + 9.5t_7$

subject to

$$g_1(t) = 67.01t_1^{-1} + 294.55t_2^{-1} + 2.97t_3^{-1} + 0.66t_4^{-1} \leq 1$$

$$g_2(t) = 119.83t_2^{-1} + 2.15t_3^{-1} + 312.20t_5^{-1} + 23.65t_6^{-1} \leq 1$$

$$g_3(t) = 0.27t_3^{-1} + 0.10t_4^{-1} + 3.38t_6^{-1} + 193.10t_7^{-1} \leq 1$$

where $t_h = n_h$ = sample size in stratum h.

DUAL PROBLEM

Maximize $\ln v(\delta)$ =

$$\sqrt{636.27\delta_1} + \sqrt{3387.27\delta_1 + 1378.01\delta_2} + \sqrt{40.15\delta_1 + 29.00\delta_2 + 3.59\delta_3} + \sqrt{7.56\delta_1 + 1.10\delta_3} + \sqrt{2965.87\delta_2} + \sqrt{271.96\delta_2 + 38.87\delta_3} + \sqrt{1834.47\delta_3} + \Lambda(\delta_1 + \delta_2 + \delta_3 - 1)$$

The δ's are weights, one for each constraint, which must add to one. Their coefficients are costs multiplied by variance constraint coefficients. This expression is derived analytically in this special case from the general dual expression in Exhibit 1. Computational methods for solving dual problems are described in Rosen (1960) Dinkel, Kochenberger and McCarl (1974) and Dinkel, Elliott and Kochenberger (1977). A slow, but simple and serviceable, method iterates to a solution:

$\delta_1 = .34232$ $\delta_2 = .54582$ $\delta_3 = .11186$

In substituting these values of the δ's into $\ln v(\delta)$, each term is the quantity used to calculate the optimum sample size and its cost:

Stratum number	$\sqrt{\ }$ term	C_h	$t_h = n_h = (\sqrt{\text{term}})(\Sigma\sqrt{\text{terms}})/C_h$
1	14.76	9.5	206
2	43.73	11.5	504
3	5.47	13.5	54
4	1.65	11.5	19
5	40.23	9.5	561
6	12.36	11.5	142
7	14.32	9.5	200
sum	132.52		1686

Optimum sample sizes are calculated by multiplying each $\sqrt{\ }$ term by their sum and dividing each by C_h, its cost. The total sample size is 1686. The total cost is 17,563 hours, equal to $(\Sigma\sqrt{\ }\text{terms})^2$ except for rounding up the sample sizes to integers. Checking the primal problem, using t_h, $g_0(t)$ = 17,563 hours, $g_1(t) = 0.99945$, $g_2(t) = 1.00062$, $g_3(t) = 0.99957$, so the variance constraints are satisfied.

Programs written in Basic are available for this special case, which was used as a test case and an expository example.

ALLOCATION OF REDETERMINATION REVIEW SYSTEM SAMPLES

Exhibit 2 shows allocations of samples developed by geometric programming. The sample size per month and region is minimized subject to several constraints on variances of proportions of defective cases taken over several months. In addition to variance constraints, sample sizes per region per month are required to be equal over months, to stabilize workloads. To achieve this, a common upper bound t* on the monthly sample size for each region is minimized. The monthly sample sizes for areas within a region are the variables t_h in the primal problem. They are not necessarily equal over months, but they are constrained by "posynomial" constraints to add to a sum less than or equal to t*. The constraints are:

$$t_1 t^{*-1} + t_2 t^{*-1} + \ldots + t_9 t^{*-1} \leq 1$$

The resulting primal geometric program is more intricate and larger than the expository problem drawn from Schwartz (1978), but the same methods are used to solve it.

In Exhibit 2, allocated sample sizes are shown. Total sample sizes per region are nearly equal, for each month, as specified. At the end of Exhibit 2, expected sampling errors are shown for each area over nine months and also over the six sets of four consecutive month "rolls". The samples were designed to produce 4% expected sampling errors over nine months. Those calculated from the allocations range from 3.96% to 3.99%.

COMPUTER PROGRAMS

Several programs have been written in APL to produce these allocations and similar ones. Write for details to Dr. Miles Davis, 1214 Bolton Street, Baltimore, Maryland 21217, U.S.A.

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EXHIBIT 1

from Duffin, Peterson & Zener (1967) pp. 78-81.

1. PRIMAL PROGRAMS AND DUAL PROGRAMS

We begin this section by presenting the most general primal program.

