SAMPLING ON TWO OCCASIONS WITH PPSWOR

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SUMMARY

A theory of sampling on two occasions with unequal probabilities and without replacement is presented. Fellegi's (1963) method, which yields the same selection probabilities for a given unit on each occasion, is used to select the units for the rotation sample. The variances of composite estimators of the population total on the second occasion are developed. Numerical results are presented for small sample sizes and efficiency comparisons are made with a competing strategy.

1. INTRODUCTION

In surveys of a repetitive nature there are advantages to using a partial replacement sampling scheme both from the point of view of efficiency of estimation as well as reduction of respondent's burden. Essentially, after each sampling occasion a fraction of the units is rotated out of the sample and is replaced by a fresh subsample from the population. The literature abounds with discussions of sampling procedures and estimators when sampling on two or more occasions with equal probability. But of particular practical importance is the situation where units are selected on a given occasion with unequal probabilities. Thus, consider a finite population of N units $\{1,2,\ldots,N\}$ and two sampling occasions 1 (the previous occasion) and 2 (the current occasion). Let y_{1i} and y_{2i} denote the values of a characteristic y borne by the i-th unit on occasions 1 and 2 and let Y_1 and Y_2 denote the respective population totals. A size measure x_i is known for each of the units in the population.

Raj (1965) considered the following pps (probabilities proportional to size) sampling scheme: on the first occasion a sample s of size n is selected with probabilities p_i proportional to the x_i values and with replacement (wr). On the second occasion a simple random sample s_1 of m units is selected from s without replacement (wor) and an independent pps sample s_2 of $u = n - m$ is selected wr from the entire population. Y_1 and Y_2 are then respectively estimated by

$$\hat{Y}_1 = \sum_s y_{1i}/(np_i) \quad \text{and} \quad \hat{Y}^*_{2R} = Q^*\hat{Y}_{2u} + (1-Q^*)\hat{Y}'_2 \, ,$$

where $\hat{Y}_{2u} = \sum_{s_2} y_{2i}/(up_i)$, $\hat{Y}'_2 = \hat{Y}_1 + \sum_{s_1} (y_{2i}-y_{1i})/(mp_i)$,

and Q^* is a weight, $0 \le Q^* \le 1$.

The minimum variance of $\hat{Y}_{2R}$ was developed under the assumption that

$$V_{pps}(y_t) = \sum_{i=1}^{N} p_i(y_{ti}/p_i - Y_t)^2$$

is the same for $t = 1$ and 2.

The problem of sampling with ppswor on one occasion has attracted considerable attention in the literature. A major difficulty lies in the specification of feasible procedures which lead to specified probabilities at each and every draw. Fellegi (1963) has proposed a method such that the probability that unit i is selected on each of the n draws is p_i by determining $n-1$ sets of "working probabilities". This is an extremely desirable feature for rotating samples where it is essential that the usual pps estimator be unbiased for Y_2; this will not be true for any partial replacement design that does not feature a constant p_i for each of the n draws. The calculations inherent in Fellegi's scheme have, until recently, been prohibitive for $n > 2$. Choudhry (1981) has developed an iterative procedure for implementing Fellegi's scheme and prepared a computer program to evaluate the working probabilities when $n \le 5$. Although the convergence is fast in terms of the number of iterations, the amount of computation increases at the rate N^n. The program also computes the joint probabilities for the inclusion of both units i and j in the sample for variance calculation purposes.

Rao, Hartley and Cochran (1962) devised the "random group method" for selecting a sample with ppswor. The population of N units is split into n groups of sizes $N_1,N_2,\ldots,N_n$ where $\Sigma N_h = N$ and a sample of one unit is drawn independently from each group with probabilities proportional to the p_i's. Ghangurde and Rao (1969) extended the random group method to sampling on two occasions. For simplicity, the N units were split into n groups each of size N/n (assumed to be an integer). On occasion 1, one unit is drawn from each random group as above, giving a sample s of n units. On the second occasion a simple random sample s_1 of m matched units is selected from the n units wor and an independent sample s_2 of $u = n-m$ units is drawn from the whole population by the method used in obtaining s. They form a composite estimator $\hat{Y}'_{2G}$ of Y_2 and obtain its minimum variance under an optimum choice of the weight Q. The optimum value of $\lambda = m/n$ is then determined. The authors remarked that it would likely be more efficient to select s_2 from the $N-n$ units in the population that are not included in s.

