Four sampling and estimation methods for estimating the number of red-cockaded woodpecker colonies on National Forests in the Southeast were compared, using samples chosen from simulated populations based on the observed sample. The methods included (1) simple random sampling without replacement using a mean per sampling unit estimator, (2) simple random sampling without replacement with a ratio per pine area estimator, (3) probability proportional to "size" sampling with replacement, and (4) probability proportional to "size" without replacement using Murthy's estimator. The survey sample of 274 National Forest compartments (1000 acres each) constituted a superpopulation from which simulated stratum populations were selected with probability inversely proportional to the original probability of selection. Compartments were originally sampled with probabilities proportional to the probabilities that the compartments contained woodpeckers ("size"). These probabilities were estimated with a discriminant analysis based on tree species and tree age. The ratio estimator would have been the best estimator for this survey based on the observed sample. Murthy's estimator would have been doubled if woodpeckers had been found and halved if they had not.

Murthy's estimator (Gehan, 1977: 263-266) was used based on a review by Rao (1978: 75) who cited studies "which indicate that Murthy's method might be preferable over other methods ... when a stable estimator as well as a stable variance estimator are required." Because of the computational requirements of Murthy's estimator, no more than 10 samples per stratum could be analyzed. CPU times on a Hewlett Packard 3000 minicomputer were:

<table>
<thead>
<tr>
<th>Samples</th>
<th>Seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>27</td>
</tr>
<tr>
<td>5</td>
<td>29</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
</tr>
<tr>
<td>7</td>
<td>37</td>
</tr>
<tr>
<td>8</td>
<td>85</td>
</tr>
<tr>
<td>9</td>
<td>533</td>
</tr>
<tr>
<td>10</td>
<td>5231</td>
</tr>
</tbody>
</table>

CPU seconds can be estimated by 26 + 0.0014339n! where n is the number of samples. It took about 1.5 hours to analyze 10 samples, and would have taken about 16 hours to analyze 11 samples. While it is clear from the variance formula that computational times are related to n!, guidelines on reasonable sample sizes are not readily available. When more than 10 samples had been drawn, the national forest-ranger district strata were poststratified into approximately equal sized substrata. Six of the 30 strata had more than 10 samples and had to be poststratified, 4 strata into 2 substrata and 2 into 3 substrata. Poststratification increases the variance because samples are not distributed proportionally to the poststrata (Gehan 1976: 135). The increase is about (L-1)/Ln where L is the number of poststrata and n is the mean number of sampling units per poststratum. For 2 poststrata with 6 sampling units per strata, this is about an 8% increase.

**EMPIRICAL STUDY**

With the advantage of hindsight, we compared four sampling and analytic methods that we could have used for the survey:

1. Mean per unit estimator with simple random sampling, equal probability and without replacement (see Gehan 1977: 21-26).

\[ \hat{Y} = N \sum_{i=1}^{n} \frac{y_i}{n} \]  

\[ v(\hat{Y}) = N^2 \frac{(1-n/N)}{N^2} \sum_{i=1}^{n} (y_i - \bar{y})^2 / (n(n-1)) \]  

where 

\[ Y = \text{total number of colonies in stratum} \]  
\[ Y_i = \text{number of colonies in ith compartment} \]  
\[ \bar{y} = \frac{\sum_{i=1}^{n} y_i}{n} \]  
\[ N = \text{number of compartments in stratum} \]  
\[ n = \text{number of compartments in sample} \]  

\[ v(\hat{Y}) = \frac{N^2}{n^2} \frac{(1-n/N)}{N} \sum_{i=1}^{n} (y_i - \bar{y})^2 / (n(n-1)) \]  

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where 

\[ Y = \text{total number of colonies in stratum} \]  
\[ Y_i = \text{number of colonies in ith compartment} \]  
\[ \bar{y} = \frac{\sum_{i=1}^{n} y_i}{n} \]  
\[ N = \text{number of compartments in stratum} \]  
\[ n = \text{number of compartments in sample} \]  

144
2. Ratio per pine area with simple random sampling, equal probability and without replacement (see Cochran 1977: 150-156).

