COMPARISON OF AUXILIARY VARIABLES FOR SATELLITE-DATA REGRESSION ESTIMATORS IN CROP SURVEYS

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The Statistical Reporting Service (SRS) of the U.S. Department of Agriculture uses data from earth-observation satellites to decrease the variance of survey estimates of crop areas. This is accomplished by using a regression estimator in which the enumerated survey item is the primary variable and some transformation of the satellite data is the auxiliary variable. The auxiliary variable currently used by SRS is the sample-unit aggregation of the results from discriminant analysis performed on the satellite data. This study investigates an alternative auxiliary variable--the sample-unit aggregation of estimated posterior probabilities. We derive formulas for the expected variance of the regression estimator in terms of population characteristics and information measures for the two auxiliary variables. The two auxiliary variables are compared using the developed formulas.

I. INTRODUCTION

Data Descriptions. Since the mid-1970's, the U.S. Department of Agriculture's Statistical Reporting Service (SRS) has been a regular user of satellite data. SRS uses these data in two different ways: in construction of area sampling frames [1,2] and as auxiliary data in survey estimation of crop areas [3,4,5,6]. The latter application is the focus of this paper.

The U.S. Landsat satellites provide the satellite data used by SRS. The current, Landsat IV satellite contains two different instruments for collecting earth observation data. Called the Multispectral Scanner (MSS) and the Thematic Mapper (TM), respectively, these two instruments operate on the same basic principle. That is, over certain intervals of the electromagnetic spectrum both measure the amount of energy originating from picture elements (pixels) on the earth's surface. MSS measures four spectral intervals; whereas TM measures seven. Pixel sizes are 0.32 hectares (0.80 acres) and 0.09 hectares (0.22 acres) for MSS and TM, respectively.

Annually SRS conducts a June Enumerative Survey (JES). The JES sample units, called segments by SRS, are randomly selected from a stratified area-sampling frame. In agricultural strata, segments are typically 1 square mile (260 hectares) in size.

Data Notation. At the pixel level, $Z(h,i,j)$ denotes the MSS or TM measurement vector for pixel j of segment i in stratum h. An information function, g, is a real-valued function over the Z measurement space.

At the segment level, $Y(h,i)$ denotes the amount of land planted to the crop of interest in segment i of stratum h, whereas $X(h,i;g)$ denotes the auxiliary variable resulting from the aggregation of $g[Z(h,i,j)]$ over pixels in segment i of stratum h. The total number of potential segments in stratum h is denoted by $N(h)$ and the number of sampled segments by $n(h)$.

At the sample level, $Y(h;s)$ and $\bar{X}(h;g,s)$ are the sample means of $Y(h,i)$ and $X(h,i;g)$, respectively. $B(h;g,s)$ is the sample regression slope for $Y(h,i)$ on $X(h,i;g)$.

At the stratum level, $Y(h)$ is the total amount of land planted to the crop of interest. This is the quantity that is estimated. $\bar{X}(h;g)$ is the stratum mean-per-segment of the auxiliary variable $X(h,i;g)$. $\bar{X}(h;g)$ is known because all the $Z(h,i,j)$'s are known.

Estimators. In geographical areas where Landsat data are not available, SRS estimates $Y(h)$ via a direct expansion estimator, denoted $DE(h)$, using only ground data. Where both Landsat and JES data are available, SRS calculates the following regression estimator:

$$Reg(h;g) = N(h)\,[\bar{Y}(h;s) + B(h;g,s)\,[\bar{X}(h;g) - \bar{X}(h;g,s)]]$$

The approximate, large-sample variance is

$$(1)\quad V[Reg(h;g)] \doteq [1-R^2(h)]\;V[DE(h)],$$

where $R^2(h;g)$ is the square of the stratum correlation between $X(h;g)$ and $Y(h)$, and $V[DE(h)]$ is the variance of $DE(h)$ for the same geographical area.