Primal Program A. *Find the minimum value of a function $g_0(\mathbf{t})$ subject to the constraints*

$$t_1 > 0, \quad t_2 > 0, \quad \ldots, \quad t_m > 0 \tag{1}$$

and

$$g_1(\mathbf{t}) \leq 1, \quad g_2(\mathbf{t}) \leq 1, \quad \ldots, \quad g_p(\mathbf{t}) \leq 1. \tag{2}$$

Here

$$g_k(\mathbf{t}) = \sum_{i \in J[k]} c_i t_1^{a_{i1}} t_2^{a_{i2}} \cdots t_m^{a_{im}}, \qquad k = 0, 1, \ldots, p, \tag{3}$$

where

$$J[k] = \{m_k, m_k + 1, m_k + 2, \ldots, n_k\}, \qquad k = 0, 1, \ldots, p, \tag{4}$$

and

$$m_0 = 1, \quad m_1 = n_0 + 1, \quad m_2 = n_1 + 1, \quad \ldots, \quad m_p = n_{p-1} + 1, \quad n_p = n. \tag{5}$$

The exponents a_{ij} are arbitrary real numbers, but the coefficients c_i are assumed to be positive. Thus the functions $g_k(\mathbf{t})$ are posynomials.

The posynomial to be minimized, namely $g_0(\mathbf{t})$, is termed the ***primal function***, and the variables $t_1, t_2, \ldots, t_m$ are called ***primal variables***. The constraints imposed by (1) are termed ***natural constraints***, whereas those imposed by (2) are called ***forced constraints***. Collectively, these constraints are referred to as ***primal constraints***.

The matrix (a_{ij}) is termed the ***exponent matrix***. It has n rows and m columns.

The dual program corresponding to primal program A is the following:

Dual Program B. *Find the maximum value of a product function*

$$v(\boldsymbol{\delta}) = \left[\prod_{i=1}^{n} \left(\frac{c_i}{\delta_i}\right)^{\delta_i}\right] \prod_{k=1}^{p} \lambda_k(\boldsymbol{\delta})^{\lambda_k(\boldsymbol{\delta})}, \tag{6}$$

where

$$\lambda_k(\boldsymbol{\delta}) = \sum_{i \in J[k]} \delta_i, \qquad k = 1, 2, \ldots, p. \tag{7}$$

Here

$$J[k] = \{m_k, m_k + 1, m_k + 2, \ldots, n_k\}, \qquad k = 0, 1, \ldots, p, \tag{8}$$

where

$$m_0 = 1, \quad m_1 = n_0 + 1, \quad m_2 = n_1 + 1, \quad \ldots, \quad m_p = n_{p-1} + 1, \quad n_p = n. \tag{9}$$

The factors c_i are assumed to be positive and the vector variable $\boldsymbol{\delta} = (\delta_1, \ldots, \delta_n)$ is subject to the linear constraints:

$$\delta_1 \geq 0, \quad \delta_2 \geq 0, \quad \ldots, \quad \delta_n \geq 0, \tag{10}$$

$$\sum_{i \in J[0]} \delta_i = 1, \tag{11}$$

and

$$\sum_{i=1}^{n} a_{ij}\delta_i = 0, \qquad j = 1, 2, \ldots, m. \tag{12}$$

Here the coefficients a_{ij} are real numbers.

In evaluating the product function $v(\boldsymbol{\delta})$, it is to be understood that $x^x = x^{-x} = 1$ for $x = 0$. This will make $v(\boldsymbol{\delta})$ continuous over its domain of definition.

The product function $v(\boldsymbol{\delta})$ is termed the ***dual function***, and the variables $\delta_1, \delta_2, \ldots, \delta_n$ are called ***dual variables***. Relation (10) is termed the ***positivity condition***, (11) is called the ***normality condition***, and (12) constitutes the ***orthogonality condition***. Collectively, these conditions are referred to as ***dual constraints***.

Notice how dual program B is obtained from its corresponding primal program A. The factors c_i appearing in the dual function $v(\boldsymbol{\delta})$ are the coefficients of the posynomials $g_k(\mathbf{t})$, $k = 0, 1, 2, \ldots, p$. We say that δ_i is associated with the ith term $c_i t_1^{a_{i1}} \cdots t_m^{a_{im}}$ of primal program A, so that each term of $g_k(\mathbf{t})$, $k = 0, 1, 2, \ldots, p$, is associated with one and only one of the dual variables $\delta_1, \delta_2, \ldots, \delta_n$. Each factor $\lambda_k(\boldsymbol{\delta})^{\lambda_k(\boldsymbol{\delta})}$ of $v(\boldsymbol{\delta})$ comes from a forced constraint $g_k(\mathbf{t}) \leq 1$. Notice that no such factor appears from the primal function because the normality condition forces $\lambda_0(\boldsymbol{\delta})$ to be one. The normality condition is the only part of dual program B that distinguishes between the primal function $g_0(\mathbf{t})$ and those posynomials $g_k(\mathbf{t})$, $k = 1, 2, \ldots, p$, that appear in the forced constraints. Finally, it should be noted that the coefficient matrix (a_{ij}) appearing in the orthogonality condition is simply the exponent matrix of primal program A.