\[ \hat{Y}_2 = \hat{R} X \]  

\[ v(\hat{Y}_2) = N^2 (1-n/N) \frac{\sum_i (y_i - \hat{R} x_i)^2}{n(n-1)} \]  

where 
- \( X \) = total pine acres in stratum
- \( x_i \) = pine acres in ith compartment
- \( \hat{R} = \frac{\sum_i y_i}{\sum_i x_i} \)


\[ \hat{Y}_3 = \sum_i y_i/z_i^n \]  

\[ v(\hat{Y}_3) = \sum_i \frac{(y_i/z_i - \hat{Y}_3)^2}{n(n-1)} \]  

where
- \( z_i \) = probability of selecting ith compartment

4. Murthy's estimator with probability proportional to "size" and without replacement (see Cochran 1977: 263-265).

\[ \hat{Y}_4 = \sum_i P(s|i) y_i / P(s) \]  

\[ v(\hat{Y}_4) = \sum_{i,j} \frac{z_i z_j P(s|i)P(s|j)}{P(s)} (y_i/z_i - y_j/z_j)^2/ \frac{1}{P(s)} \]  

where
- \( P(s) = \sum_{i,j} z_i z_j / (1-z_1) \) \( z_k / (1-z_1-z_2) \)
- \( P(s|i) = \sum_j z_j (1-z_1) z_k / (1-z_1-z_2) \)
- \( P(s|j) = \sum_i z_i (1-z_2) z_k / (1-z_1-z_2) \)
- \( P(s|i,j) = z_k / (1-z_1-z_2) \)

An empirical approach was used to investigate these 4 methods for the specific conditions of the red-cockaded woodpecker survey. The actual sampled compartments from all strata were used as a combined superpopulation from which the artificial stratum populations were drawn. A Tausworthe random number generator was used (Kennedy and Gentle, 1980: 155). Sampling from the superpopulation was with replacement and inversely proportional to the probability that the original compartments were selected. This selection reversed the over-representation of compartments that were originally assigned high selection probabilities so that the simulated populations were as similar to the real population as possible. Fifty trials each with 25 replications (1250 replications in all) were run for 9 situations. The situations included combinations of 3 population sizes (20, 100, and 500 compartments), and 3 numbers of strata (1, 10, and 20); each strata had 4 sample compartments. For each trial, the bias, estimated variance and mean square error, and the proportion of 90% and 95% confidence intervals that enclosed the true value were output to a disk file for summarization. These five values were used as criteria for evaluating the four sampling and estimation methods. The relative bias, relative mean square error and relative variance were defined as

\[ \text{Rel. bias} = \frac{\sum_r (\hat{Y}_r - Y_r)}{\sum_r Y_r} \]  

\[ \text{Rel. MSE} = \frac{\sum_r (\hat{Y}_r - Y_r)^2}{\sum_r Y_r^2} \]  

\[ \text{Rel. var.} = \frac{\sum_r v(\hat{Y}_r)}{\sum_r Y_r^2} \]  

where \( r=1,...,m \) indexes replications.

Note that the relative mean square error and relative variance have been multiplied by the number of observations \( n \) so that they would be comparable over trials with different sample sizes.

Unfortunately the discriminant analysis predictions of which compartments contained woodpeckers were not as good as we had hoped. Additional trials were conducted to see how the non-uniform probability methods would perform if accurate predictions of the probability that a compartment contains woodpeckers were available. In these trials, the sampling units contained woodpeckers only when the posterior probability from the discriminant analysis was greater than 0.5. In that case, the probabilities of compartments containing 1, 2, ..., or 11 woodpecker colonies were 0.44, 0.17, 0.14, 0.11, 0.03, 0.05, 0.02, 0.02, 0.01, 0, 0.01, respectively, based on the observed frequency distribution.

RESULTS

For the red-cockaded woodpecker survey, equal probability sampling with a ratio to pine area would have been the best method as judged by the mean square error (Table I). Unequal
probability sampling with Murthy's method that we used for the survey was second best. However, Murthy's method requires extensive computations which effectively limit one to 10 or fewer samples per stratum. Although none of the estimators is seriously biased, the ratio estimator underestimated the number of colonies by about 2 percent. Note that unbiased and reduced bias ratio estimators are available (see Cochran 1977: 174-177). The variance of the ratio estimator underestimated the mean square error more than the other methods.

