OPTIMAL SELECTION OF THE T BEST OF A SEQUENCE WITH SAMPLING COST

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ABSTRACT

This paper deals with the problem of selecting the t best of n independent and identically distributed random variables which are observed sequentially with sampling cost c per unit. Assume that a decision for acceptance or rejection must be made after each sampling and that the reward for each observation with value x is given by px - c where p is 1 if the observation is accepted or 0 otherwise. The optimal decision procedure (strategy) for maximizing the total expected reward is obtained. The critical numbers which are necessary to carry out the optimal decision procedure is presented by two recursive equations. The limit values of the critical numbers and the expected sample size are also studied.

KEY WORDS: Sequential decision problem, total expected reward, dynamic programming, sampling cost.

1. INTRODUCTION

Let $X_1, X_2, \ldots, X_n$ be a sequence of independent and identically distributed (i.i.d.) random variables, each with cumulative distribution function (c.d.f.) F(x), and $E(X)<\infty$. It is assumed that X's are observed sequentially and that the sampling cost for each observation is a known constant c.

This paper deal with the problem of selecting the t best of X's. Indeed, the sequential decision model studied here is the one for which a decision for acceptance or rejection must be made after each sampling. Once rejected an observation can not be reconsidered. When an observation $X_i = x_i$ appears, the reward is given by $p_i x_i - c$ where $p_i = 1$ if the observation is decided to be selected, and $p_i = 0$, otherwise; the p's have sum t, $1 \le t \le n$. When t observations are selected, the search process should be terminated.

The aim of this paper is to find an optimal decision procedure (strategy) which maximizes the total expected reward. Problems similar to the one described but with t=1 have been treated by many authors, such as MacQueen and Miller (1960), Sakaguchi (1961), Chow and Robbins (1961), DeGroot (1970) and Dhariyal and Dudewicz (1981) etc.. On the other hand, in a paper by Derman, Lieberman and Ross (1972), a sequential stochastic assignment model which concerns with the optimal assignment of n men (p_i's), $0 \le p_i \le 1$, to the n jobs (X_i's) is presented and their results have been extended by Albright (1974, 1976 and 1977). However, these papers discussed the assignment problem only, hence the sampling cost was not considered.

In Section 2, the problem is formalized and an optimal decision procedure is described and proved. The formulas for the optimal total expected reward and for the critical numbers which are necessary to carry out the procedure are obtained. The limit values of the critical numbers as n approaches to infinity, and the expected sample size (stopping time) are discussed in Section 3 and 4, respectively.

2. OPTIMAL DECISION PROCEDURE

Suppose that the search process progresses through a series of stages S(j,m), $j \le t$, $0 \le j \le m$, $0 \le m \le n$. At initial stage S(t,n), $1 \le t \le n$, the decision-maker observes an observation $x_1 = x$ and needs to make a decision between acceptance and rejection. The search process enters stage S(t-1,n-1) with reward x-c if the observation is accepted, and enters stage S(t,n-1) with reward -c if it is rejected. Using the functional equation technique in the theory of dynamic programming, Bellman(1957) we can easily determine the structure of the optimal decision procedure for the problem.

Let us define two sequences of functions:

$r_{j,m}(x)$ = the total conditional experted reward obtained by using an optimal decision procedure with the first observed value x at stage S(j,m), and $r_{j,m}$ = the expectation of $r_{j,m}(X)$.

Here $0 \le j \le m$, $j \le t$, $0 \le m \le n$.

The following theorem which will be proven by induction gives an optimal decision procedure that maximizes the total expected reward for any t,n and $c(\ge 0)$. For simplicity, all X's will be assumed to be either discrete or continuous random variables. Let us denote $F(d^-) = P(X<d)$. It is clear $F(d) = F(d^-)$ if F is absolutely continuous.