2. THE DUALITY THEORY

We say that a program (either primal or dual) is ***consistent*** if there is at least one point (vector) that satisfies its constraints. Primal program A is said to be ***superconsistent*** if there is at least one vector $\mathbf{t}^*$ that has positive components and the property

$$g_k(\mathbf{t}^*) < 1, \qquad k = 1, 2, \ldots, p. \tag{1}$$

It should be noted that primal program A can be consistent without being superconsistent but that each superconsistent program is consistent.

In terms of the preceding concepts we state Theorem 1, which is called the first duality theorem of geometric programming and is the main theorem of the present formulation of geometric programming.

Theorem 1. *Suppose that primal program A is superconsistent and that the primal function $g_0(\mathbf{t})$ attains its constrained minimum value at a point that satisfies the primal constraints. Then*

(i) *The corresponding dual program B is consistent and the dual function $v(\boldsymbol{\delta})$ attains its constrained maximum value at a point which satisfies the dual constraints.*

(ii) *The constrained maximum value of the dual function is equal to the constrained minimum value of the primal function.*

(iii) *If $\mathbf{t}'$ is a minimizing point for primal program A, there are nonnegative Lagrange multipliers μ_k', $k = 1, 2, \ldots, p$, such that the Lagrange function*

$$L(\mathbf{t}, \boldsymbol{\mu}) = g_0(\mathbf{t}) + \sum_{k=1}^{p} \mu_k[g_k(\mathbf{t}) - 1] \tag{2}$$

has the property

$$L(\mathbf{t}', \boldsymbol{\mu}) \leq g_0(\mathbf{t}') = L(\mathbf{t}', \boldsymbol{\mu}') \leq L(\mathbf{t}, \boldsymbol{\mu}') \tag{3}$$

for arbitrary $t_j > 0$ and arbitrary $\mu_k \geq 0$. Moreover, there is a maximizing vector $\boldsymbol{\delta}'$ for dual program B whose components are

$$\delta_i' = \begin{cases} \dfrac{c_i t_1'^{a_{i1}} \cdots t_m'^{a_{im}}}{g_0(\mathbf{t}')}, & i \in J[0], \\[2ex] \dfrac{\mu_k' c_i t_1'^{a_{i1}} \cdots t_m'^{a_{im}}}{g_0(\mathbf{t}')}, & i \in J[k], \quad k = 1, \ldots, p, \end{cases} \tag{4}$$

where $\mathbf{t} = \mathbf{t}'$ and $\boldsymbol{\mu} = \boldsymbol{\mu}'$. Furthermore,

$$\lambda_k(\boldsymbol{\delta}') = \frac{\mu_k'}{g_0(\mathbf{t}')}, \qquad k = 1, 2, \ldots, p. \tag{5}$$

(iv) *If $\boldsymbol{\delta}'$ is a maximizing point for dual program B, each minimizing point $\mathbf{t}'$ for primal program A satisfies the system of equations*

$$c_i t_1'^{a_{i1}} \cdots t_m'^{a_{im}} = \begin{cases} \delta_i' \, v(\boldsymbol{\delta}'), & i \in J[0], \\[1ex] \dfrac{\delta_i'}{\lambda_k(\boldsymbol{\delta}')}, & i \in J[k], \end{cases} \tag{6}$$

where k ranges over all positive integers for which $\lambda_k(\boldsymbol{\delta}') > 0$.

Relation (4) provides a formula for computing a maximizing vector $\boldsymbol{\delta}'$ from the knowledge of a minimizing vector $\mathbf{t}'$ and appropriate Lagrange multipliers μ_k', $k = 1, 2, \ldots, p$. On the other hand, (6) gives a method for finding a minimizing vector $\mathbf{t}'$ from the knowledge of a maximizing vector $\boldsymbol{\delta}'$. It should be mentioned that (6) is easily reduced to a system of linear equations in the variables $\log t_j$, $j = 1, 2, \ldots, m$, by taking the logarithm of both sides of each equation. Thus a minimizing point $\mathbf{t}'$ is easily found from a maximizing point $\boldsymbol{\delta}'$. Finally, it should be noted from (5) that the numbers $\lambda_k(\boldsymbol{\delta}')$, aside from a constant factor, are the Lagrange multipliers for primal program A.

