Alternative Estimators to the Current Composite Estimator

Patricia Breau and Lawrence Ernst, Bureau of the Census

I. Introduction

For the Current Population Survey (CPS), a monthly household survey conducted by the Census Bureau, data are collected on labor force items, demographic characteristics, and other characteristics of the non-institutionalized civilian population (Hanson 1978). Every month, these data provide estimates of these labor force characteristics. The survey is based on a rotating sample partitioned into eight rotation groups. Households in a rotation group are interviewed for four months, dropped for eight months, and then interviewed for an additional four months.

At any period in time, each of the rotation groups in the sample should ideally provide estimates of a characteristic with the same expected values. However, this is not the case. This problem of rotation group bias is discussed by Bailar (1975).

In this paper, alternative estimators to the previously studied ratio, current composite, simple composite, and AK composite estimators are considered. These alternative estimators are compared to each other and to the previously studied estimators in terms of variance, bias, and mean squared error.

The ratio estimator only uses data from the current month in estimation. Composite estimators make use of data from the current and previous months. The current composite estimator is an average of the ratio estimator for the current month and a second estimator which is the sum of the current composite estimator for the previous month and the estimated change from the previous to the current month (Bailar 1975). The current composite estimator is one member (K = .5) of a class of estimators of the form:

$$y_h' = (1 - K)y_h + K(y'_{h-1} + \delta_{h,h-1}), \quad 0 \le K \le 1,$$

where y'_{h-1} is this estimator for month h-1, y_h is the ratio estimator for month h and $\delta_{h,h-1}$ estimates the change in level from month h-1 to month h and is based on the six rotation groups which are common to both months. Estimators in this class are called simple composite estimators. AK composite estimators, by adding an extra parameter A, define a broader class of estimators than the class of estimators described above. The extra parameter A allows for additional differential weighting of the rotation groups in sample for the first and fifth time and the other rotation groups.

The ratio, current composite, simple composite, and AK composite estimators will be discussed in more detail in Section 2. The alternative estimators which will be studied in this paper, beginning in Section 3, are the generalized composite, minimum variance, and revised generalized composite estimators.

The generalized composite estimator for the current month is a general linear combination of the rotation group totals for the current month, the rotation group totals for the previous month, and the generalized composite estimator for the previous month. This estimator, which is discussed in Section 3, allows for different weights for each of the rotation groups in the current and previous months.

Minimum variance estimators are discussed in Sections 4 and 5. In Section 4 we assume that there are no rotation group bias effects, while in Section 5 we assume that there are such effects. The minimum variance estimator for the current month is the linear combination of all of the rotation group totals available for estimation with the smallest variance. The minimum variance estimator for any previous month uses rotation group totals through the current month in estimation. This is a feature which distinguishes minimum variance estimators from all composite estimators previously discussed, none of which revise the estimates as additional data becomes available.

However, the revised composite estimator, discussed in Section 6, imitates the minimum variance estimator, in part, by using data from the current month for estimating the previous month.

In Section 7, a general discussion of results is given.

This is an abbreviated version of the paper. A longer version of the paper (including references) is available from the authors.

II. Review of Previously Studied Estimators

The ratio estimator for monthly level for month h is of the form

$$y_h = \sum_{i=1}^{8} \left(x_{h,i}\right)/8,$$

where $x_{h,i}$ is the corresponding estimate of level obtained from the rotation group which is in its ith month in sample.

The current composite estimator is of the form:

$$y'_h = .5y_h + .5(y'_{h-1} + \delta_{h,h-1}),$$

where y_h is the ratio estimator for monthly level for month h, y'_{h-1} is the current composite estimator for monthly level for month h-1, and $\delta_{h,h-1}$ is an estimate of the difference in level between months h and h-1, obtained from the rotation groups common to the two months, that is

$$\delta_{h,h-1} = 1/6\left((x_{h,2} + x_{h,3} + x_{h,4} + x_{h,6} + x_{h,7} + x_{h,8}) - (x_{h-1,1} + x_{h-1,2} + x_{h-1,3} + x_{h-1,5} + x_{h-1,6} + x_{h-1,7})\right)$$

This estimator is now being used in the CPS.