Chotai (1974) modified the Ghangurde-Rao (G-R) design on the second occasion; the n units in s are split at random into m groups of size n/m (assumed to be an integer). One unit is selected from each of the m groups with probability proportional to p_i, yielding a sample s_1. A sample s_2 is obtained as in the G-R method. The optimum variance of his composite estimator $\hat{Y}'_{2C}$ is derived, the optimum λ determined and relative efficiency comparisons of $\hat{Y}'_{2C}$ with respect to G-R's and Raj's optimal estimators are made. $\hat{Y}'_{2C}$ was found to be always more efficient than $\hat{Y}'_{2R}$ and, in many cases, $\hat{Y}'_{2G}$ as well. A brief discussion of the case when n/m is not an integer is provided. It is worth noting that because λ is not a continuous function, the optimum λ should really be determined using integer programming methods.

2. SAMPLING STRATEGY

2.1 Sampling Procedure

From the population of N units, $(1,2,\ldots,N)$, select a sample of $n+u$ units, $u < n$, draw by draw and without replacement using Fellegi's Method such that the probability of selecting the i-th unit at each draw is p_i, $i=1,2,\ldots,N$, $\Sigma p_i = 1$. On the first of the two occasions, the first n units are observed from the $n+u$ selected; on the second occasion the first u units are dropped from the sample and the unused set of u units is rotated into the sample. Thus $m = n-u$ units are observed on both occasions. The n units observed on the first occasion are referred to as s, those units observed on both occasions as s_1 (where $s_1 \subset s$) and the set of unmatched units observed only on the second occasion as s_2. Note that Fellegi's scheme guarantees that the selection probabilities for a given unit i are the same on each draw and hence the same on both occasions. By restricting his attention to a sub-class of non-homogeneous linear model-design unbiased estimators, Chaudhri (1980) has shown that the foregoing sampling scheme yields an optimal strategy. This is a further motivation for using Fellegi's method.

2.2 Estimation Theory

In what follows, composite estimators of Y_2, the current occasion total, are proposed and their variances determined using an indicator variable approach.

Let ${}_ra_i = 1$ if unit i, $i=1,2,\ldots,N$, is selected at draw r, $r=1,2,\ldots,n+u$ and ${}_ra_i = 0$ otherwise. Since the expectation of ${}_ra_i$ is p_i, an unbiased estimator of the first occasion population total Y_1 is

$$\hat{Y}_1 = \frac{1}{n} \sum_{r=1}^{n} \sum_{i=1}^{N} {}_ra_i y_{1i}/p_i .$$

Then $$\hat{Y}_2' = \hat{Y}_1 + \frac{1}{m} \sum_{r=u+1}^{n} \sum_{i=1}^{N} {}_ra_i (y_{2i}-y_{1i})/p_i$$

is an unbiased estimator of the second occasion total Y_2. An unbiased estimator of Y_2 based on the current observations is

$$\hat{Y}_2 = \frac{1}{n} \sum_{r=u+1}^{u+n} \sum_{i=1}^{N} {}_ra_i y_{2i}/p_i .$$

A composite estimator of Y_2 is the weighted sum

$$\hat{Y}_{2c} = Q\hat{Y}_2' + (1-Q)\hat{Y}_2 ,$$

where $0 \le Q \le 1$.

The variance of $\hat{Y}_{2c}$, $\mathrm{Var}(\hat{Y}_{2c}) = Q^2\mathrm{Var}(\hat{Y}_2') + (1-Q)^2\mathrm{Var}(\hat{Y}_2) + 2Q(1-Q)\mathrm{Cov}(\hat{Y}_2',\hat{Y}_2)$, is derived by using the following properties of the indicator variable ${}_ra_i$:

$$\mathrm{Var}({}_ra_i) = p_i(1-p_i) \quad (i=1,2,\ldots,N,\ r=1,2,\ldots,n+u),$$

$$\mathrm{Cov}({}_ra_i, {}_ta_i) = -p_i^2 \quad (r \neq t),$$

$$\mathrm{Cov}({}_ra_i, {}_ra_j) = -p_ip_j \quad (i \neq j),$$

$$\mathrm{Cov}({}_ra_i, {}_ta_j) = E({}_ra_i \cdot {}_ta_j) - p_ip_j \quad (i \neq j,\ r \neq t),$$

where $E(\cdot)$ denotes the expected value with respect to the probability design. Now $E({}_ra_i \cdot {}_ta_j) = P({}_ra_i \cdot {}_ta_j = 1) = P({}_ra_i = 1, {}_ta_j = 1)$ where $P(\cdot)$ denotes probability.