Statement of the Problem. This paper investigates how the choice of g, the information function, affects the variance of the described regression estimator. We consider two different information functions: the classification function, denoted $a(.)$, and the posterior-probability function, denoted $o(.)$. These two functions are described in detail in Section III. From equation (1) it follows that the effect of choosing between these two information functions is characterized by the following efficacy ratio:

$$E(h) = R^2(h;a)/R^2(h;o).$$

Section V presents formulas by which the asymptotic value of $E(h)$ can be computed in terms of population parameters. Section VI establishes the validity of these formulas, and Section VII presents the implications of the derived formulas. In addition to the results of this paper, Hung [7] describes related work on this problem.

II. MODEL ASSUMPTIONS

Use of Models. The direct-expansion and regression estimators described above are design-based estimators. Thus no models need be developed in order to calculate the estimates or to estimate variances after the survey data have been collected. Our purpose in using models is to obtain insight into the choice of the information function prior to performing the survey. We develop two interconnected models: a Landsat reflectance model and a super-population model for the finite geographical region of interest. The latter model is composed of two submodels: a pixel submodel and a segment submodel.

Landsat Reflectance Model. There is general consensus (within the remote sensing community) that for an individual ground cover a mixture of multivariate normal (MVN) density functions adequately models Landsat reflectance. The

individual MVN distributions in such a mixture are said to correspond to spectral classes; whereas the various ground covers are said to correspond to information classes.

Let f(.;i,j) denote the MVN density for spectral class j of information class i; f(.;i), the mixture of MVN densities for information class i; and f(.), the overall mixture density for the collection of Landsat data. Information class 1 is called the target information class or (in agricultural applications) the crop of interest. The information classes other than information class 1 are collectively referred to as the confusion information class or information class 0.

We assume (spectral class) conditional independence between the Landsat reflectances of neighboring pixels. Admittedly, in practice the reflectance density functions are not known. We assume, however, that they can be estimated with sufficient accuracy and precision that density estimation effects can be ignored.

Super-Population Model. The super-population model is a sequence of realizations of the finite, geographical region of interest. These realizations are generated by pixel and segment submodels.

For realization i' the pixel submodel generates N'(i') pure pixels. (A pixel is said to be pure if it is a member of only one information class.) The probability that a pixel is a member of information class 1 is the same for all pixels in a stratum. This common value is denoted by the super-population parameters p = p(1) = 1 - p(0). The probabilities of membership in spectral classes are also identical for every pixel in a stratum. The notation p(i,j) is the joint probability of membership in spectral class j of information class i. (Note: The stratum subscript will be omitted from model parameters when omission will not cause confusion.)

The segment submodel then groups together pixels to form sample units. M(h,i) is the number of pixels in segment i of stratum h. Y(h,i,j) indicates membership in information class 1 (1=yes, 0=no) of pixel j in segment i of stratum h. (For each pixel there are as many Y(h,i,j)'s as there are crops of interest. Though only one classification is performed, each crop of interest is a separate estimation problem with its own information class 1.) The M(h,i)'s are independently distributed with respect to each other and with respect to the Y(h,i,j)'s. The segment submodel assumes that

$$\text{corr*}\,[Y(h,i,j),\ Y(h',i',j')] = \begin{cases} c & h=h' \\ & i=i' \\ & j\neq j \\ 0 & h\neq h' \text{ or } i\neq i', \end{cases}$$

where corr* denotes super-population correlation and c is a super-population parameter.

Recall that Y(h,i) is the amount of land planted to the crop of interest in segment i of stratum h. If the unit of areal measurement is the size of a pixel, then Y(h,i) is the aggregation of Y(h,i,j) over j (i.e. pixels).

Other super-population parameters which describe the segment submodel are the following:

N(i') = number of potential segments in realization i',

n(i') = number of sampled segments in realization i',

m = E*[M(h,i)],

v = V*[M(h,i)],

and

R*(h;g) = corr*[Y(h,i), X(h,i;g)],

where E* and V* are expectations and variance, respectively, with respect to the super-population.

We assume that as i' approaches infinity that N(i'), n(i'), and N(i') - n(i') do likewise. Then equation (1) becomes exact and R(h;g) converges to R*(h;g).