The disappointing performance of Murthy's method may be due to our inability to predict the presence of woodpeckers (estimate the "size") as well as we had hoped. The trials with improved "size" estimates showed greatly improved performance with Murthy's method. If more accurate predictions of woodpecker presence had been available, Murthy's estimator would have been the best. The squared correlation between the probability and the number of colonies in the sample was only 9.7%. The discriminant function had the following classification rates with the original training compartments that were available for planning the survey and with the National Forest sample compartments (using modified probabilities if the compartment had been previously searched):

<table>
<thead>
<tr>
<th>Training Compartments</th>
<th>Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed</td>
<td>Absent</td>
</tr>
<tr>
<td>Absent</td>
<td>244(76%)</td>
</tr>
<tr>
<td>Present</td>
<td>99(39%)</td>
</tr>
<tr>
<td>Total</td>
<td>343(60%)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sample Compartments</th>
<th>Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed</td>
<td>Absent</td>
</tr>
<tr>
<td>Absent</td>
<td>66(46%)</td>
</tr>
<tr>
<td>Present</td>
<td>21(16%)</td>
</tr>
<tr>
<td>Total</td>
<td>87(32%)</td>
</tr>
</tbody>
</table>

The 64% correct classification rate for the sample compartments compares favorably with the 69% correct classification rate for the training compartments. Although 84% of the sample compartments with woodpeckers were correctly classified, only 46% of the compartments without woodpeckers were correctly classified. The low correct classification rates for compartments without woodpeckers results in poor overall predictions because few compartments have woodpeckers. We can only speculate on the reason for our poor predictions. It is possible that the compartments that were predicted to have woodpeckers have good habitat but that much of the habitat is not occupied by this endangered species. It is also possible that we did not have a representative training sample of compartments without woodpeckers because the training compartments were not selected randomly.

Confidence interval widths were underestimated especially with small sample sizes. With larger sample sizes the widths were improved. With 60 degrees of freedom for error, the 90% confidence intervals included the true value about 87% of the samples.

The authors would like to thank Dr. B. Kenneth Williams, also of Patuxent Wildlife Research Center, for his review of earlier drafts of this paper and his many helpful suggestions.

REFERENCES


KERNIGHAN, B.W. and P.J. Plauger (1976), Software Tools, Addison-Wesley.


Table 1. Results of empirical study of red-cockaded woodpecker survey, sampling from a super-

population consisting of the observed samples. Marginals for population size and number of

strata (each with 4 samples) are shown. Estimates are followed by their standard errors which

were calculated among trials each of which had 50 replicates. In some trials, the estimate of

"size" was improved by generating a nonzero number of woodpecker colonies whenever the
discriminant analysis predicted their presence.