Let $\Sigma_{(k-2;i,j)}$ denote summation over all possible ordered $(k-2)$-tuples of different units $\{i_1,i_2,\ldots,i_{r-1}, i_{r+1},\ldots,i_{k-2},i_{k-1}\}$ included in the sample from the first k draws selected from the $N-2$ units in the set $\{1,2,\ldots,i-1, i+1,\ldots, j-1,j+1,\ldots,N\}$ such that the i-th unit is selected at draw r and the j-th unit at draw k. There are $(N-2)(N-3)\ldots(N-k+1)$ terms involved in the summation.

As in Fellegi (1963), let $\{p_i(\ell);\ i=1,2,\ldots, N\}$ be the set of "working probabilities" for selecting a unit at draw ℓ, $\ell=1,2,\ldots,n+u$. For draws k and r with $k > r$,

$$E({}_ra_i \cdot {}_ka_j) = \sum_{(k-2;i,j)} p_{i_1}(1) \frac{p_{i_2}(2)}{1-p_{i_1}(2)} \cdots \times \frac{p_{i_{r-1}}(r-1)}{1-\sum_{\ell=1}^{r-2} p_{i_\ell}(r-1)} \times \frac{p_i(r)}{1-\sum_{\ell=1}^{r-1} p_{i_\ell}(r)} \times \frac{p_{i_{r+1}}(r+1)}{1-\sum_{\ell=1}^{r-1} p_{i_\ell}(r+1) - p_i(r+1)} \times \cdots \times \frac{p_j(k)}{1-\sum_{\ell=1}^{r-1} p_{i_\ell}(k) - p_i(k) - \sum_{\ell=r+1}^{k-1} p_{i_\ell}(k)} .$$

Now

$$\mathrm{Var}(\hat{Y}_2') = \frac{1}{n^2}\mathrm{Var}\left(\sum_{r=1}^{n}\sum_{i=1}^{N} {}_ra_i y_{1i}/p_i\right) + \frac{1}{m^2}\mathrm{Var}\left(\sum_{r=u+1}^{n}\sum_{i=1}^{N} {}_ra_i (y_{2i}-y_{1i})/p_i\right) + \frac{2}{mn}\mathrm{Cov}\left(\sum_{r=1}^{n}\sum_{i=1}^{N} {}_ra_i y_{1i}/p_i,\ \sum_{r=u+1}^{n}\sum_{i=1}^{N} {}_ra_i (y_{2i}-y_{1i})/p_i\right) .$$

Using the previously cited properties of the indicator variables ${}_ra_i$ it may be verified that

$$\frac{1}{n^2}\,\mathrm{Var}\left(\sum_{r=1}^{n}\sum_{i=1}^{N}{}_r a_i y_{1i}/p_i\right)=\frac{1}{n}\sum_{i=1}^{N}p_i z_{1i}^2 + \frac{1}{n^2}\sum_{i\neq j}\sum P(i,j\epsilon s)\, z_{1i}z_{1j}-Y_1^2,$$

$$\frac{1}{m^2}\,\mathrm{Var}\left(\sum_{r=u+1}^{n}\sum_{i=1}^{N}{}_r a_i (y_{2i}-y_{1i})/p_i\right)$$

$$=\frac{1}{m}\sum_{i=1}^{N}p_i(z_{2i}-z_{1i})^2+\frac{1}{m^2}\sum_{i\neq j}\sum P(i,j\epsilon s_1)(z_{2i}-z_{1i})\cdot (z_{2j}-z_{1j})-(Y_2-Y_1)^2,$$

where $z_{tj}=y_{tj}/p_j$, $t=1,2$ and $n-u=m$.

Also,

$$\frac{1}{mn}\,\mathrm{Cov}\left(\sum_{r=1}^{n}\sum_{i=1}^{N}{}_r a_i y_{1i}/p_i,\ \sum_{u+1}^{n}\sum_{1}^{N}{}_r a_i (y_{2i}-y_{1i})/p_i\right)$$

$$=\frac{1}{n}\sum_{1}^{N}p_i z_{1i}(z_{2i}-z_{1i})+\frac{1}{mn}\sum_{i\neq j}\sum P(i\epsilon s, j\epsilon s_1)\, z_{1i}(z_{2j}-z_{1j}) - Y_1(Y_2-Y_1).$$