Theorem 1. In any S(t,n), $1 \le t \le n$, $c \ge 0$, stage search process there exist numbers

$$-\infty = d_{0,n} \le d_{1,n} \le \cdots \le d_{n-1,n} \le d_{n,n} = +\infty$$

such that the optimal choice at the initial stage S(t,n) is to select the random observation $x_1 = x$ and enters stage S(t-1,n-1) if $x \ge d_{n-t,n}$, otherwise reject it and enters stage S(t,n-1). The search process will be terminated at stage S(0,m) for some m, $0 \le m \le n-t$, i.e. when t observations are selected. Furthermore, if an optimal decision procedure is followed, the optimal expected reward for a S(t,n-1) stage search process is

$$r_{t,n-1} = \sum_{j=1}^{t} d_{n-j,n}. \tag{1}$$

Proof: Suppose that there exist numbers $d_{j,m}$, j=1,...,m+1; m=1,...,n-1, such that the optimal choice in the initial stage of a S(j,m) stage search process is to select the observation x and enters stage S(j-1,m-1) if $x \ge d_{m-j,m}$, otherwise reject it and enters stage S(j,m-1). Then in the S(t,n), $1 \le t \le n$, stage search process the total conditional expected reward given $x_1 = x$ under an optimal decision procedure is

$$r_{t,n}(x) = \max_d \{(x + r_{t-1,n-1}) I_{\{x \ge d\}}(x) + r_{t,n-1} I_{\{x<d\}}(x)\} - c. \tag{2}$$

Its expectation, the total expected reward, is

$$r_{t,n} = \max_d \{\int_d^\infty x dF(x) + r_{t,n-1} F(d^-) + r_{t-1,n-1}(1-F(d^-))\} - c. \tag{3}$$

For any j,n, $1 \le j \le n-2$, let $d_{n-j,n}$ be defined as the expected incremental value of the total expected rewards under an optimal decision procedure between S(j,n-1) stage and S(j-1,n-1) stage search processes, that is

$$d_{n-j,n} = r_{j,n-1} - r_{j-1,n-1}. \quad (4)$$

Then the total expected reward $r_{t,n-1}$ of a S(t, n-1) stage search process is given by (1).

On the other hand, no matter X's are discrete or continuous random variables, the right hand side of (3) can be rewritten as

$$\max_d \{\int_d^\infty x dF(x) + \int_{-\infty}^{d^-} (r_{t,n-1} - r_{t-1,n-1}) dF(x) + r_{t-1,n-1}\} - c. \quad (5)$$

It is easy to see that

$$d = r_{t,n-1} - r_{t-1,n-1} = d_{n-t,n} \quad (6)$$

gives a maximum value of (5). Because the result is trivial for case n=1 completes the induction.

The following corollary indicate how to obtain the critical numbers $d_{t,n+1}$, $1 \le t \le n+1$, $1 \le n$.

<u>Corollary 1</u>. Define $d_{0,n} = -\infty$, $d_{n,n} = +\infty$. Then

$$d_{n,n+1} = \int_{d_{n-1,n}}^{d_{n,n}} x\, dF(x) + d_{n-1,n}F(d^-_{n-1,n}) - c \quad (7)$$

and

$$d_{i,n+1} = \int_{d_{i-1,n}}^{d^-_{i,n}} x\, dF(x) + d_{i-1,n}F(d^-_{i-1,n}) + d_{i,n}(1-F(d^-_{i,n})) \quad (8)$$

for i=1,...,n-1, where $-\infty \cdot 0$ and $\infty \cdot 0$ are defined to be 0.

Proof: Recall that $d_{n-j,n} = r_{j,n-1} - r_{j-1,n-1}$, $1 \le j \le n-2$, $2 \le n$ and $r_{0,m} = 0$ for all m. Hence we have by induction that

$$d_{n,n+1} = r_{1,n} - r_{0,n} = r_{1,n}$$

$$= \int_{d_{n-1,n}}^\infty x\, dF(x) + r_{1,n-1}F(d_{n-1,n}) + 0 \cdot (1-F(d^-_{n-1,n})) - c$$

$$= \int_{d_{n-1,n}}^\infty x\, dF(x) + d_{n-1,n}F(d^-_{n-1,n}) - c$$

and

$$d_{i,n+1} = r_{n+1-i,n} - r_{n-i,n}$$

$$= \int_{d_{i-1,n}}^\infty x\, dF(x) + d_{i-1,n}F(d^-_{i-1,n}) + r_{n-i,n-1} - c - (\int_{d_{i,n}}^\infty x\, dF(x) + d_{i,n}F(d^-_{i,n}) + r_{n-i-1,n-1} - c)$$

$$= \int_{d_{i-1,n}}^{d^-_{i,n}} x\, dF(x) + d_{i-1,n}F(d^-_{i-1,n}) + r_{n-i,n-1} - r_{n-i-1,n-1} - d_{i,n}F(d^-_{i,n})$$

$$= \int_{d_{i-1,n}}^{d^-_{i,n}} x\, dF(x) + d_{i-1,n}F(d_{i-1,n}) + d_{i,n}(1-F(d^-_{i,n}))$$

for i=1,...,n-1.