EXHIBIT 2

REGION 1
BOSTON

CHANGE CASE
DEF RATE = 63 PERCENT

SAMPLE SIZE

AREA	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	SUM	AVE
1	59	50	56	52	53	45	39	46	48	448	50
2	43	58	53	60	56	66	75	75	77	563	63
3	55	60	58	63	63	63	73	67	57	559	62
4	73	63	65	57	60	57	45	44	49	513	57
SUM	230	231	232	232	232	231	232	232	231	2083	231

POPULATION SIZE

AREA	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	SUM	AVE
1	165	180	202	243	197	186	173	172	191	1709	190
2	475	862	817	1222	394	1198	1469	1209	1268	9414	1046
3	539	784	774	1119	868	978	1225	929	822	8038	893
4	353	397	409	464	388	414	347	282	338	3392	377
SUM	1532	2223	2202	3048	2347	2776	3214	2592	2619	22553	2506

REGION 2
NEW YORK

CHANGE CASE
DEF RATE = 42 PERCENT

SAMPLE SIZE

AREA	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	SUM	AVE
1	63	62	54	47	78	62	60	60	60	546	61
2	44	46	53	84	71	65	63	82	70	578	64
3	78	66	64	54	60	57	53	58	51	541	60
4	57	64	63	77	69	59	71	57	51	568	63
5	67	42	59	62	60	74	85	59	57	565	63
6	83	55	64	74	40	52	66	60	67	561	62
7	45	88	67	30	41	74	52	65	74	536	60
8	60	75	74	71	80	56	50	58	68	592	66
SUM	497	498	498	499	499	499	500	499	498	4487	499

POPULATION SIZE

AREA	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	SUM	AVE
1	264	445	332	336	407	309	240	200	171	2704	300
2	233	420	413	767	467	413	319	344	256	3632	404
3	307	449	371	361	292	264	197	182	138	2561	285
4	276	542	459	650	419	343	330	221	171	3411	379
5	334	362	437	526	374	438	403	232	195	3301	367
6	379	433	432	575	225	280	285	217	210	3036	337
7	169	574	377	166	190	336	187	194	193	2386	265
8	331	822	676	766	631	418	298	289	297	4598	511
SUM	2343	4047	3517	4147	3005	2801	2259	1879	1631	25629	2848

REGION 3
PHILADELPHIA

CHANGE CASE
DEF RATE = 54 PERCENT

SAMPLE SIZE

AREA	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	SUM	AVE
1	71	55	56	54	50	65	57	60	65	533	59
2	56	70	51	80	63	50	56	51	50	527	59
3	30	51	51	30	36	45	65	65	71	444	49
4	59	60	67	72	71	64	50	39	50	532	59
5	39	54	60	62	52	59	58	57	51	492	55
6	67	77	57	65	84	66	55	60	61	592	66
7	86	48	60	52	75	52	57	53	39	522	58
8	54	46	61	47	30	61	64	77	76	516	57
SUM	462	461	463	462	461	462	462	462	463	4158	462

POPULATION SIZE

AREA	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	SUM	AVE
1	365	368	303	316	239	254	182	189	212	2428	270
2	263	428	256	434	279	181	167	147	149	2304	256
3	86	219	177	75	112	114	134	131	148	1196	133
4	305	399	368	423	342	252	160	123	162	2534	282
5	152	274	250	278	192	178	143	135	125	1727	192
6	597	891	539	672	708	450	809	826	346	4838	538
7	415	297	305	286	339	191	170	154	117	2274	253
8	248	278	301	247	118	218	186	218	223	2037	226
SUM	2431	3154	2499	2731	2329	1838	1451	1423	1462	19338	2149

REGION 4
ATLANTA

CHANGE CASE
DEF RATE = 48 PERCENT

SAMPLE SIZE

AREA	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	SUM	AVE
1	76	79	78	80	44	58	68	60	50	598	66
2	69	84	45	68	68	64	60	75	62	590	66
3	57	65	46	62	50	83	61	53	84	561	62
4	61	49	69	58	57	68	74	64	63	563	63
5	60	81	59	58	61	60	69	73	62	583	65
6	60	75	65	57	78	52	57	72	79	595	66
7	45	68	58	72	70	75	69	61	63	581	65
8	64	62	67	87	49	65	52	76	51	573	64
9	66	65	61	59	67	62	70	76	65	591	66
10	62	81	61	50	88	59	49	60	61	521	58
11	75	53	75	78	59	65	65	53	60	583	65
12	66	62	80	67	69	49	79	55	52	579	64
13	69	56	63	37	68	69	57	51	78	548	61
SUM	830	830	827	828	829	829	830	829	830	7461	829

POPULATION SIZE

AREA	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	SUM	AVE
1	778	1051	958	1180	502	667	617	540	412	6705	745
2	670	1057	525	884	752	696	519	644	488	6235	693
3	362	539	349	565	357	588	341	296	439	3836	426
4	415	428	558	565	434	515	443	384	346	4088	454
5	521	919	614	738	602	584	535	567	441	5516	613
6	647	1036	835	879	951	627	538	683	687	6883	765
7	372	735	575	859	661	692	506	445	426	5271	586
8	462	582	579	910	402	525	880	492	304	4586	510
9	645	822	710	836	745	680	603	654	516	6211	690
10	291	170	341	336	463	309	200	248	233	2591	288
11	650	593	779	977	583	632	502	409	425	5550	617
12	541	668	789	792	644	446	571	401	345	5192	577
13	396	419	435	302	441	446	288	257	366	3350	372
SUM	6750	9014	8047	9818	7537	7407	5998	6020	5428	66014	7335