The AK composite estimator, (Gurney and Daly 1965, and Huang and Ernst 1981), is of the form:

$$y''_h = 1/8\ \big((1-K-A)\ (x_{h,1} + x_{h,5}) + (1-K-A/3)\ (x_{h,2} + x_{h,3} + x_{h,4}$$

$$+ x_{h,6} + x_{h,7} + x_{h,8})) + K(y''_{h-1} + \delta_{h,h-1}),\ 0 \leq A \leq 1,\ 0 \leq K \leq 1$$

where y''_{h-1} is the AK composite estimator for monthly level for month h-1.

Throughout this paper, we assume the following covariance structure, based on the 4-8-4 rotation plan:

1) $V(x_{jk}) = \sigma^2$ for all j,k.
2) The covariances between different rotation groups in the same month are zero. That is,

$$COV(x_{jk},\ x_{jk'}) = 0 \text{ for } k \neq k' = 1, \ldots 8.$$

3) All covariances between different rotation groups in different months are zero except for the following:

$COV(x_{j,k+1},\ x_{j-1,k}) = \rho_1\sigma^2, k = 1, 2, 3, 5, 6, 7,$

$COV(x_{j,k+2},\ x_{j-2,\ k}) = \rho_2\sigma^2,\ k = 1, 2, 5, 6,$

$COV(x_{j,k+3},\ x_{j-3,\ k}) = \rho_3\sigma^2,\ k = 1, 5,$

$COV(x_{j,5},\ x_{j-9,4}) = \rho_9\sigma^2,$

$COV(x_{j,k+2},\ x_{j-10,\ k}) = \rho_{10}\sigma^2,\ k = 3, 4,$

$COV(x_{j,k+3},\ x_{j-11,\ k}) = \rho_{11}\sigma^2,\ k = 2, 3, 4,$

$COV(x_{j,k+4},\ x_{j-12,\ k}) = \rho_{12}\sigma^2,\ k = 1, 2, 3, 4,$

$COV(x_{j,k+5},\ x_{j-13,\ k}) = \rho_{13}\sigma^2,\ k = 1, 2, 3,$

$COV(x_{j,k+6},\ x_{j-14,\ k}) = \rho_{14}\sigma^2,\ k = 1, 2,$

$COV(x_{j,8},\ x_{j-15,1}) = \rho_{15}\sigma^2.$

Estimates of correlations assumed to be zero were examined and were indeed found to be close to zero.

With the stated assumptions, we have

$$V(y_h) = V\left(\sum_{i=1}^{8} x_{h,i}/8\right) = \sigma^2/8.$$

The complicated algebraic expressions for $V(y_h')$ and $V(y_h'')$ can be found in Huang and Ernst (1981) and are not reproduced here. In Table 2.1, for the characteristics civilian labor force and unemployed, for monthly level, month-to-month change, and annual average, the variance of the current composite estimator relative to the variance of the ratio estimator is given. The variance of the simple composite estimator depends on the value of K and the variance of the AK composite estimator depends on the values of A and K. For civilian labor force and unemployed, and for monthly level, month-to-month change, and annual average, the values of A and K which minimize the variance of the simple composite and AK composite estimators were determined. These values, along with the corresponding variances (relative to the variance of the ratio estimator) are also in Table 2.1.

Now, we will examine the effect of rotation group bias on these estimators. Let T_h be the true monthly level or total for month h. Let a_{hi} be the rotation group bias for month h associated with the rotation group in its ith month in sample. Then $a_{hi} = E(x_{h,i} - T_h)$. Throughout this paper it will be assumed that

$$a_{hi} = a_i \text{ for all } h, \tag{1}$$

and unless otherwise stated, that

$$\sum_{i=1}^{8} a_{hi} = 0. \tag{2}$$

The validity of these assumptions has not definitely been determined. However, if assumption (1) is accepted but not assumption (2), then all the estimates of mean squared error that are to be presented assuming (1) and (2) retain an important meaning. Without assumption (2), these numbers are no longer estimates of mean squared error, but still estimate the expected squared deviation from the expected value of the ratio estimator. In the absence of information on which to model the rotation group bias, this is a possible criterion on which estimators may be compared.