<table>
<thead>
<tr>
<th>Population size</th>
<th>Number strata</th>
<th>Number trials</th>
<th>Equal-probability Mean per unit area</th>
<th>Relative mean square error, standard error with actual &quot;size&quot; estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>450</td>
<td>2.59, 0.07</td>
<td>2.09, 0.06</td>
<td>2.59, 0.07 2.43, 0.07</td>
</tr>
<tr>
<td>100</td>
<td>150</td>
<td>2.10, 0.15</td>
<td>1.84, 0.11</td>
<td>2.38, 0.17 1.89, 0.13</td>
</tr>
<tr>
<td>500</td>
<td>150</td>
<td>2.78, 0.10</td>
<td>2.34, 0.10</td>
<td>2.70, 0.11 2.60, 0.11</td>
</tr>
<tr>
<td>1</td>
<td>150</td>
<td>2.88, 0.08</td>
<td>2.41, 0.09</td>
<td>2.80, 0.09 2.80, 0.09</td>
</tr>
<tr>
<td>10</td>
<td>150</td>
<td>2.56, 0.14</td>
<td>2.18, 0.12</td>
<td>2.69, 0.16 2.48, 0.14</td>
</tr>
<tr>
<td>20</td>
<td>150</td>
<td>2.76, 0.12</td>
<td>2.38, 0.11</td>
<td>2.54, 0.10 2.31, 0.09</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Unequal-probability Probability Murthy’s proportional method to &quot;size&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative mean square error, standard error with actual &quot;size&quot; estimates</td>
</tr>
<tr>
<td>2.59, 0.07 2.43, 0.07</td>
</tr>
<tr>
<td>2.38, 0.17 1.89, 0.13</td>
</tr>
<tr>
<td>2.70, 0.11 2.60, 0.11</td>
</tr>
<tr>
<td>2.80, 0.09 2.80, 0.09</td>
</tr>
<tr>
<td>2.69, 0.16 2.48, 0.14</td>
</tr>
<tr>
<td>2.54, 0.10 2.31, 0.09</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Estimated relative variance, standard error with actual &quot;size&quot; estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.59, 0.07 2.43, 0.07</td>
</tr>
<tr>
<td>2.38, 0.17 1.89, 0.13</td>
</tr>
<tr>
<td>2.70, 0.11 2.60, 0.11</td>
</tr>
<tr>
<td>2.80, 0.09 2.80, 0.09</td>
</tr>
<tr>
<td>2.69, 0.16 2.48, 0.14</td>
</tr>
<tr>
<td>2.54, 0.10 2.31, 0.09</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Relative bias, standard error with actual &quot;size&quot; estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001, 0.005 -0.018, 0.005</td>
</tr>
<tr>
<td>0.005, 0.007 -0.013, 0.008</td>
</tr>
<tr>
<td>-0.003, 0.009 -0.024, 0.009</td>
</tr>
<tr>
<td>0.002, 0.009 -0.017, 0.009</td>
</tr>
<tr>
<td>0.004, 0.014 -0.010, 0.013</td>
</tr>
<tr>
<td>-0.005, 0.005 -0.025, 0.007</td>
</tr>
<tr>
<td>0.005, 0.003 -0.019, 0.004</td>
</tr>
<tr>
<td>-0.014, 0.008 -0.006, 0.005</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Proportion of 90% confidence intervals with true value, standard error with actual &quot;size&quot; estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.945, 0.008 0.909, 0.007</td>
</tr>
<tr>
<td>0.862, 0.006 0.816, 0.007</td>
</tr>
<tr>
<td>0.892, 0.005 0.840, 0.006</td>
</tr>
<tr>
<td>0.821, 0.009 0.796, 0.008</td>
</tr>
<tr>
<td>0.873, 0.006 0.871, 0.006</td>
</tr>
<tr>
<td>0.892, 0.005 0.894, 0.005</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Proportion of 95% confidence intervals with true value, standard error with actual &quot;size&quot; estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.990, 0.004 0.962, 0.004</td>
</tr>
<tr>
<td>0.909, 0.005 0.873, 0.005</td>
</tr>
<tr>
<td>0.934, 0.004 0.900, 0.005</td>
</tr>
<tr>
<td>0.886, 0.008 0.854, 0.007</td>
</tr>
<tr>
<td>0.925, 0.004 0.940, 0.004</td>
</tr>
<tr>
<td>0.909, 0.004 0.854, 0.007</td>
</tr>
</tbody>
</table>

147
SUBROUTINE MURTHY(NSAMP, Y, Z, ESTIMATE, VARIANCE)

# Subroutine to calculate Murthy's estimates, written in RATFOR (Kernighan and Plauger, 1976), a FORTRAN preprocessor that translates the structured source into a FORTRAN subroutine. A copy of the resulting FORTRAN program is available on request. A "#" indicates that the remainder of the line is a comment. A " " signals the continuation of a statement. "DO I=1,N statement" specifies that the statement is to be executed N times with I=I,2,...,N. Compound statements may be used. They are indicated by "$( statement-1 