Under any affine transformation Y = aX + b where $0 < a < \infty$ and $-\infty < b < \infty$ and c' = ac, it can easily be shown by induction that the critical numbers $b_{i,n}$ corresponding to Y's satisfy the relations $b_{i,n} = ad_{i,n} + b$ for all $1 \le i \le n$ where $d_{i,n}$'s are the critical numbers associated with X or distribution function F and sampling cost c per unit. Hence if Y's have location-scale parameters μ and σ where $0 < \sigma < \infty$ and $-\infty < \mu < \infty$, and the sampling cost per unit is c, then we can standardize Y's and use the critical numbers $d_{i,n}$'s based on the standardized distribution and new sampling cost c' = c/σ per unit to progress the search process.

3. BEHAVIOR OF $d_{i,n}$'s

Some basic behavior of the $d_{i,n}$'s, the critical numbers, are studied in this section under the assumptions $c \ge 0$ and the c.d.f. of X's has finite mean.

<u>Lemma 1</u>. $\sum_{i=1}^{n} d_{i,n+1} = n(E(X) - c)$ for all n. (9)

Proof: The proof will be omitted since it is straight forward induction proof on n by using the recursive formulas (7) and (8).

<u>Lemma 2</u>. For any fixed value of s, $0 \le s \le n$, $d_{n-s,n+1}$ is increasing in n. Note that by Theorem 1 $d_{n-s,n+1}$ is decreasing in s for fixed n

Proof: We will prove this lemma by using induction on n. First we consider the case s = 0. In this case, let $b_n = d_{n,n+1}$, then when n = 1

$$b_1 = \int_{-\infty}^\infty x\, dF(x) - c > -\infty = b_0.$$

Suppose $b_{n-1} > b_{n-2}$ is true for some $n \ge 2$, then

$$b_n = \int_{b_{n-1}}^\infty x\, dF(x) + b_{n-1}F(b^-_{n-1}) - c$$

$$= \int_{b_{n-2}}^\infty x\, dF(x) - \int_{b_{n-2}}^{b^-_{n-1}} x\, dF(x) + \int_{-\infty}^{b^-_{n-1}} b_{n-1} dF(x) - c$$

$$= \int_{b_{n-2}}^\infty x\, dF(x) + \int_{-\infty}^{b^-_{n-2}} b_{n-1} dF(x) + \int_{b_{n-2}}^{b^-_{n-1}} (b_{n-1}-x)\, dF(x) - c$$

$$> \int_{b_{n-2}}^\infty x\, dF(x) + \int_{-\infty}^{b^-_{n-2}} b_{n-2}\, dF(x) - c$$

$$= b_{n-1}.$$

The inequality above is a result of the induction hypothesis. Therefore the proof is completed for the case $s = 0$.

For the case $s \neq 0$, $1 \leq s \leq n$, the proof is almost the same, hence it is omitted.

Let us now introduce a nonnegative, convex and strictly decreasing function

$$A(x) = \int_x^\infty (y - x)\, dF(y), \quad -\infty < x < \infty \tag{10}$$

Because

$$A(x) = \int_0^\infty t\, dF(x + t)$$

$$= \int_0^\infty (1 - F(x+t))\, dt,$$

we have

$$A(x) \to 0 \text{ as } x \to \infty. \tag{11}$$

Theorem 2. If $c > 0$, then the sequence $d_{n,n+1}$ converges and the limit value α is given by

$$A(x) = c. \tag{12}$$

Proof: With function $A(x)$, (7) can be rewritten as

$$d_{n,n+1} = A(d_{n-1,n}) + d_{n-1,n} - c, \text{ for all } n \geq 1, \tag{13}$$

or

$$d_{n,n+1} - d_{n-1,n} = A(d_{n-1,n}) - c, \text{ for all } n \geq 1. \tag{14}$$

By Lemma 2, $d_{n,n+1}$ is increasing in n, so the left hand side of (14) is nonnegative for all n. On the other hand, if $d_{n,n+1}$ diverges, then $d_{n,n+1} \to \infty$ as $n \to \infty$. This implies the right hand side of (14) has limit value $-c < 0$, contradicting what we just proved. Therefore $d_{n,n+1}$ converges with limit value, say α. Let $n \to \infty$ on both sides of (14), the proof is finished.

