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I. INTRODUCTION

We shall develop some extensions to the models proposed by Drew and Fuller (1980) and Proctor (1977) in their studies of the responsenonresponse of individuals given repeated opportunities to respond to a questionnaire. The analysis follows in the spirit of Deming (1953) and bears some methodological resemblance to the work of Politz and Simmons (1949, 1950) and Thomsen and Siring (1979). The concept of Poisson sampling as given by Hajek (1957) is also appropriate in our development.

Suppose that the population is partitioned into K categories based on the values of a discrete random variable. Associated with each unit in the kth category is a response probability $q_{i}\varepsilon$ [0,1] which is the conditional probability that a unit furnishes a response when sampled. Those units which have zero response probabilities are handled as follows: a proportion 1-Y of the population is composed of hard core nonrespondents who will never answer the survey. The relative categorical composition of this group must be assumed or estimated from other data, and we shall suppose in the sequel that the relative categorical composition of the hard core nonrespondents is the same as that of the rest of the population.

Two potentially undesirable features of this model are that: (1) the response probability is required to be constant over a category, and (2) the response probability is a function of the unit's category only, and is thus not dependent on the survey circumstances under which a response is solicited. The first assumption seems to be necessary in order to avoid any parametric modeling of the response probabilities, but may be weakened if the categorization is fine enough to admit only slight changes in the response probabilities of units in a given category. The second assumption can be largely

eliminated by the incorporation of parameters into the model which represent interviewer, questionnaire, or callback effects. These parameters will not be used in the sequel, but an approach in this area was made by Drew and Fuller (1980) and Thomsen and Siring (1979).

The general survey situation requires a selection of units according to a given sampling design. If some of the units do not respond when contacted, those units are recontacted in a second call. After R calls, the number of sampled units not responding to any call of the survey is recorded. We give the appropriate notation below for various sampling designs.

II. SIMPLE RANDOM SAMPLING

Let a simple random sample of n units be selected from a population of N units. Let n_{rk} be the number of sampled units observed in the k^{th} category on the r^{th} call, and let n_{o} be the number of sampled units unobserved after R calls. Let f_{k} be the proportion of units in the k^{th} category. Under the assumptions given above, the data $n_{o} = (n_{11}, \dots, n_{RK}, n_{o})$ satisfy a multinomial model with cell probabilities $n_{o} = (n_{11}, \dots, n_{RK}, n_{o})$, where:

$$\pi_{rk} = \Upsilon(1 - q_k)^{r-1} q_k f_k, r = 1, 2, ..., R;$$

and

$$\pi_{o} = 1 - \Upsilon + \Upsilon \sum_{k=1}^{K} (1 - q_{k})^{R} f_{k},$$

and we set

$$f_K = 1 - \sum_{j=1}^{K-1} f_j$$
.

Thus, π_{rk} is the probability that an individual in category k will respond on call r, and π_{o} is the probability that a sampled individual will not have responded by the Rth call. The associated log likelihood differs by a constant from:

$$\log L = \sum_{r=1}^{R} \sum_{k=1}^{K} n_{rk} \log \pi_{rk} + n_{o} \log \pi_{o}$$
 (1)

The solutions to the likelihood equations can be verified to be

$$\hat{q}_1, \hat{q}_2, \dots, \hat{q}_k, \hat{f}_1, \dots, \hat{f}_{K-1}, \hat{\gamma},$$
 (2)

where

1) \hat{q}_k is the solution to the Rth degree polynomial equation

$$\sum_{r=1}^{R} n_{.k}^{-1} n_{rk} (1-rq_k) = [1-(1-q_k)^R]^{-1} R q_k (1-q_k)^R,$$
(3)

2)
$$\hat{\Upsilon} = n^{-1} \sum_{k=1}^{K} [1 - (1 - \hat{q}_k)^R]^{-1} n_{.k},$$
 (4)

3)
$$\hat{f}_k = n^{-1} \hat{\Upsilon}^{-1} [1 - (1 - \hat{q}_k)^R]^{-1} n_{.k}$$
 (5)

and

$$n_{.k} = \sum_{r=1}^{R} n_{rk}$$

If the maximum likelihood estimates $f_1,\dots,f_{K-1}, \hat{q}_1,\dots,\hat{q}_K,\hat{T}$ are in the interval (0,1), then they are roots of the likelihood equations. Otherwise the roots must be found numerically. Using a result of Rao (1973, p. 361) it can be demonstrated that $(\hat{q}_1,\dots,\hat{q}_K,\hat{f}_1,\dots,\hat{f}_{K-1},\hat{T})$ is consistent for $(q_1,\dots,q_K,f_1,\dots,f_{K-1},\hat{T})$.