III. INFORMATION FUNCTIONS

We consider two information functions: the classification function and the posterior-probability function.

Classification Function. The classification function is the indicator function for classification into information class 1. There are two variants of this function depending on the decision rule. The Bayes rules classifies into a spectral class on the basis of the maximum value of p(i,j)f(Z;i,j); the maximum-likelihood rule, on the basis of f(Z;i,j). Both rules then classify into the containing information class.

Of course, any monotone transformation of the decision criteria can be used to perform an identical classification. The advantage of the two step procedure -- first classification into spectral class, then aggregation into information class -- is that since the spectral class distributions are MVN, the decision rules simplify to a quadratic discriminant. The winning information class could be determined directly on the basis of the f(Z;i)'s but the required exponentiations increase the computational effort and for Landsat data the increases in classification accuracy are small.

Posterior Probability Function. The posterior probability function is given by o(Z)=[pf(Z;1) + (1-p) f(Z;0)] / f(Z).

Choice of Information Functions. The classification function is of practical interest because of the computational simplicity of the quadratic discriminant. Also it creates a classification map which may satisfy non-statistical, pictorial information needs. On the other hand, the posterior probability function is of theoretical interest because in limiting situations (e.g., M(h,i) always 1 or c=0) it is known that R*(h;g) is maximized by g(.)=o(.) [8, pp 264-265].

IV. INFORMATION IN REMOTELY SENSED DATA

We show in the next section that correlations involving information functions can be expressed in terms of Fisher information for p.

Marco [9] derives

I_o = information content in unclassified data

= (1 - b) / p(1-p)

and

I_a = information content in classified data

= k/ p(1-p),

where

b = $\int[f(z;1)\ f(z;0)\ /f(z)]dz$

k = $p(1-p)\ (1-d-e)^2/q\ (1-q)$

d = probability of misclassification of a pixel from the target information class into the confusion information class

e = probability of misclassification from the confusion information class into the target information class, and

q = $p(1-d) + (1-p)e$
= probability of classification into the target information class

These two information measures are equal to zero when f(.;1) and f(.;0) are identical and are equal to $1/p(1-p)$ when f(.;1) and f(.;0) have non-overlapping support.

Marco [9] shows that $I_a \leq I_o$. The quantity b is also the asymptotic error rate for nearest neighbor classification of f(.;1) versus f(.;0). The quantity k is called the reliability by Tenebein [10] and it assumes values in the unit interval.

V. MODEL FORMULAS

Pixel-level correlations. From simple moment calculations we get the following intra-segment correlations in order of decreasing absolute value:

corr*[Y(h,i,j), Y(h,i,j')] = c,

corr*(o[Z(h,i,j)], o[Z(h,i,j')] = c(1-b)
= $c\ p(1-p)\ I_o$

corr*(a[Z(h,i,j)],a[Z(h,i,j')] = ck
= $cp(1-p)I_a$ for j ≠j'.

These correlations are for the same random variable but different pixels. For the opposite situation let
r*(h;g) = corr*(Y(h,i),g[Z(h,i,j)]). For correlations between different random variables at the same pixel we obtain

$[r^*(h;o)]^2 = 1-b = p(1-p)\ I_o$.

$[r^*(h;a)]^2 = k = p(1-p)\ I_a$,

Since $I_a \leq I_o$, it follows that
$r^2(h;a) \leq r^2(h;o)$.

Segment-level correlations. If v=0, then

$[R^*(h;g)]^2 = [r^*(h;g)]^2\ t(h;g)$
$= p(1-p)\ I_g\ t(h;g)$

where

$t(h;g) = [1 + (m-1)\ c]/[1 + (m-1)cp(1-p)\ I_g]$.

Hence, $[R^*(h;g)]^2$ increases with increasing p(1-p), m, c, and I.

The sign of c determines the rate of change of $[R^*(h;g)]^2$ as a function of increasing I. Other effects of the sign of c are the following:

. If c=0, then R*(h;g) = r*(h;g).

. t(h;g) is greater or less than 1 as c is greater or less than 0.