Under these assumptions, it then follows that

$$E(y_h) = T_h,$$

$$E(y_h') = T_h + K/6(1-K)\ [(a_4 + a_8) - (a_1 + a_5)], \text{ and}$$

$$E(y_h'') = T_h + (K/6(1-K))\ ((a_4 + a_8) - (a_1 + a_5)) + (A/8(1-K))\ ((a_1 + a_5) - 1/3(a_2 + a_3 + a_4 + a_6 + a_7 + a_8)).$$

The current composite and AK composite estimators of month-to-month change (change from the previous month to the current month) are unbiased (with or without assumption (2)) while these estimators of annual average have the same bias as the corresponding estimators of monthly level.

The a_i's can be estimated by using rotation group bias indices. A rotation group bias index is computed by dividing the total number of persons in a given rotation group having the characteristic of interest by the average number of persons having the characteristic over all eight rotation groups, and then multiplying by 100.

In Table 2.2, for unemployed and civilian labor force, values of the variance, squared bias, and mean squared error of monthly level are given for the ratio estimator, current composite estimator, simple composite estimator, and for the AK composite estimator. The corresponding values for annual average are given in the longer version of the paper.

These results given in Tables 2.1 and 2.2 are discussed in detail in Huang and Ernst (1981). These results will be compared with results obtained from the alternative estimators discussed in the next few sections.

III. Generalized Composite Estimator

The generalized composite estimator is of the form:

$$y_h''' = \sum_{i=1}^{8} A_i\ x_{h,i} - K\sum_{i=1}^{8} B_i\ x_{h-1,i} + K\ y_{h-1}''',$$

where $\sum_{i=1}^{8} A_i = 1$ and $\sum_{i=1}^{8} B_i = 1.$

Different weights are allowed for each rotation group in the current and previous months in an effort to achieve a reduction in variance. The generalized composite estimator is an estimator of monthly level, but can also be used to estimate month-to-month change and annual average. A recursive method described by Gurney and Daly (1965) is used in computing the variances of these estimators of monthly level, month-to-month change and annual average. In computing these variances, estimates of ρ_1, ρ_2, ρ_3, and ρ_9-ρ_{15} are used. Therefore, these variances are functions of the unknown A_i's, B_i's and K, subject to the constraints that $\sum_{i=1}^{8} A_i = 1$ and $\sum_{i=1} B_i = 1$. For civilian labor force and unemployed, the values of the A_i's, B_i's, and K which minimize variance were determined separately for monthly level, month-to-month change, and annual average. The variances (relative to the variance of ratio estimator) corresponding to these optimal coefficients are presented in Table 2.1.

The AK composite estimator is one specific case of the generalized composite estimator. Because of this, the minimum variance of the generalized composite estimator must be less than or equal to the minimum variance of the AK composite estimator for any labor force characteristic. From Table 2.1, we can see that the reductions in variances which result from using the generalized composite instead of the AK composite are fairly small for monthly level and month-to-month change, and somewhat larger for annual average.

Now, we compare the generalized composite estimator with the previously studied estimators in terms of bias and mean squared error. Under the rotation group bias assumptions in Section 2, the bias formulas for the generalized composite estimators for monthly level and annual average are the same, that is,

$$\text{Bias} = \sum_{i=1}^{8} \frac{(A_i - KB_i)}{1 - K} a_i \qquad (3)$$

The generalized composite estimator for month-to-month change is unbiased under the assumptions in Section 2.

In Table 2.2, for monthly level and for unemployed and civilian labor force, the biases and mean squared errors of the estimators which minimize variance are given. The corresponding values for annual average are given in the longer version of the paper. Looking at the values in Table 2.2, (as well as the values for annual average in the longer version of the paper) we find that for unemployed, for both monthly level and annual average, the generalized composite estimator does fairly well compared to the other composite estimators in terms of bias and mean squared error. However, for civilian labor force, the exact opposite is true. This is unfortunate, because for annual average, the generalized composite is more effective in reducing variance, compared with the AK composite, for civilian labor force than it is in reducing variance for unemployed.