REGION 5
CHICAGO

CHANGE CASE
DEF RATE = 48 PERCENT

SAMPLE SIZE

AREA	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	SUM	AVE
1	30	32	62	72	56	54	56	53	53	468	52
2	98	80	42	46	53	69	79	80	69	561	62
3	46	62	54	72	60	57	46	60	52	509	57
4	92	69	48	62	68	65	43	48	70	560	62
5	95	94	59	56	88	81	66	52	46	587	65
6	54	69	65	75	84	86	45	49	66	543	60
7	52	49	90	44	48	73	58	52	77	543	60
8	84	47	65	62	84	60	70	61	65	548	61
9	130	72	70	44	30	38	48	61	52	545	61
10	58	85	73	53	99	60	47	48	61	529	59
11	46	105	56	84	54	68	64	45	58	580	64
12	80	99	68	62	50	53	64	123	50	594	66
13	49	47	69	61	80	58	75	49	58	496	55
14	87	38	80	51	88	71	82	63	66	521	58
SUM	841	843	841	844	842	843	843	844	843	7584	843

POPULATION SIZE

AREA	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	SUM	AVE
1	90	138	234	290	150	142	122	115	110	1391	155
2	690	204	317	369	285	362	346	346	282	3201	356
3	203	318	244	342	190	177	119	154	126	1873	208
4	597	518	282	434	319	297	164	183	250	3044	338
5	802	914	503	505	529	173	327	256	211	4220	469
6	299	448	366	449	136	335	145	157	208	2538	282
7	298	329	526	272	199	295	195	173	246	2533	281
8	207	331	402	409	365	255	248	216	219	2652	295
9	732	472	398	267	90	150	158	198	161	2626	292
10	286	219	399	305	386	226	148	151	181	2501	256
11	344	915	428	679	292	357	282	197	240	3734	415
12	114	928	515	537	287	302	303	583	228	3792	421
13	200	225	287	270	59	167	181	117	131	1637	182
14	201	204	143	297	543	268	260	200	196	2112	235
SUM	5068	6163	5044	5425	3630	3506	2998	3046	2779	37654	4184

REGION 6
KANSAS CITY

CHANGE CASE
DEF RATE = 30 PERCENT

SAMPLE SIZE

AREA	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	SUM	AVE
1	45	56	44	45	49	49	52	40	41	421	47
2	38	57	59	55	51	41	40	44	48	433	48
3	59	49	42	61	50	55	50	53	49	468	52
4	54	36	51	35	45	52	54	60	58	445	49
SUM	196	198	196	196	195	197	196	197	196	1767	196

POPULATION SIZE

AREA	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	SUM	AVE
1	155	273	196	223	196	214	168	108	106	1639	182
2	142	302	283	290	221	193	139	129	135	1834	204
3	315	366	282	462	308	374	249	220	198	2774	308
4	229	215	280	214	223	284	214	202	189	2050	228
SUM	841	1156	1041	1189	948	1065	770	659	628	8297	922

REGION 7
DALLAS

CHANGE CASE
DEF RATE = 41 PERCENT

SAMPLE SIZE

AREA	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	SUM	AVE
1	49	59	75	81	64	70	59	60	62	579	64
2	52	65	54	51	59	66	69	73	79	568	63
3	49	52	81	60	76	62	74	65	57	576	64
4	92	63	62	64	61	71	61	67	58	599	67
5	37	69	56	72	94	61	66	69	57	581	65
6	50	66	66	54	63	61	67	79	85	591	66
7	92	73	61	67	56	60	75	39	61	584	65
8	98	71	63	69	45	67	48	66	59	586	65
SUM	519	518	518	518	518	518	519	518	518	4664	518

POPULATION SIZE

AREA	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	SUM	AVE
1	535	753	840	1052	605	624	434	427	435	5705	634
2	477	701	511	553	475	499	431	437	467	4551	506
3	523	656	901	768	716	549	545	457	400	5520	613
4	1570	1264	1071	1294	905	993	699	739	633	9168	1019
5	428	928	655	983	983	577	509	516	422	5951	661
6	656	1025	883	848	718	663	599	674	726	6792	755
7	1115	1025	740	951	577	593	611	296	472	6380	709
8	1192	1010	779	991	472	668	390	516	458	6476	720
SUM	6501	7362	6380	7440	5401	5166	4218	4062	4013	50543	5616

REGION 8
DENVER

CHANGE CASE
DEF RATE = 60 PERCENT

SAMPLE SIZE

AREA	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	SUM	AVE
1	55	55	55	55	55	55	55	55	55	495	55
2	54	54	54	54	54	54	54	54	54	486	54
3	51	52	52	52	52	52	51	51	51	464	52
SUM	160	161	161	161	161	161	160	160	160	1445	161