Combining the foregoing 3 terms gives

$$\mathrm{Var}(\hat{Y}_2')=\sum_{1}^{N}p_i\left[\frac{z_{2i}^2}{n}+(z_{2i}-z_{1i})^2\left(\frac{1}{m}-\frac{1}{n}\right)\right] + \sum_{i\neq j}\sum\left[\frac{P(i,j\epsilon s)}{n^2}z_{1i}z_{1j} + \frac{P(i,j\epsilon s_1)}{m^2}(z_{2i}-z_{1i})(z_{2j}-z_{1j}) + \frac{2P(i\epsilon s,\ j\epsilon s_1)}{nm}z_{1i}(z_{2j}-z_{1j})\right]-Y_2^2. \quad (1)$$

Also,

$$\mathrm{Var}(\hat{Y}_2)=\frac{1}{n}\sum_{1}p_i z_{2i}^2+\frac{1}{n^2}\sum_{i\neq j}\sum P(i,j\epsilon s^*)\, z_{2i}z_{2j}-Y_2^2, \quad (2)$$

where s^* is the set of n units observed on the second occasion, and

$$\mathrm{Cov}(\hat{Y}_2',\hat{Y}_2)=\sum_{i\neq j}\sum\left[\frac{P(i\epsilon s, j\epsilon s^*)}{n^2}z_{1i}z_{2j} + \frac{P(i\epsilon s_1, j\epsilon s^*)}{nm}(z_{2i}-z_{1i})z_{2j}\right] + \frac{1}{n}\sum_{i}p_i z_{2i}\left(z_{2i}-\frac{u}{n}z_{1i}\right)-Y_2^2. \quad (3)$$

Expressions (1), (2) and (3), when combined, yield $\mathrm{Var}(\hat{Y}_{2c})$.

The optimum value of the weight Q which minimizes $\mathrm{Var}(\hat{Y}_{2c})$ is

$$Q_{opt}=[\mathrm{Var}(\hat{Y}_2)-\mathrm{Cov}(\hat{Y}_2',\hat{Y}_2)]/[(\mathrm{Var}(\hat{Y}_2')+\mathrm{Var}(\hat{Y}_2) - 2\,\mathrm{Cov}(\hat{Y}_2',\hat{Y}_2)].$$

The corresponding minimum variance is

$$\mathrm{Var}(\hat{Y}_{2c})=[\mathrm{Var}(\hat{Y}_2')\cdot\mathrm{Var}(\hat{Y}_2)-(\mathrm{Cov}(\hat{Y}_2',\hat{Y}_2))^2]/[\mathrm{Var}(\hat{Y}_2') + \mathrm{Var}(\hat{Y}_2) - 2\,\mathrm{Cov}(\hat{Y}_2',\hat{Y}_2)].$$

An alternative composite estimator $\hat{Y}_{2c}^*$ of Y_2 is

$$\hat{Y}_{2c}^*=Q^*\hat{Y}_2'+(1-Q^*)\hat{Y}_{2u},$$

where

$$\hat{Y}_{2u}=\sum_{r=n+1}^{n+u}\sum_{i=1}^{N}{}_r a_i y_{2i}/(up_i).$$

The variance of $\hat{Y}_{2c}^*$ is found by combining (1) with

$$\mathrm{Var}(\hat{Y}_{2u})=\frac{1}{u}\sum_{i}p_i z_{2i}^2+\frac{1}{u^2}\sum_{i\neq j}\sum P(i,j\epsilon s_2)\, z_{2i}z_{2j}-Y_2^2,$$

$$\mathrm{Cov}(\hat{Y}_2',\hat{Y}_{2u})=\frac{1}{nu}\sum_{i\neq j}\sum P(i\epsilon s, j\epsilon s_2)\, z_{1i}z_{2j} + \frac{1}{mu}\sum_{i\neq j}\sum P(i\epsilon s_1, j\epsilon s_2)(z_{2i}-z_{1i})z_{2j}-Y_2^2.$$

2.3 Special Case

As a check on the calculations, consider the case of simple random sampling without replacement.

Then $\hat{Y}_2'=N(\bar{y}_1+(\bar{y}_{2m}-\bar{y}_{1m}))$ where $\bar{y}_1$, $\bar{y}_{1m}$ are, respectively, the sample means based on all the sampled units and all matched units on the first occasion, and $\bar{y}_{2m}$ is the sample mean based on all matched units on the second occasion. A direct evaluation gives

$$\mathrm{Var}(\hat{Y}_2')=N^2\left[\left(\frac{1}{m}-\frac{1}{n}\right)(S_1^2-2S_{12})+\left(\frac{1}{m}-\frac{1}{N}\right)S_2^2\right]$$

where, e.g.,

$$S_{12}=\sum_{i=1}^{N}(y_{1i}-\bar{Y}_1)(y_{2i}-\bar{Y}_2)/(N-1).$$

This agrees with the result given by (1) with $p_i=1/N$ and $P(i,j\epsilon s)=n(n-1)/N(N-1)$ $(i\neq j)$.