Because of our concern with the bias of the generalized composite estimator, suppose that we put restrictions on the coefficients of the generalized composite estimator so that it will be unbiased under assumptions (1) and (2). By equation (3), the generalized composite estimator is unbiased under assumptions (1) and (2) if and only if

$$\frac{(A_i - KB_i)}{(1 - K)} = \frac{(A_j - KB_j)}{(1 - K)} \text{ for all } i,j = 1, \ldots 8. \qquad (4)$$

However, since $\sum_{i=1}^{8} \frac{(A_i - KB_i)}{(1 - K)} = 1$, equation (4) is equivalent to $\frac{(A_i - KB_i)}{(1 - K)} = 1/8$, $i = 1, \ldots 8$ or $B_i = (K - 1 + 8A_i)/8K$. For monthly level, for civilian labor force and unemployed, the coefficients which minimize variance were determined under the restrictions mentioned above. The variances (relative to the variance of the ratio estimator) corresponding to these coefficients are .934 for civilian labor force and .975 for unemployed. Comparing these results to the results in Table 2.1, we see that this restricted generalized composite estimator does not do very well in reducing variance compared to the unrestricted generalized composite estimator.

IV. Minimum Variance Estimators With No Rotation Group Bias

In this section, we will discuss minimum variance estimators by using the linear models approach described by Wolter (1979). Let $\underset{\sim}{y}_h = [x_{h,1}, x_{h,2}, \ldots x_{h,8}]'$, where $x_{h,i}$, as defined in the Introduction, is an estimate of the monthly total for month h based on the rotation group which is in its ith month in sample. Suppose that h months of data are available for use in estimation. Then, let $\underset{\sim}{Y}_h = [\underset{\sim}{y}'_h, \underset{\sim}{y}'_{h-1}, \ldots \underset{\sim}{y}'_1]'$, of order 8hx1. Let $\underset{\sim}{\beta}_h = [T_h, T_{h-1}, \ldots T_1]'$ be the vector of true monthly totals. Let $\underset{\sim}{X}$ be the 8hxh design matrix of 1's and 0's which relates the estimated totals in $\underset{\sim}{Y}_h$ to the true totals in $\underset{\sim}{\beta}_h$, i.e, the matrix with 1's only in the first 8 rows of the first column, the second 8 rows of the second column, etc. The linear model to be used, is:

$\underset{\sim}{Y}_h = \underset{\sim}{X}\,\underset{\sim}{\beta}_h + \underset{\sim}{\varepsilon}_h$, where $\underset{\sim}{\varepsilon}_h$ is the vector of error terms. Initially, we assume that $E(\underset{\sim}{\varepsilon}_h) = 0$. Let $\underset{\sim}{V}$ be the variance - covariance matrix of $\underset{\sim}{Y}_h$. The covariance structure of the elements of $\underset{\sim}{Y}_h$ was given in Section 2. Under the assumption that $E(\underset{\sim}{\varepsilon}_h) = 0$, the minimum variance linear unbiased estimator of $\underset{\sim}{\beta}_h$, is $\hat{\underset{\sim}{\beta}}_h = (\underset{\sim}{X}'\,\underset{\sim}{V}^{-1}\,\underset{\sim}{X})^{-1}\underset{\sim}{X}'\,\underset{\sim}{V}^{-1}\,\underset{\sim}{Y}_h$, with the variance covariance matrix of $\hat{\underset{\sim}{\beta}}_h = (\underset{\sim}{X}'\,\underset{\sim}{V}^{-1}\,\underset{\sim}{X})^{-1}$ (Searle 1971).

The form of $\hat{\underset{\sim}{\beta}}_h$ is such that the estimate of the current month is a linear combination of all of the rotation group totals, not just the rotation group totals for the current month. In this linear combination, the sum of the coefficients of the rotation group totals for the current month equals 1 and the sum of the coefficients of the rotation group totals for every other month equals 0. In general, when there are h months of data available for estimation, then the estimate for month J (J<h) is a linear combination of all of the rotation group totals, with the sum of the coefficients of the rotation group totals for month J being equal to 1 and the sum of the coefficients of the rotation group totals for every other month being equal to 0.

For the composite estimators which have been discussed in Sections 2 and 3, an estimator for month k only uses data from months k, k-1, ...,1 in estimation, even if h (h > k) months of data are available. In section 6, a composite estimator which uses data from the current month for estimating the previous month will be introduced.