POPULATION SIZE

AREA	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	SUM	AVE
1	200	313	258	271	219	241	154	156	126	1938	215
2	184	288	237	249	201	221	142	143	116	1781	198
3	157	246	202	212	171	189	121	122	99	1519	169
SUM	541	847	697	732	591	651	417	421	341	5238	582

REGION 9
SAN FRANCISCO

CHANGE CASE
DEF RATE = 58 PERCENT

SAMPLE SIZE

AREA	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	SUM	AVE
1	64	57	77	68	64	87	62	82	58	564	63
2	42	37	31	49	57	30	67	31	32	376	42
3	68	58	69	58	88	55	74	65	52	577	64
4	48	89	50	49	52	99	48	49	66	550	61
5	82	49	57	65	39	34	67	105	63	561	62
6	48	58	68	71	49	60	61	62	94	571	63
7	75	85	69	55	72	96	34	41	49	576	64
8	63	61	53	63	58	74	79	54	73	583	65
9	53	54	69	74	64	58	51	56	51	530	59
SUM	543	543	543	542	543	543	543	545	543	4888	543

POPULATION SIZE

AREA	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	SUM	AVE
1	445	494	672	667	578	358	534	761	542	5051	561
2	82	89	63	144	146	61	161	73	78	897	100
3	574	558	726	688	982	643	781	733	589	6274	697
4	276	644	363	427	392	302	343	375	513	4135	459
5	531	393	458	648	329	306	540	911	553	4669	519
6	373	563	661	847	500	644	587	643	984	5802	645
7	614	870	711	690	774	1095	334	447	537	6072	675
8	614	737	645	937	738	999	951	699	1020	7340	816
9	245	306	396	522	383	374	292	341	316	3175	353
SUM	3754	4654	4695	5570	4822	5282	4523	4983	5132	43415	4824

REGION 10
SEATTLE

CHANGE CASE
DEF RATE = 65 PERCENT

SAMPLE SIZE

AREA	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	SUM	AVE
1	57	50	58	59	50	56	58	55	57	495	55
2	56	53	57	56	52	48	44	40	50	456	51
3	43	53	47	42	55	53	54	61	49	457	51
SUM	156	156	157	157	157	157	156	156	156	1408	156

POPULATION SIZE

AREA	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	SUM	AVE
1	246	354	329	398	252	272	237	200	204	2492	277
2	181	279	262	275	194	170	131	109	133	1734	193
3	143	284	217	210	211	192	166	170	134	1727	192
SUM	570	917	808	883	657	634	534	479	471	5953	661

EXHIBIT 2 (Continued)

NATION

CHANGE CASE
DEF RATE = 51 PERCENT

SAMPLE SIZE

REGION	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	SUM	AVE
1	230	231	232	232	232	231	232	232	231	2083	231
2	497	498	498	499	499	499	500	499	498	4487	499
3	462	461	463	462	461	462	462	462	463	4158	462
4	830	830	827	828	828	829	830	829	830	7461	829
5	841	843	841	844	842	843	843	844	843	7584	843
6	196	198	196	196	195	197	196	197	196	1767	196
7	519	518	518	518	513	518	519	518	518	4664	518
8	160	161	161	161	161	161	160	160	160	1445	161
9	543	543	543	542	543	543	543	545	543	4888	543
10	156	156	157	157	157	157	156	156	156	1408	156
NATION	4434	4439	4436	4439	4436	4440	4441	4442	4438	39945	4438

POPULATION SIZE

REGION	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	SUM	AVE
1	1532	2223	2202	3048	2347	2776	3214	2592	2619	22553	2506
2	2343	4047	3517	4147	3005	2801	2259	1879	1631	25629	2848
3	2431	3154	2499	2731	2329	1838	1451	1423	1482	19338	2149
4	6750	9014	8047	9818	7537	7407	5993	6020	5428	66014	7335
5	5063	6163	5044	5425	3630	3506	2998	3046	2779	37654	4184
6	841	1156	1041	1189	948	1065	770	659	628	8297	922
7	6501	7362	6380	7440	5401	5166	4213	4062	4013	50543	5616
8	541	847	697	732	591	651	417	421	341	5238	582
9	3754	4654	4695	5570	4822	5282	4523	4983	5132	43415	4824
10	570	917	808	883	657	634	534	479	471	5953	661
NATION	30326	39537	34930	40983	31267	31126	26377	25564	24524	284634	31626