Also, under simple random sampling, $\hat{Y}_2=N\bar{y}_2$ (where $\bar{y}_2$ is the sample mean based on all n sampled units on the second occasion) with variance

$$\mathrm{Var}(\hat{Y}_2)=N(N-n)S_2^2.$$

An evaluation of $\mathrm{Var}(\hat{Y}_2)$ from (2) gives the same result. Finally

$$\mathrm{Cov}(\hat{Y}_2',\hat{Y}_2)=-NS_2^2$$

from either a direct evaluation or from (3). Similarly, $\text{Var}(\hat{Y}_{2c}^*)$ may also be checked.

3. NUMERICAL EXAMPLES

The composite estimators $\hat{Y}_{2c}$ and $\hat{Y}_{2c}^*$ with their optimum Q and Q^* values which minimize their respective variances are compared in efficiency with the pps estimator $\hat{Y}_2$ which is based on the current occasion information only. Because closed forms for $\text{Var}(\hat{Y}_{2c})$ and $\text{Var}(\hat{Y}_{2c}^*)$ are not available to permit analytic comparisons to be made, small populations of variate values were employed to affect these contrasts. (The populations studied were necessarily small, like those one might encounter in stratified sampling, since the differential affect of sampling with and without replacement is evident only when the sampling fractions are not negligible.) Four rotation sampling plans were applied to each population: $(n,m) = (2,1)$, $(3,2)$, $(3,1)$ and $(4,3)$. Two of the populations are given in Murthy (1967) where his single population of 34 villages was subdivided into two populations of sizes 16 and 17 (one outlier unit being discarded). The size measure characteristic is x = cultivated acreage in 1961 with y_1 and y_2 being the acreage under wheat in 1963 and 1964 respectively. A third population is a set of 14 farms in the province of Saskatchewan with x = 1980 farm acreage and y_1 and y_2 the 1980 and 1981 cropland acreages respectively. Two additional real data sets relating to populations of sizes 15 and 16 respectively are also analyzed.

Table 1 reports the relative efficiencies of $\hat{Y}_{2c}$ and $\hat{Y}_{2c}^*$ with respect to $\hat{Y}_2$ for each of these 5 populations and 4 sampling plans. A crucial parameter in each comparison is the correlation ρ_z between $z_{1i} = y_{1i}/p_i$ and $z_{2i} = y_{2i}/p_i$:

$$\rho_z = \frac{\sum_{i=1}^{N} p_i z_{1i} z_{2i} - Y_1 Y_2}{\sqrt{\sum_{i=1}^{N} p_i z_{1i}^2 - Y_1^2} \cdot \sqrt{\sum_{i=1}^{N} p_i z_{2i}^2 - Y_2^2}} .$$

The populations studied yielded ρ_z values ranging from 0.940 to 0.213. The optimum Q and Q^* values are also cited.

We note the following from these empirical studies: (1) The optimum Q values tend to be larger when ρ_z is large and as ρ_z decreases, the optimum Q tends to decrease in both $\hat{Y}_{2c}$ and $\hat{Y}_{2c}^*$. (2) The optimum Q value for $\hat{Y}_{2c}^*$ always exceeds that for $\hat{Y}_{2c}$. (3) As ρ_z decreases, the efficiency of $\hat{Y}_{2c}$ with respect to $\hat{Y}_2$ decreases (as expected), approaching unity as a lower bound under an optimum choice of Q. On the other hand, no such distinct behaviour for $\hat{Y}_{2c}^*$ is evident since $\text{Var}(\hat{Y}_{2c}^*)$ is not a monotone function of ρ_z. For small ρ_z values, small efficiency gains and losses relative to $\hat{Y}_2$ are both recorded. (4) $\hat{Y}_{2c}^*$ is more efficient than $\hat{Y}_{2c}$ for larger ρ_z values whereas $\hat{Y}_{2c}$ is more efficient than $\hat{Y}_{2c}^*$ for smaller ρ_z values. (5) If $\lambda = m/n$ is small, e.g., the (3,1) plan, then large efficiency gains using $\hat{Y}_{2c}^*$ in preference to $\hat{Y}_2$ result for large ρ_z's. For smaller ρ_z values, the (4.3) plan yields the largest gains using $\hat{Y}_{2c}$; the other three schemes give about the same gains.