The minimum variance estimator for month-to-month change (change from the previous month to the current month) is the difference between the first two elements of $\hat{\underset{\sim}{\beta}}_h$. In general, the coefficients of the minimum variance estimator of a linear combination of months are obtained by taking this same linear combination of the minimum variance estimators for each month (Gurney and Daly 1965). This is a desirable property of minimum variance estimators because it means that both estimates of monthly level and month-to-month change can be obtained from one estimator, $\hat{\underset{\sim}{\beta}}_h$.

The name minimum variance implies that these estimators should have the smallest variance of all estimators. This statement is true, provided that all available months of data are used for estimation.

Since the computation of minimum variance estimators becomes progressively more difficult as the number of months of data increases, (unlike composite estimators which are computed recursively) in practice it would be necessary to limit the number of months of data used in the estimation to the current month and a fixed number of most recent prior months. To determine the effect such limitation has on the variance, minimum variance estimators were computed for civilian labor force and unemployed for 2, 3, 4, ...12 months of data. As the number of months used in estimation increases, the variance of an estimator should decrease. The variances of monthly level (current) and month-to-month change are presented in the longer version of the paper for up to 11 months. In general, the variances decrease quickly for the first two to six months, decrease very little from the seventh to the tenth month, and then begin to decrease somewhat more quickly again after the tenth month. This pattern of variance reduction can be attributed to the 4-8-4 rotation scheme. The minimum variance estimator can be easily compared to the estimators discussed in Section 2 (in terms of variance) by looking at Table 2.1.

It appears that the minimum variance estimator is more effective in reducing variance (compared to composite estimators) for month-to-month change than for monthly level. This is explained by the fact that the minimum variance estimator, in estimating month-to-month change, uses data from the current month in estimating the previous month, while the composite estimators previously discussed do not revise an estimate for the previous month by using data from the current month.

The assumption was made in the beginning of this section that $E(\underset{\sim}{\varepsilon}) = 0$. This assumption implies that there is no rotation group bias. However, in the presence of rotation group bias, the minimum variance estimators of monthly level, annual average, and month-to-month change are biased under assumptions (1) and (2). In the case of month-to-month change the bias results from the fact that the estimator of current monthly level and the revised estimator of previous monthly level have different coefficients (unlike the composite estimators discussed in Sections 2 and 3). In Table 2.2, for unemployed and civilian labor force, estimates of the $(\text{bias})^2$ and mean squared error of the minimum variance estimator of monthly level (after 12 months) are presented (again assuming (1) and (2)). Comparing these biases and mean squared errors with those of the optimal generalized and AK composite estimators, we see that for both civilian labor force and unemployed, the bias and mean squared error of the minimum variance estimator are greater than the bias and mean squared error of either the generalized composite or AK composite estimator. For unemployed, the bias and mean squared error of the minimum variance estimator are less than the bias and mean squared error of the current composite estimator, while for civilian labor force, the opposite is true. In the longer version of the paper, estimates of the $(\text{bias})^2$ and mean squared error of the minimum variance estimator of annual average (after 12 months) are given. In general, the minimum variance estimator of annual average does fairly well in terms of bias and mean squared error compared to the other estimators.

Now, we will discuss the possible advantages and disadvantages of using the minimum variance estimator. An obvious advantage of the minimum variance estimator is, of course, that this estimator has the smallest variance of all estimators, provided that enough months of data are used in estimation. However, it may not always be computationally feasible to use enough months of data so that the minimum variance estimator achieves an appreciable reduction in variance over other estimators.

One disadvantage in using the minimum variance estimator of month-to-month change is that it is biased under assumption (1), while the estimators of month-to-month change which have been previously discussed are unbiased.

For the generalized composite estimator, and the other composite estimators, coefficients which minimize variance were determined separately for monthly level and month-to-month change. An advantage of minimum variance estimators is that the coefficients which minimize the variance of monthly level also minimize the variance of month-to-month change. However, this is only true if the estimator for the previous month is revised using the data from the current month.