9 MONTH ROLL SAMPLING ERRORS
CHANGE CASE
DEF RATE = 51 PERCENT

4 MONTH ROLL SAMPLING ERRORS
CHANGE CASE
DEF RATE = 51 PERCENT

REGION	AND	AREA	MAR-NOV	REGION	AND	AREA	MAR-JUN	APR-JUL	MAY-AUG	JUN-SEP	JUL-OCT	AUG-NOV
1	1	BOSTON	3.97	1	1	BOSTON	5.71	5.78	5.87	6.16	6.19	6.29
1	2	BOSTON	3.99	1	2	BOSTON	6.58	6.27	6.15	5.88	5.70	5.49
1	3	BOSTON	3.98	1	3	BOSTON	6.19	6.02	5.98	5.80	5.74	5.30
1	4	BOSTON	3.97	1	4	BOSTON	5.62	5.73	5.82	6.09	6.24	6.42
2	1	NEW YORK	3.98	2	1	NEW YORK	6.17	5.91	5.86	5.80	5.50	5.62
2	2	NEW YORK	3.98	2	2	NEW YORK	6.26	5.38	5.65	5.59	5.45	5.40
2	3	NEW YORK	3.98	2	3	NEW YORK	5.70	5.84	5.91	6.07	5.81	5.83
2	4	NEW YORK	3.98	2	4	NEW YORK	5.82	5.64	5.67	5.63	5.65	5.30
2	5	NEW YORK	3.98	2	5	NEW YORK	6.21	6.23	5.31	5.56	5.43	5.42
2	6	NEW YORK	3.98	2	6	NEW YORK	5.63	6.06	6.09	6.11	6.04	5.63
2	7	NEW YORK	3.98	2	7	NEW YORK	6.03	6.04	6.12	6.29	5.72	5.29
2	9	NEW YORK	3.98	2	9	NEW YORK	5.70	5.46	5.63	5.93	5.95	6.05
3	1	PHILADELPHIA	3.98	3	1	PHILADELPHIA	5.93	6.23	6.02	6.01	5.71	5.36
3	2	PHILADELPHIA	3.97	3	2	PHILADELPHIA	5.64	5.57	5.73	5.70	5.81	5.74
3	3	PHILADELPHIA	3.98	3	3	PHILADELPHIA	6.77	6.54	6.46	5.91	5.32	4.67
3	4	PHILADELPHIA	3.98	3	4	PHILADELPHIA	5.67	5.55	5.46	5.65	5.86	5.93
3	5	PHILADELPHIA	3.98	3	5	PHILADELPHIA	6.02	5.84	5.70	5.71	5.48	5.24
3	6	PHILADELPHIA	3.98	3	6	PHILADELPHIA	5.83	5.66	5.75	5.81	5.76	5.87
3	7	PHILADELPHIA	3.98	3	7	PHILADELPHIA	5.75	5.89	5.79	5.83	5.64	5.35
3	8	PHILADELPHIA	3.98	3	8	PHILADELPHIA	6.24	6.66	6.32	6.23	5.49	4.93
4	1	ATLANTA	3.99	4	1	ATLANTA	5.47	5.76	5.99	6.16	6.31	6.21
4	2	ATLANTA	3.99	4	2	ATLANTA	5.98	5.97	6.22	6.07	5.84	5.38
4	3	ATLANTA	3.98	4	3	ATLANTA	6.22	6.30	6.05	5.90	5.89	5.49
4	4	ATLANTA	3.98	4	4	ATLANTA	6.15	6.18	5.94	5.91	5.74	5.64
4	5	ATLANTA	3.99	4	5	ATLANTA	5.98	5.95	6.21	6.12	5.84	5.30
4	6	ATLANTA	3.99	4	6	ATLANTA	6.04	5.82	6.08	6.23	5.97	5.92
4	7	ATLANTA	3.99	4	7	ATLANTA	6.15	5.84	5.76	5.68	5.70	5.75
4	8	ATLANTA	3.98	4	8	ATLANTA	5.69	5.83	5.30	6.01	6.01	5.96
4	9	ATLANTA	3.98	4	9	ATLANTA	6.10	6.06	6.10	6.04	5.76	5.75
4	10	ATLANTA	3.98	4	10	ATLANTA	6.41	6.02	5.68	5.83	5.60	5.85
4	11	ATLANTA	3.99	4	11	ATLANTA	5.73	5.88	5.76	5.90	6.10	6.06
4	12	ATLANTA	3.99	4	12	ATLANTA	5.77	5.72	5.36	5.92	5.95	6.12
4	13	ATLANTA	3.97	4	13	ATLANTA	6.22	6.22	6.03	6.12	5.87	5.70
5	1	CHICAGO	3.96	5	1	CHICAGO	6.18	5.82	5.49	5.51	5.21	5.12
5	2	CHICAGO	3.98	5	2	CHICAGO	6.41	7.17	6.47	5.94	5.33	5.14