. If c > 0, then t(h;g) decreases with increasing I. If c < 0, the opposite is true.

Let $E^*(h) = [R^*(h;a)]^2/[R^*(h;o)]^2$ be the asymptotic limit of E(h),
$e^*(h) = [r^*(h;a)]^2/[r^*(h;o)]^2$ and
u(h) = t(h;a) / t (h;o). Then
$E^*(h) = e^*(h)\ u(h)$.
The quantity e*(h) is always less than unity. The quantity u(h) has the following properties:

. u(h) is not less or not greater than 1 as c is greater or less than 0.

. u(h) is an increasing function of c and of m.

. As cm approaches infinity, u(h) approaches $1/e^*(h)$; hence, E*(h) approaches 1.

When v≠0, the expressions for R*(h;g) become very involved. The variances and covariances needed to calculate segment-level correlations for v≠0 are given in Table 1. It can be shown that if c=0 but v≠0, then R*(h;g) is an increasing function of v.

VI. SUPER-POPULATION MODEL VALIDITY

Using the derived formulas, segment-level correlations can be predicted from parameters of the reflectance and super-population models. We analyzed an SRS data set to test the validity of the assumed models, estimate model parameters, and determine the ability of the models to predict segment-level correlations.

The analyzed data set consisted of JES and Landsat data from 1981 for 41 SRS segments. These 41 segments were contained within a single Landsat pass (185 kilometers by 350 kilometers) within Iowa. The Landsat data for pure pixels of corn, soybeans, pasture, and "other" were decomposed into component MVN densities using the CLASSY algorithm [11].

Because of the large size of the pixel data set (31,576 pixels), the remainder of the analysis was performed in only one agricultural stratum -- the largest one, containing 19 segments. The data set was further reduced by selecting a systematic subsample consisting of every fourth pixel. (Pixels sorted by Landsat row and column.) The resulting number of pixels was 3724 pixels with an average of 196 pixels per segment.

First, we tested the validity of the super-population model. Instead of testing the Landsat reflectance model, we used simulated Landsat data which were mixtures of generated MVN variates. Parameters for the simulated Landsat data were those estimated by CLASSY on the larger data set. Thus, by analyzing simulated Landsat data in conjunction with actual ground data, we were able to examine the validity of the super-population model alone.

The Bayes-classification and posterior-probability functions for corn, soybeans, pasture, and "other" were evaluated for all pixels in the reduced data set. The parameters required by these functions were estimated from the larger data set.

In addition to the JES assignment of pixels to segments (which we call Assignment A), two artificial assignments of pixels were made to create data sets with different c and v values. In Assignment B the pixels in the reduced data set were randomly assigned to one of the 19 segments according to a multinomial distribution with proportions identically equal to 1/19. In Assignment C the pixels were assigned systematically to segments--every 19th pixel to segment 1, etc.

The parameters m and v were estimated from segment-size data (with the unit of measurement equal to the size of a pixel) by the method of moments. The parameters p and c were estimated

from the ground-cover label data also by the method of moments. The parameter b was estimated by $1-obs([r^*(h;o)]^2)$, where $obs([r^*(h;o)]^2)$ is the observed value of $[r^*(h;o)]^2$. This was calculated by the pixel-level product-moment formula for all 3724 pixels in the reduced data set. The estimates of d and e were the observed misclassification rates. Table 2 lists the parameter estimates.

Predicted segment-level correlations were calculated using the estimated parameters and the moment formulas in Table 1. Observed segment-level correlations were calculated using product-moment formulas for the 19 segment aggregations in the reduced data set. The observed squared segment-level correlations were adjusted to unbiasedness under the assumption that $[X(h,i;.), Y(h,i)]$ is distributed bivariate normal [12].

There was a close agreement between predicted and observed segment-level correlations. The coefficients of determination between predicted and observed values was 0.93 and 0.89 for $[R^*(h;a)]^2$ and $[R^*(h;o)]^2$ respectively. When observed values were regressed on the predicted values the slopes were not significantly different from 1.0. The 95 percent confidence intervals for the corresponding intercepts were [.02, .22] and [.01, .25] for $[R^*(h;a)]^2$ and $[R^*(h,o)]^2$, respectively.