V. Minimum Variance Estimators with Assumptions about Rotation Group Bias

In this section, we find minimum variance estimators under a linear model which includes rotation group bias effects. In this section we assume (1) but only (2) when explicity stated. The linear model is:

$$\tilde{y}_h = \tilde{X}\,\tilde{\beta}_h + \tilde{\varepsilon}_h.$$

As in Section 4, $\tilde{y}_h$ is the vector of estimated monthly rotation group totals, $\tilde{V}$ is the variance - covariance matrix, and $\tilde{\varepsilon}_h$ is the vector of error terms. $\tilde{\beta}_h = [T_h, T_{h-1}, \ldots T_1, a_1, \ldots a_8]$, where $T_h, \ldots T_1$ are the true monthly totals and $a_1, \ldots a_8$ are the true rotation group bias effects. $\tilde{X}$ is now an 8h x (h + 8) design matrix where the first h columns are identical to those of the design matrix of Section 4, while the last 8 columns can be partitioned into h identity matrices of order 8. In the previous section, the minimum variance estimator of β_h was $\hat{\tilde{\beta}}_h = (\tilde{X}'\,\tilde{V}^{-1}\,\tilde{X})\;\tilde{X}'\,\tilde{V}^{-1}\,\tilde{y}_h$. However, in the present case, $(\tilde{X}'\,\tilde{V}^{-1}\tilde{X})$ is singular.

The quantities T_h, T_{h-1}, $\ldots T_1$, a_1, $\ldots a_8$ are nonestimable (Searle 1971). Imposing a nonestimable constraint on the parameters allows us to obtain a solution $\tilde{\beta}^o = (\tilde{X}'\tilde{V}^{-1}\tilde{X})^{-1}\,\tilde{X}'\tilde{V}^{-1}\tilde{y}_h$. Two different "reasonable" constraints were imposed to obtain two different solutions. The first constraint imposed was the $a_1=0$, implying that an estimate obtained from the rotation group which is in its first month in sample is unbiased. Under this constraint, for unemployed and civilian labor force, the variances (relative to the variance of the ratio estimator) of monthly level were determined when 6 months of data were used in estimation. These variances are 2.05 for civilian labor force and 2.14 for unemployed. Since it it not known whether the assumption that $a_1 = 0$ is correct, one should be cautious about assuming this model which yields such large variances.

The second constraint imposed was that the sum of the rotation group bias effects = 0. For unemployed and civilian labor force, the variances (relative to the variance of the ratio estimator) of monthly level were determined after 6 months. These variances are .904 for civilian labor force and .959 for unemployed, and are quite a bit larger than the variances of the AK and generalized composite estimators, because of the extra constraints.

It should also be noted that, although the estimator of total for month h obtained from this model will not be unbiased without the assumption that $\sum_{i=1}^{8} a_i = 0$, it will still be the MVLU of $1/8 \sum_{i=1}^{8} E(x_{hi})$, i.e., it will have the smallest expected squared deviation from the expected value of the ratio estimator among all linear estimators with that expected value. In the absence of information on which to model the rotation group bias, this is a possible criterion on which estimators may be compared.

VI. Revised Generalized Composite Estimator

The revised generalized composite estimator for the previous month h-1 is a general linear combination of the rotation group totals for month h, the rotation group totals for month h-1, the rotation group totals for month h-2, and the revised generalized composite estimator for month h-2. The form of the estimator is:

$$y''''_{h-1} = \sum_{i=1}^{8} C_i x_{h,i} + \sum_{i=1}^{8} A_i x_{h-1,i} - K \sum_{i=1}^{8} B_i x_{h-2,i} + K\, y''''_{h-2},$$

where

$$\sum_{i=1}^{8} C_i = 0,\ \sum_{i=1}^{8} A_i = 1,\ \sum_{i=1}^{8} B_i = 1, \text{ and } 0 \le K \le 1.$$

This estimator was introduced in order to imitate the minimum variance estimator by using data from the current month in estimating the previous month. For unemployed and civilian labor force, the variances (relative to the variance of the ratio estimator) of previous monthly level were determined. These variances are .663 for civilian labor force and .883 for unemployed. This revised generalized composite estimator should be particularly useful in estimating month-to-month change. Unfortunately, computational difficulties have been encountered in attempting to find the coefficients which minimize the variance of the expression for month-to-month change. Presently, these coefficients have not been found. An alternative to finding the coefficients which minimize the variance of month-to-month change is to use the coefficients which minimize the variance of monthly level (current) for the generalized composite, and the coefficients which minimize the variance of monthly level (previous) for the revised generalized composite estimator in the expression for the variance of month-to-month change. The variances which result from using these coefficients are .614 for civilian labor force and .902 for unemployed. In terms of variance, this estimator does not do as well as the generalized composite and minimum variance estimators. It does better than the AK composite estimator for unemployed, and does not do as well as the AK composite estimator for civilian labor force. It does better than the current composite estimator for both characteristics. One advantage of this estimator, like the minimum variance estimator, is that the coefficients which minimize level (current and previous) are also used in the expression for the variance of month-to-month change. For the AK and generalized composite estimators, coefficients which minimize the variance of monthly level and the variance of month-to-month change are determined separately.