Example:

(a) Standard normal distribution $N(0,1)$

$$\phi(\alpha) + \alpha\Phi(\alpha) = c \tag{15}$$

where ϕ and Φ are p.d.f. and c.d.f. of standard normal distribution, respectively.

(b) Exponential distribution $\varepsilon(\lambda)$ with p.d.f. $\lambda \exp(-\lambda x)$

$$\alpha = -\lambda^{-1}\ln(\lambda c). \tag{16}$$

(c) Uniform distribution $U(0,1)$

$$\alpha = 1 - (2c)^{1/2} \tag{17}$$

(d) Poisson distribution with frequency function $e^{-\lambda}\lambda^x/x!$

$$\lambda P_\lambda(X \geq \alpha) - \alpha P_\lambda(X \geq \alpha + 1) = c. \tag{18}$$

Theorem 3. For any fixed nonnegative integer s, if $c > 0$ then $d_{n-s,n+1}$ converges to α as $n \to \infty$.

Proof: Since $d_{n,n+1}$ has limit value α and $d_{n-s,n+1}$ is less than or equal to $d_{n,n+1}$ for all n, we have

$$\lim_{n\to\infty} d_{n-s,n+1} \equiv d(s) \leq \alpha \text{ for all fixed } s,$$

$0 \leq s \leq n$. and $d(s)$ is decreasing in s. By (7) and (8) we have

$$d(s) = \int_{d(s)}^{d(s-1)} x\, dF(x) + d(s)F(d(s)^-) +$$

$$d(s-1)(1-f(d(s-1)^-))$$

that is

$$0 = \int_{d(s)}^{d(s-1)} (x-d(s))\, dF(x) + (d(s-1) - d(s))$$

$$(1-F(d(s-1)^-)).$$

But this can be true only if $d(s) = d(s-1)$. Since the value of s is arbitrary, the proof is finished.

4. EXPECTED STOPPING TIME (SAMPLE SIZE)

Let random variable $N_{t,n}$ be the stopping time (or, equivalently, the sample size) for which the search process started at stage $S(t,n)$ will be terminated under the optimal decision procedure. If the sampling cost c for each observation is positive and is less than $E(x)$, then the expected stopping time $E(N_{t,n})$ is given by

$$r_{t,n} = R_{t,n} - cE(N_{t,n}) \tag{18}$$

where $r_{t,n}$ is defined as before and $R_{t,n}$ is the total expected reward which is calculated as if there were no sampling cost.

The following theorem gives a recursive formula for calculating $E(N_{t,n})$ for all $1 \leq t \leq n$.

Theorem 4. $N_{0,n} = 0$, $N_{n,n} = n$ and

$$E(N_{t,n}) = F(d^-_{n-t,n})E(N_{t,n-1}) + (1-F(d^-_{n-t,n}))$$

$$E(N_{t-1,n-1})$$

$$+ 1 \text{ if } 1 \leq t \leq n-1. \tag{19}$$

Proof: $N_{0,n} = 0$ and $N_{n,n} = n$ are trivial. It can be seen that for all $1 \leq t \leq n-1$

$$R_{t,n} = \int_{d_{n-t,n}}^\infty x\, dF(x) + R_{t,n-1}F(d^-_{n-t,n}) +$$

$$R_{t-1,n-1}(1-F(d^-_{n-t,n})). \tag{20}$$

From (18) and (20) we have by induction

$$cE(N_{t,n}) = R_{t,n} - r_{t,n}$$

$$= (R_{t,n-1} - r_{t,n-1})F(d^-_{n-t,n}) +$$

$$(R_{t-1,n-1} - r_{t-1,n-1})(1-F(d_{n-t,n})) + c$$

$$= cE(N_{t,n-1})F(d^-_{n-t,n}) + cE(N_{t-1,n-1})$$

$$(1-F(d_{n-t,n})) + c.$$

Dividing both sides by c we have (19).