5	3	CHICAGO	3.97	5	3	CHICAGO	5.81	5.68	5.67	5.73	5.44	5.44
5	4	CHICAGO	3.98	5	4	CHICAGO	5.68	6.01	6.01	6.00	5.88	5.79
5	5	CHICAGO	3.98	5	5	CHICAGO	5.41	5.54	6.21	6.11	5.89	6.41
5	6	CHICAGO	3.98	5	6	CHICAGO	5.63	5.89	5.66	5.89	5.85	5.40
5	7	CHICAGO	3.98	5	7	CHICAGO	5.97	6.06	5.73	6.06	5.66	5.27
5	8	CHICAGO	3.98	5	8	CHICAGO	6.39	5.78	5.60	5.52	5.24	5.36
5	9	CHICAGO	3.99	5	9	CHICAGO	5.14	6.32	6.84	7.27	6.34	5.97
5	10	CHICAGO	3.98	5	10	CHICAGO	6.21	5.66	5.36	5.56	5.36	5.70
5	11	CHICAGO	3.98	5	11	CHICAGO	5.50	5.46	5.82	5.75	5.90	5.80
5	12	CHICAGO	3.98	5	12	CHICAGO	6.02	5.74	6.27	6.27	5.29	5.25
5	13	CHICAGO	3.98	5	13	CHICAGO	5.85	6.22	5.99	5.82	5.36	5.02
5	14	CHICAGO	3.97	5	14	CHICAGO	7.41	6.42	5.79	5.23	4.88	4.99
6	1	KANSAS CITY	3.98	6	1	KANSAS CITY	5.94	5.36	5.94	5.31	5.79	5.37
6	2	KANSAS CITY	3.97	6	2	KANSAS CITY	5.72	5.53	5.71	6.01	6.09	6.08
6	3	KANSAS CITY	3.98	6	3	KANSAS CITY	5.91	6.01	5.91	5.82	5.89	5.89
6	4	KANSAS CITY	3.97	6	4	KANSAS CITY	6.32	6.47	6.16	6.12	5.69	5.48
7	1	DALLAS	3.98	7	1	DALLAS	5.82	5.67	5.57	5.78	5.85	5.82
7	2	DALLAS	3.98	7	2	DALLAS	6.29	6.21	6.18	6.01	5.62	5.37
7	3	DALLAS	3.98	7	3	DALLAS	6.07	5.77	5.66	5.77	5.58	5.74
7	4	DALLAS	3.99	7	4	DALLAS	5.73	6.09	6.00	6.07	5.91	5.91
7	5	DALLAS	3.99	7	5	DALLAS	6.19	5.57	5.65	5.59	5.48	5.32
7	6	DALLAS	3.98	7	6	DALLAS	6.20	6.06	6.11	6.15	5.72	5.46
7	7	DALLAS	3.98	7	7	DALLAS	5.54	5.94	6.09	5.98	6.17	6.06
7	8	DALLAS	3.98	7	8	DALLAS	5.47	6.04	6.10	6.35	6.23	6.00
8	1	DENVER	3.98	8	1	DENVER	5.95	5.93	5.84	5.86	5.78	5.67
8	2	DENVER	3.97	8	2	DENVER	5.95	5.93.	5.88	5.85	5.70	5.63
8	3	DENVER	3.97	8	3	DENVER	5.96	5.94	5.82	5.83	5.68	5.61
9	1	SAN FRANCISCO	3.98	9	1	SAN FRANCISCO	5.38	5.86	6.10	6.30	6.02	6.10
9	2	SAN FRANCISCO	3.97	9	2	SAN FRANCISCO	6.18	5.96	6.08	5.50	5.62	5.98
9	3	SAN FRANCISCO	3.98	9	3	SAN FRANCISCO	6.17	5.89	5.87	5.82	5.67	6.03
9	4	SAN FRANCISCO	3.98	9	4	SAN FRANCISCO	6.09	6.02	5.91	5.94	5.92	5.76
9	5	SAN FRANCISCO	3.98	9	5	SAN FRANCISCO	5.96	6.51	6.76	6.60	5.99	5.72
9	6	SAN FRANCISCO	3.99	9	6	SAN FRANCISCO	6.12	6.08	6.06	6.14	6.23	5.70
9	7	SAN FRANCISCO	3.98	9	7	SAN FRANCISCO	5.69	5.69	5.59	5.96	6.11	6.42
9	8	SAN FRANCISCO	3.99	9	8	SAN FRANCISCO	6.25	6.28	6.11	5.82	5.89	5.68
9	9	SAN FRANCISCO	3.98	9	9	SAN FRANCISCO	5.33	5.63	5.65	5.85	6.03	6.22
10	1	SEATTLE	3.98	10	1	SEATTLE	6.00	6.06	5.94	5.87	5.72	5.56
10	2	SEATTLE	3.98	10	2	SEATTLE	5.75	5.77	5.78	5.94	5.93	5.33
10	3	SEATTLE	3.97	10	3	SEATTLE	6.31	6.09	6.00	5.86	5.40	5.36