Table 1. Efficiencies of composite wor estimators relative to ppswor estimator

Population	N	(n,m)	ρ_z	Q_{opt}	RE $\hat{Y}_2$ wrt $\hat{Y}_{2c}$	Q^*_{opt}	RE $\hat{Y}_2$ wrt $\hat{Y}^*_{2c}$
Murthy set 1	17	(2,1)	0.940	0.443	1.337	0.640	1.476
		(3,2)		0.456	1.230	0.738	1.367
		(3,1)		0.419	1.524	0.545	1.656
		(4,3)		0.462	1.188	0.793	1.309
Murthy set 2	16	(2,1)	0.867	0.377	1.205	0.615	1.364
		(3,2)		0.402	1.151	0.727	1.296
		(3,1)		0.336	1.282	0.499	1.459
		(4,3)		0.431	1.191	0.786	1.255
Acreages	14	(2,1)	0.546	0.181	1.083	0.466	0.927
		(3,2)		0.215	1.070	0.646	0.927
		(3,1)		0.137	1.092	0.295	0.933
		(4,3)		0.279	1.117	0.736	0.931
Data set 1	15	(2,1)	0.392	0.113	1.019	0.506	1.013
		(3,2)		0.140	1.017	0.670	1.013
		(3,1)		0.082	1.020	0.340	1.013
		(4,3)		0.330	1.170	0.752	1.012
Data set 2	16	(2,1)	0.213	0.061	1.007	0.451	0.898
		(3,2)		0.078	1.007	0.636	0.896
		(3,1)		0.042	1.007	0.280	0.908
		(4,3)		0.285	1.142	0.730	0.901

Table 2. Efficiencies of composite wor estimators relative to Raj's composite estimator

Population	N	(m,m)	ρ_z	RE $\hat{Y}^*_{2R}$ wrt $\hat{Y}_{2c}$	RE $\hat{Y}^*_{2R}$ wrt $\hat{Y}^*_{2c}$
Murthy set 1	17	(2,1)	0.940	1.038	1.146
		(3,2)		1.127	1.252
		(3,1)		1.215	1.320
		(4,3)		1.248	1.375
Murthy set 2	16	(2,1)	0.867	1.001	1.133
		(3,2)		1.106	1.246
		(3,1)		1.130	1.287
		(4,3)		1.309	1.380
Acreages	14	(2,1)	0.546	1.244	1.065
		(3,2)		1.330	1.151
		(3,1)		1.351	1.154
		(4,3)		1.507	1.257
Data set 1	15	(2,1)	0.392	1.095	1.089
		(3,2)		1.197	1.192
		(3,1)		1.200	1.192
		(4,3)		1.522	1.317
Data set 2	16	(2,1)	0.213	1.197	1.068
		(3,2)		1.293	1.152
		(3,1)		1.280	1.154
		(4,3)		1.589	1.254

It is worth remarking that even if one has a good correlation between the y_{1i} and y_{2i} values, composite estimation using $\hat{Y}_{2c}^*$ can still lead to efficiency losses compared to the use of the pps estimator $\hat{Y}_2$ based on current occasion data only. The critical factor is the correlation ρ_z between the z_{1i} and the z_{2i} values. One cannot lose using $\hat{Y}_{2c}$ under an optimum choice of Q for $\hat{Y}_{2c} = \hat{Y}_2$ with $Q = 0$.

Table 2 provides the relative efficiencies of $\hat{Y}_{2c}$ and $\hat{Y}_{2c}^*$ in the ppswor design with the estimator $\hat{Y}_{2R}^*$ used by Raj (1965) in his ppswr design described earlier. For more valid comparisons, it was not assumed that $V_{pps}(y_t)$ was the same for occasions $t = 1$ and 2; the optimum Q^* values for the given (n,m) combinations were utilized. In all cases, as expected, the estimators $\hat{Y}_{2c}$ and $\hat{Y}_{2c}^*$ in the wor design are more efficient than $\hat{Y}_{2R}^*$ in Raj's design. As n increases for a given ρ_z , the efficiency gain using the wor strategy increases. Finally, we note that Raj realized efficiency gains compared with no matching only when $\rho_z > 0.5$ whereas efficiency gains always resulted using $\hat{Y}_{2c}$ for any ρ_z in the wor situation.

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