Corollary 2. If X's have an uniform distribution $U(0,1)$, then

$$E(N_{t,n}) = d_{n-t,n}E(N_{t,n-1}) + (1 - d_{n-t,n})$$

$$E(N_{t-1,n-1}) + 1.$$

Example: If X's are $U(0,1)$ distributed and $c=0.1$, then

$$E(N_{1,5}) = E(N_{1,4})d_{4,5} + 1$$

$$= (E(N_{1,3})d_{3,4} + 1)d_{4,5} + 1$$

$$= (((d_{1,2} + 1)d_{2,3} + 1)d_{3,4} + 1)d_{4,5} + 1$$

$$= 1.9916 \doteq 2.$$

$$E(N_{2,4}) = E(N_{2,3})d_{2,4} + E(N_{1,3})(1-d_{2,4}) + 1$$
$$= (2d_{1,3} + (1+d_{1,2})(1-d_{1,3}) + 1)d_{2,4} + ((1+d_{1,2})d_{2,3} + 1)(1-d_{2,4}) + 1$$
$$= 3.0545 \doteq 3.$$

The critical numbers used in this example are listed in Table I.

Let $\beta_t = \lim_{n\to\infty} R_{t,n}$ and $\gamma_t = \lim_{n\to\infty} E(N_{t,n})$ then (12) and (20) gives

$$\beta_1 = \int_\alpha^\infty x\, dF(x) \Big/ \int_\alpha^\infty dF(x) \qquad (21)$$

and

$$\gamma_1 = (1 - F(\alpha^-))^{-1}. \qquad (22)$$

Although our assumptions and goal are different from Chow and Robbins (1961), the results about the limit values of $R_{1,n}$ and $E(N_{1,n})$ are the same.

If $t \neq 1$ but fixed and small then from (20) we have

$$\beta_t - \beta_{t-1} = \int_\alpha^\infty x\, dF(x) \Big/ \int_\alpha^\infty dF(x)$$

hence

$$\beta_t = t\beta_1 \quad \text{and} \quad \gamma_t = t\gamma_1. \qquad (23)$$

5. COMMENT:

The main results of Theorem 1 and Corollary 1 go through in the same manner when the cost c_i of sampling x_i are unequal and X's are independent but not necessary identically distributed.

TABLE I

Critical values d(j,m) under U(0,1) distribution with sampling cost C per unit

	j=1	2	3	4	5	6	7	8	9	10
					C = 0.05					
m= 2	0.4500									
m= 3	0.3488	0.5512								
m= 4	0.2879	0.4601	0.6019							
m= 5	0.2465	0.3957	0.5266	0.6312						
m= 6	0.2161	0.3478	0.4663	0.5707	0.6492					
m= 7	0.1928	0.3107	0.4180	0.5165	0.6013	0.6607				
m= 8	0.1742	0.2810	0.3789	0.4705	0.5539	0.6232	0.6683			
m= 9	0.1590	0.2567	0.3466	0.4316	0.5112	0.5824	0.6392	0.6733		
m=10	0.1464	0.2364	0.3195	0.3985	0.4737	0.5435	0.6045	0.6509	0.6767	
m=11	0.1357	0.2192	0.2964	0.3702	0.4409	0.5080	0.5695	0.6218	0.6596	0.6789
					C = 0.1					
m= 2	0.4000									
m= 3	0.3200	0.4800								
m= 4	0.2688	0.4160	0.5152							
m= 5	0.2327	0.3656	0.4690	0.5327						
m= 6	0.2056	0.3258	0.4259	0.5008	0.5419					
m= 7	0.1845	0.2939	0.3883	0.4661	0.5205	0.5468				
m= 8	0.1675	0.2677	0.3561	0.4328	0.4936	0.5328	0.5495			
m= 9	0.1534	0.2459	0.3285	0.4026	0.4655	0.5127	0.5404	0.5510		
m=10	0.1417	0.2274	0.3048	0.3755	0.4382	0.4896	0.5258	0.5452	0.5518	
m=11	0.1316	0.2116	0.2842	0.3514	0.4127	0.4657	0.5074	0.5348	0.5482	0.5522
					C = 0.2					
m= 2	0.3000									
m= 3	0.2550	0.3450								
m= 4	0.2225	0.3180	0.3595							
m= 5	0.1977	0.2922	0.3454	0.3646						
m= 6	0.1782	0.2691	0.3285	0.3578	0.3665					
m= 7	0.1623	0.2487	0.3107	0.3477	0.3633	0.3672				
m= 8	0.1491	0.2310	0.2934	0.3356	0.3578	0.3658	0.3674			
m= 9	0.1380	0.2154	0.2770	0.3223	0.3501	0.3629	0.3668	0.3675		
m=10	0.1285	0.2017	0.2619	0.3087	0.3407	0.3583	0.3654	0.3672	0.3675	
m=11	0.1202	0.1896	0.2479	0.2954	0.3303	0.3522	0.3628	0.3666	0.3674	0.3675