VII. MODEL IMPLICATIONS

Sensitivity Analysis. By evaluating the model formulas at several different parameter values, the importance of different parameters in predicting segment-level correlations can be determined. This was done to produce Table 3. In column 3 of the table, the model formulas are evaluated at the (Assignment A) estimated parameter values given in Table 2. In column 4, the same parameter values are used except v=0; in column 5, c=0; and in column 6, c=v=0. Table 3 indicates the following:

- . v has a very small effect on predicted $[R^*(h;o)]^2$ and $E^*(h)$ values.
- . The effect of v on predicted values is least for large values of $[R^*(h;o)]^2$.
- . E(h) at v=0 is a lower bound for $E^*(h)$ at v > 0.
- . c has a very large effect on the predicted values.
- . $E^*(h)$ is an increasing function of c over the observed values of c.

Two-class/common-covariance case. The estimated k and 1-b values in Table 2 are larger than would be encountered in practice. The reason for this is that only pure pixels were present in the simulated Landsat data.

Smaller corresponding k and 1-b values can be determined from the I_o and I_a values given by Marco [9]. These values are for the case of f(.;1) and f(.;0) both MVN with common variance-covariance matrix and are tabled by D, the square-root of the Mahalanobis distance between the two distributions.

Using m,c,p, and v from Table 2 and also v=0; k and b based on Marco's I_g values; and d as a function of D [13, page 12]; we evaluated the model formulas at D=0.5, 1.0, 2.0, and 3.0 for the four crops. Table 4 lists the results for D=0.5 and 2.0 for corn and for "other". The results for soybeans were similar to those for corn, and those for pasture similar to "other." These results indicate the following:

- . The maximum-likelihood classifier has higher $[R^*(h;a)]^2$ values than the Bayes classifier for D small; whereas, the Bayes classifier gives slightly higher values for D large.
- . The effect of v (estimated v versus v=0) is large only in its influence on $[R^*(h;o)]^2$ at small values of D. In this situation, $[R^*(h;o)]^2$ for v estimated is smaller than for v=0 -- so much so, that $E^*(h)$ can exceed 1. In all other cases, the effect of v is small.
- . $E^*(h)$ is an increasing function of v. Except for the case of the Bayes classifier at small values of D, $E^*(h)$ exceeds 0.90 at the levels of v estimated for the four crops.

VIII. CONCLUSIONS

The analyses performed on the Iowa data set (with simulated Landsat data) and on the two-class/common-covariance case support the following conclusions:

1. The postulated super-population model reliably predicts segment-level correlations.
2. Using the model formulas with v (segment-size variability within stratum) set to zero instead of using an estimated value has the following effects:
   - . Small effect in predicting segment-level correlations under conditions of high spectral separability.
   - . For low spectral separability, small effect in predicting $R^*(h;a)$, segment-level correlation between ground-truth and classification results, and significant over-prediction effect in predicting $R^*(h;o)$, segment-level correlation between ground-truth and posterior probability.
3. The maximum-likelihood classifier has higher $[R^*(h;a)]^2$ values than the Bayes classifier for low spectral separability; whereas, the Bayes classifier gives slightly higher values for high spectral separability.
4. $E^*(h)$, the efficacy ratio of $[R^*(h;a)]^2$ (classification) to $[R^*(h;o)]^2$ (posterior probability), is an increasing function of m (average segment size by stratum), v (segment size variability by stratum), and of c (intra-segment correlation). Because of these effects, low efficacies at the pixel level are considerably increased at the segment level. For corn, soybeans, pasture, and "other" studied in one agricultural stratum of Iowa, the segment-level efficacies exceeded 0.90 under a wide range of conditions of spectral separability.

IX. RECOMMENDATIONS

On the basis of this study, the authors make the following recommendations:

1. Because of classification's high segment-level efficacy and low computational

effort, classification should be the auxiliary variable of choice under conditions similar to the ones encountered in this study.