One disadvantage of this estimator is that like the minimum variance estimator in Section 4, the expression for the variance of month-to-month change will in general be biased under the assumptions in Section 2 because of the difference in the form of the current month's and previous month's estimators. A second obvious problem is the requirement of revision of monthly estimates.

VII. Discussion

In the previous sections, the results of the various estimators have been discussed. Now, we will give a brief review of these results. The generalized composite estimator shows more improvement (in terms of variance) over the other composite estimators for annual average than for monthly level or month-to-month change. However, for certain characteristics, this improvement in variance may be more than offset by an increase in bias. Similarly, the reduction in variance which results from using the minimum variance estimator may be more than offset by increases in bias. Also, there are potential computational difficulties in using the minimum variance estimator with more than a few months of data. However, the minimum variance estimator does have the desirable property of yielding one estimation procedure that is optimal for both monthly level and month-to-month change.

For the generalized composite estimator, separate coefficients were determined for monthly level, month-to-month change and annual average and for unemployed and civilian labor force. In practice, we would not want to do this. For monthly level, month-to-month change, and annual average, the optimal generalized composite coefficients for unemployed were substituted into the variance expression for civilian labor force (and vice versa). The variances which result from doing this are in the longer version of the paper. From these results, the conclusion is that it would be better to use the optimal coefficients for unemployed in a general estimator than the optimal coefficients for civilian labor force.

Similarly, for the generalized composite estimator, for civilian labor force and unemployed, we looked at how well (in terms of variance) the optimal estimator for one quantity (for example, monthly level) does for another quantity (for example, annual average). These results are in the longer version of the paper From these results it can be seen that the optimal estimator for annual average does poorly for monthly level and month-to-month change. Also, the optimal estimator for monthly level does better generally than the optimal estimators for month-to-month change and annual average.

It would be better if a compromise estimator could be found--an estimator which is not necessarily optimal for monthly level, month-to-month change, annual average or any characteristic but performs well in general. This will be a subject of future research.

Table 2.1 Optimal Variances (Relative to Variance of Ratio Estimator)

Characteristic	Current Composite	Simple K	Simple Composite Variance	AK A	AK K	AK Composite Variance	Generalized Composite	Minimum Variance (12 Months)
Monthly Level								
C.L.F.	.812	.6	.789	.4	.7	.731	.727	.667
Unemploued	.996	.3	.958	.4	.5	.928	.923	.907
Month-to-Month Change								
C.L.F.	.674	.8	.607	.1	.8	.599	.592	.573
Unemployed	.923	.4	.920	.2	.5	.913	.903	.891
Annual Average								
C.L.F.	1.038	.2	.998	.7	.5	.975	.845	.956
Unemployed	1.197	.1	1.000	.5	.3	.986	.960	.978

Table 2.2 Variance, Squared Bias, and Mean Squared Error of the Optimal Estimators for Monthly Level

Characteristic and Estimator	Variance	$(Bias)^2$	Mean Squared Error
	10^9	10^9	10^9
C.L.F			
Ratio	55.255	0	55.255
Current Composite	44.890	80.656	125.546
Simple Composite, K=.6	43.584	181.476	225.06
AK Composite, K=.7, A=.4	40.366	58.243	98.609
Generalized Composite	40.170	132.569	172.738
Minimum Variance	36.874	166.796	203.67
Unemployed			
Ratio	12.983	0	12.983
Current Composite	12.938	16.384	29.322
Simple Composite, K=.3	12.437	3.009	15.446
AK Composite, k=.5, A=.4	12.054	.602	12.656
Generalized Composite	11.983	2.039	14.022
Minimum Variance	11.776	4.074	15.85

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