TABLE II

Critical values d(j,m) under N(0,1) distribution with sampling cost C per unit

	j=1	2	3	4	5	6	7	8	9	10
					C = 0.05					
m= 2	-0.0500									
m= 3	-0.4244	0.3244								
m= 4	-0.6466	-0.0354	0.5320							
m= 5	-0.8028	-0.2606	0.1934	0.6701						

TABLE II (continued)

m= 6	-0.9224	-0.4231	-0.0276	0.3528	0.7703					
m= 7	-1.0188	-0.5493	-0.1903	0.1382	0.4733	0.8470				
m= 8	-1.0992	-0.6520	-0.3183	-0.0227	0.2657	0.5689	0.9075			
m= 9	-1.1679	-0.7381	-0.4233	-0.1505	0.1076	0.3684	0.6472	0.9566		
m=10	-1.2278	-0.8121	-0.5119	-0.2562	-0.0193	0.2135	0.4538	0.7129	0.9970	
m=11	-1.2808	-0.8768	-0.5884	-0.3459	-0.1248	0.0882	0.3024	0.5265	0.7689	1.0308

C = 0.1

m= 2	-0.1000									
m= 3	-0.4509	0.2509								
m= 4	-0.6643	-0.0726	0.4369							
m= 5	-0.8160	-0.2846	0.1457	0.5549						
m= 6	-0.9329	-0.4404	-0.0576	0.2946	0.6363					
m= 7	-1.0274	-0.5627	-0.2116	0.1020	0.4045	0.6952				
m= 8	-1.1065	-0.6628	-0.3345	-0.0480	0.2231	0.4894	0.7393			
m= 9	-1.1742	-0.7471	-0.4362	-0.1696	0.0781	0.3193	0.5568	0.7729		
m=10	-1.2333	-0.8198	-0.5226	-0.2713	-0.0412	0.1798	0.3979	0.6116	0.7989	
m=11	-1.2857	-0.8835	-0.5974	-0.3582	-0.1420	0.0632	0.2641	0.4636	0.6567	0.8194

C = 0.2

m= 2	-0.2000									
m= 3	-0.5069	0.1069								
m= 4	-0.7026	-0.1521	0.2547							
m= 5	-0.8448	-0.3373	0.0430	0.3391						
m= 6	-0.9558	-0.4791	-0.1250	0.1687	0.3912					
m= 7	-1.0464	-0.5929	-0.2603	0.0193	0.2555	0.4247				
m= 8	-1.1226	-0.6873	-0.3719	-0.1071	0.1245	0.3176	0.4467			
m= 9	-1.1882	-0.7677	-0.4662	-0.2145	0.0079	0.2042	0.3630	0.4615		
m=10	-1.2457	-0.8374	-0.5474	-0.3069	-0.0942	0.0978	0.2659	0.3965	0.4714	
m=11	-1.2967	-0.8988	-0.6185	-0.3875	-0.1837	0.0017	0.1697	0.3142	0.4214	0.4782