2. If only one classification is to be performed, the maximum-likelihood classifier) should be the classifier of choice for regression estimation. The reason for this is superior performance under conditions of low spectral separability and only slightly inferior performance under conditions of high spectral separability.
3. The developed model formulas should be used as a planning tool in Landsat investigations. Such use would indicate potential efficiencies of Landsat regression estimation.

X. ACKNOWLEDGEMENTS

The authors wish to acknowledge Keith Crank of USDA/SRS for reviewing this paper and Patricia Joyce for manuscript word processing.

XI. REFERENCES

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Table 1: Moment Formulas

U	V	Cov(U,V)
Y(h,i)	Y(h,i)	$p[p+(1-p)c]v + p(1-p)m[1+(m-1)c]$
X(h,i;a)	X(h,i;a)	$[q^2 + p(1-p)(1-d-e)^2c]v + q(1-q)m + p(1-p)(1-d-e)^2cm(m-1)$
X(h,i;o)	X(h,i;o)	$p[p + (1-p)(1-b)^2c]v + p(1-p)(1-b)^2[((1-b)^{-1}c)m+cm^2]$
Y(h,i)	X(h,i;a)	$p[q + (1-p)(1-d-e)c]v + p(1-p)(1-d-e)m[1+(m-1)c]$
Y(h,i)	X(h,i;o)	$p[p + (1-p)(1-b)c]v + p(1-p)(1-b)m[1+(m-1)c]$

Table 2: Parameter Estimates for
Reduced Iowa Data Set with Simulated
Landsat Data

m = 196

v = 184 (Assignment A)
v = 288 (Assignment B)
v = 0 (Assignment C)

crop	p	d	e	(k)	1-b	c(A)	c(B)	c(C)
corn	.37	.13	.17	.46	.55	.09	.0076	-.0009
soybeans	.25	.23	.05	.56	.63	.13	-.0032	-.0047
pasture	.17	.32	.16	.21	.29	.15	-.0009	-.0042
other	.21	.70	.04	.13	.25	.09	.0020	-.0045

Table 3. Sensitivities of Predicted Values
To Levels of c and v in Reduced Iowa Data Set with Simulated Landsat Data

crop	predicted value	c, estimated		c=0	
		v, estimated	v=0	v, estimated	v=0
corn	$[R^*(h;a)]^2$:	.93	.94	.64	.46
	$[R^*(h;o)]^2$:	.94	.96	.71	.55
	$E^*(h)$:	.99	.98	.90	.84
soybeans	$[R^*(h;a)]^2$:	.97	.96	.65	.56
	$[R^*(h;o)]^2$:	.98	.97	.72	.63
	$E^*(h)$:	.99	.99	.90	.89
pasture	$[R^*(h;a)]^2$:	.88	.89	.32	.21
	$[R^*(h;o)]^2$:	.90	.92	.40	.29
	$E^*(h)$:	.98	.96	.78	.72
other	$[R^*(h;a)]^2$:	.73	.72	.19	.13
	$[R^*(h;o)]^2$:	.79	.85	.40	.25
	$E^*(h)$:	.92	.85	.49	.52

Table 4. Sensitivities of Predicted Values
to Levels of v and D in
Two-Class/Common-Covariance Case

		v=0				v, estimated			
crop	D	$[R^*(h;o)]^2$	a[1/]	$[R^*(h;a)]^2$	$E^*(h)$	$[R^*(h;o)$	a[1/]	$[R^*(h;a)]^2$	$E^*(h)$
corn	0.5	.51	B	.27	.52	.19	B	.29	1.49
			ML**	.43	.84		ML**	.39	2.01
	2.0	.95	B**	.94	.98	.94	B**	.94	.99
			ML	.93	.98		ML	.93	.99
other	0.5	.42	B	.05	.12	.16	B	.05	.32
			ML**	.35	.83		ML**	.30	1.88
	2.0	.95	B**	.92	.97	.94	B**	.92	.98
			ML	.91	.96		ML	.90	.96

1/ B = Bayes, ML = Maximum Likelihood
** indicates classifier with higher $[R^*(h;a)]^2$ for given v and D