SUBROUTINE MURTHY(NSAMP, Y, Z, ESTIMATE, VARIANCE)
# NSAMP = no. of samples (2<=NSAMP<-10)
# (input, integer)
# NSAMP>10 requires excessive computing
# Y = array of observed totals for sampling units I, 2, ..., NSAMP
# (input, double precision)
# Z = array of probabilities that sampling units i, 2, ..., NSAMP are selected if a single sample was drawn (input, double precision)
# ESTIMATE = estimated total
# (output, double precision)
# VARIANCE = estimated variance of total
# (output, double precision)
INTEGER NSAMP, I, II, I2, I3, I4, I5, I6, I7, I8, I9, I10,
ISU(IO)
DOUBLE PRECISION Y(10), Z(10), ESTIMATE, VARIANCE, TOTALZ, DEN1, DEN2, DEN3, DEN4, DEN5, DEN6, DEN7, DEN8, DEN9, DEN10,
PROB, PI, P2, P3, P4, P5, P6, P7, P8, P9, P10,
PIJ(10, I0)
LOGICAL INSAMP(I 0), EOF I0,
NSAMP 2, NSAMP 3, NSAMP 4, NSAMP 5, NSAMP 6, NSAMP 7,
NSAMP 8, NSAMP 9, NSAMP 10 # TRUE IF SAMPLE SIZE
IF (NSAMP>10)
$)
WRITE (6,*) ' NSAMP FOR MURTHY'S METHOD',
NSAMP, ', RETURN ZEROS'
ESTIMATE=0.0D0
VARIANCE=0.0D0
RETURN $
$)
IF (NSAMP=0)
$(
WRITE(6,*) ' NSAMP=0 FOR MURTHY'S METHOD'
ESTIMATE=0.0D0
VARIANCE=0.0D0
RETURN $
$)
IF (NSAMP=1)
$(
WRITE(6,*) ' NSAMP=1 FOR MURTHY'S METHOD'
ESTIMATE=Y(1)/Z(1)
VARIANCE=0.0D0
RETURN $
$)
IF (NSAMP4)
  $(
    P4=Z(14)
    BREAK
  )
INSAMP(14)=.TRUE.
DEN4=DEN3-Z(14)
P5=0.0
DO 15=1,NSAMP
  $(
    IF (INSAMP(15)) NEXT
  )
IF (NSAMP5)
  $(
    P5=Z(15)
    BREAK
  )
INSAMP(15)=.TRUE.
DEN5=DEN4-Z(15)
P6=0.0
DO 16=1,NSAMP
  $(
    IF (INSAMP(16)) NEXT
  )
IF (NSAMP6)
  $(
    P6=Z(16)
    BREAK
  )
INSAMP(16)=.TRUE.
DEN6=DEN5-Z(16)
P7=0.0
DO 17=1,NSAMP
  $(
    IF (INSAMP(17)) NEXT
  )
IF (NSAMP7)
  $(
    P7=Z(17)
    BREAK
  )
INSAMP(17)=.TRUE.
DEN7=DEN6-Z(17)
P8=0.0
DO 18=1,NSAMP
  $(
    IF (INSAMP(18)) NEXT
  )
IF (NSAMP8)
  $(
    P8=Z(18)
    BREAK
  )
INSAMP(18)=.TRUE.
DEN8=DEN7-Z(18)
P9=0.0
DO 19=1,NSAMP
  $(
    IF (INSAMP(19)) NEXT
  )
IF (NSAMP9)
  $(
    P9=Z(19)
    BREAK
  )
INSAMP(19)=.TRUE.
DEN9=DEN8-Z(19)

DO 110=1,NSAMP
  $(
    IF (INSAMP(110)) NEXT
    BREAK
  )
  # 110
  P9=P9+Z(19)/DEN9*Z(110)
  INSAMP(19)=.FALSE.
  # 19
  P8=P8+Z(18)/DEN8*P9
  INSAMP(18)=.FALSE.
  # 18
  P7=P7+Z(17)/DEN7*P8
  INSAMP(17)=.FALSE.
  # 17
  P6=P6+Z(16)/DEN6*P7
  INSAMP(16)=.FALSE.
  # 16
  P5=P5+Z(15)/DEN5*P6
  INSAMP(15)=.FALSE.
  # 15
  P4=P4+Z(14)/DEN4*P5
  INSAMP(14)=.FALSE.
  # 14
  P3=P3+Z(13)/DEN3*P4
  INSAMP(13)=.FALSE.
  # 13
  P2=P2+Z(12)/DEN2*P3
  PLJ(12,12)=P3DEN2
  INSAMP(12)=.FALSE.
  # 12
  P1=P1+Z(11)/DEN1*P2
  PLJ(11,11)=P2DEN1
  INSAMP(11)=.FALSE.
  # END OF 11 LOOP

# Calculate estimate of total and its variance
# PROB = unconditional probability of drawing
# sample.
# PIJ(I,I) = conditional probability of drawing
# sample given that the Ith sampling unit was
drawn first.
# PIJ(I,J) = conditional probability of drawing
# sample given that the Ith and Jth sampling
# units were drawn first.
# PIJ(I,J) = conditional probability of drawing
# sample given that the Ith and Jth sampling
# units were drawn first.
ESTIMATE=0
DO I=1,NSAMP
  ESTIMATE=ESTIMATE+PIJ(I,I)*Y(I)
PROB=P1
VARIANCE=0
DO I=1,NSAMP-1
  $(
    DO J=I+1,NSAMP
      $(
        P=Y(I)/Z(I)-Y(J)/Z(J)
        VARIANCE=VARIANCE +
        (PROB*PIJ(I,J)-PIJ(I,I)*PIJ(J,J))*
        Z(I)*Z(J)*P*P
      )
    )
  )
ESTIMATE=ESTIMATE/PROB
VARIANCE=VARIANCE/PROB/PROB
RETURN
END