TABLE III

Critical values d(j,m) under EXP(1) distribution with sampling cost C per unit

j

	1	2	3	4	5	6	7	8	9	10
					C = 0.05					
m= 2	0.9500									
m= 3	0.6133	1.2867								
m= 4	0.4584	0.8787	1.5129							
m= 5	0.3677	0.6754	1.0737	1.6832						
m= 6	0.3077	0.5511	0.8426	1.2297	1.8190					
m= 7	0.2649	0.4665	0.6968	0.9808	1.3599	1.9312				
m= 8	0.2327	0.4050	0.5955	0.8200	1.0991	1.4716	2.0261			
m= 9	0.2076	0.3581	0.5207	0.7064	0.9273	1.2027	1.5693	2.1080		
m=10	0.1875	0.3211	0.4630	0.6214	0.8042	1.0225	1.2949	1.6560	2.1795	
m=11	0.1709	0.2912	0.4171	0.5552	0.7111	0.8919	1.1083	1.3779	1.7338	2.2426
					C = 0.1					
m= 2	0.9000									
m= 3	0.5934	1.2066								
m= 4	0.4476	0.8466	1.4058							
m= 5	0.3608	0.6579	1.0303	1.5510						
m= 6	0.3029	0.5400	0.8189	1.1752	1.6630					
m= 7	0.2613	0.4588	0.6818	0.9511	1.2944	1.7526				
m= 8	0.2300	0.3993	0.5852	0.8012	1.0633	1.3951	1.8259			
m= 9	0.2054	0.3538	0.5131	0.6934	0.9047	1.1608	1.4818	1.8870		
m=10	0.1857	0.3177	0.4572	0.6119	0.7886	0.9961	1.2468	1.5575	1.9385	
m=11	0.1695	0.2884	0.4124	0.5479	0.6997	0.8738	1.0780	1.3236	1.6243	1.9824
					C = 0.2					
m= 2	0.8000									
m= 3	0.5507	1.0493								
m= 4	0.4234	0.7771	1.1995							
m= 5	0.3452	0.6185	0.9355	1.3008						
m= 6	0.2919	0.5145	0.7648	1.0556	1.3731					
m= 7	0.2532	0.4410	0.6469	0.8823	1.1503	1.4265				
m= 8	0.2237	0.3861	0.5607	0.7567	0.9795	1.2267	1.4666			
m= 9	0.2004	0.3435	0.4950	0.6623	0.8504	1.0618	1.2892	1.4973		
m=10	0.1816	0.3096	0.4432	0.5889	0.7507	0.9318	1.1321	1.3410	1.5210	
m=11	0.1661	0.2818	0.4014	0.5303	0.6718	0.8289	1.0033	1.1929	1.3841	1.5395

TABLE IV

Critical values d(j,m) under Poisson distribution with parameter 1.0 and sampling cost C per unit

	j=1	2	3	4	5	6	7	8	9	10
					C = 0.05					
m= 2	0.9500									
m= 3	0.6005	1.2995								
m= 4	0.3796	0.9322	1.5382							
m= 5	0.2400	0.7289	1.1173	1.7139						
m= 6	0.1517	0.5490	0.9313	1.2749	1.8431					
m= 7	0.9059	0.4028	0.7906	1.0474	1.4251	1.9382				
m= 8	0.0606	0.2899	0.6480	0.9355	1.1472	1.5607	2.0082			
m= 9	0.0383	0.2056	0.4163	0.8297	1.0152	1.2564	1.6774	2.0612		
m=10	0.0242	0.1440	0.4020	0.7144	0.9414	1.0789	1.3677	1.7676	2.1099	
m=11	0.0153	0.1000	0.3071	0.5995	0.8579	0.9993	1.1552	1.4733	1.8378	2.1547
					C = 0.1					
m= 2	0.9000									
m= 3	0.5689	1.2311								
m= 4	0.3596	0.9025	1.4379							
m= 5	0.2273	0.7028	1.0798	1.5901						
m= 6	0.1437	0.5279	0.9118	1.2147	1.7020					
m= 7	0.0908	0.3865	0.7705	1.0243	1.3434	1.7844				
m= 8	0.0574	0.2778	0.6293	0.9220	1.1086	1.4600	1.8450			
m= 9	0.0363	0.1967	0.4999	0.8143	1.0000	1.2014	1.5617	1.8896		
m=10	0.0229	0.1377	0.3884	0.6987	0.9317	1.0532	1.2966	1.6484	1.9224	
m=11	0.0145	0.0955	0.2962	0.5845	0.8460	0.9889	1.1175	1.3896	1.7208	1.9466
					C = 0.2					
m= 2	0.8000									
m= 3	0.5057	1.0943								
m= 4	0.3197	0.8431	1.2373							
m= 5	0.2021	0.6505	1.0050	1.3424						
m= 6	0.1277	0.4855	0.8727	1.0941	1.4198					
m= 7	0.0807	0.3539	0.7303	0.9781	1.1802	1.4768				
m= 8	0.0510	0.2534	0.5918	0.8869	1.0395	1.2586	1.5187			
m= 9	0.0323	0.1790	0.4673	0.7784	0.9688	1.0974	1.3273	1.5495		
m=10	0.0204	0.1250	0.3613	0.6639	0.8988	1.0143	1.1582	1.3860	1.5722	
m=11	0.0129	0.0865	0.2743	0.5526	0.8124	0.9665	1.0523	1.2184	1.4352	1.5889

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