The estimated population proportions $\{\hat{f}_k\}$ can be used to construct estimators of the means of variables in the survey. Let Y be the variable of interest and let \bar{y}_k , $k=1,2,\ldots$, K be the sample mean of Y for the respondents in category k computed using the responses from all calls. Assume that the probability that an individual responds to any given call is independent of the Y-value of that individual. Then $E(\bar{y}_k|\underline{n}) = \bar{Y}_k$, the populaton mean of Y for units in the k^{th} category. Consider the estimator of the population mean of Y given by

$$\hat{\bar{\mathbf{y}}} = \sum_{k=1}^{K} \hat{\mathbf{f}}_{k} \bar{\mathbf{y}}_{k} \quad . \tag{6}$$

Observe that \bar{y}_k and \bar{y}_j are uncorrelated for $k \neq j$. Then, omitting terms of order in probability n^{-2} , the asymptotic variance of \bar{Y} is:

$$V(\hat{\bar{y}}) = \sum_{k=1}^{K} n^{-1} p_k^{-1} f_k S_k^2 + \bar{y}' V(\hat{f}) \bar{y}, \qquad (7)$$

where

$$s_k^2 = (N_k - 1)^{-1} \sum_{\ell=1}^{N_k} (Y_{k\ell} - \bar{Y}_k)^2$$
,

$$\bar{\mathbf{Y}}' = (\bar{\mathbf{Y}}_1, \bar{\mathbf{Y}}_2, \dots, \bar{\mathbf{Y}}_K),$$

$$p_{k} = \Upsilon[1 - (1-q_{k})^{R}],$$

 $\bigvee_{k}(f)$ is the K x K covariance matrix of (f_1, f_2, \ldots, f_K) and N_k is the number of population units in the k^{th} category. A consistent estimate of $V(\bar{Y})$ is given by

$$\hat{V}(\hat{\bar{Y}}) = \sum_{k=1}^{K} n^{-1} \hat{p}_{k}^{-1} \hat{f}_{k} s_{k}^{2} + \bar{y}' \hat{V}(\hat{f})\bar{y},$$
 (8)

where,

$$\hat{p}_{k} = \hat{\Upsilon}[1 - (1 - \hat{q}_{k})^{R}]$$
,

$$\bar{\mathbf{y}}' = (\bar{\mathbf{y}}_1, \bar{\mathbf{y}}_2, \dots, \bar{\mathbf{y}}_K)$$
,

$$s_k^2 = (n_{.k}^{-1})^{-1} \sum_{i=1}^{n_{.k}} (y_{ki} - \bar{y}_k)^2, \text{ if } n_{.k} \neq 0$$

$$= 0$$
 , if $n_{,k} = 0$, and

 $\hat{V}(\hat{f})$ is the inverse of the estimated information matrix of \hat{f} .

It is of interest to note that under appropriate limit assumptions for an infinite superpopulation, $\hat{\overline{Y}}$ is consistent for fixed K when R \geqq 2 and Y is assumed to have finite second moments in the superpopulation.

Consistency of \bar{Y} can also be shown when K is a fixed multiple of n^{α} , $0 < \alpha < 1$, under certain regularity conditions. The details are given in Drew (1981).

III. STRATIFIED RANDOM SAMPLING

Suppose the population is partitioned into L strata, the $\boldsymbol{\ell}^{th}$ stratum being of relative size $\mathbf{W}_{\ell}.$ For each stratum, let

f_{kk} = proportion of units in the kth category

 Υ_{ϱ} = proportion of units who are not hard core nonrespondents.

Also suppose $\mathbf{q}_{\mathbf{k}^{\text{th}}}$ is the response probability of units in the \mathbf{k}^{th} category, for all strata. Note that

$$\sum_{k=1}^{K} f_{\ell k} = 1, \ \ell = 1, \ 2, \dots, \ L.$$

Let n_ℓ be the size of the sample selected in the ℓ^{th} stratum, $n_{\ell r k}$ be the number of units observed in the k^{th} category of the ℓ^{th} stratum on the r^{th} cell, and $n_{\ell o}$ be the number of units selected in the ℓ^{th} stratum who are unobserved after R calls. Then the likelihood is proportional to

$$\prod_{\ell=1}^{L} w_{\ell} n_{\ell} \left\{ \left[\prod_{k=1}^{K} \prod_{r=1}^{R} \tau_{\ell} (1 - q_{k})^{r-1} q_{k} f_{\ell k} \right]^{n_{\ell r k}} \left[1 - \tau_{\ell} + \tau_{\ell} \sum_{k=1}^{K} (1 - q_{k})^{R} f_{\ell k} \right]^{n_{\ell} o} \right\}$$

The terms involving $\{W_{\underline{\varrho}}\}$ are constants, so the estimators of the parameters have forms analogous to the simple random sampling case:

1)
$$\hat{q}_k$$
 is the solution to (10)

$$\sum_{r=1}^{R} n_{..k}^{-1} n_{.rk}^{-1} (1-rq_k) = [1-(1-q_k)^R]^{-1} q_k R(1-q_k)^R,$$

(11)

2)
$$\hat{\Upsilon}_{\ell} = n_{\ell}^{-1} \sum_{k=1}^{K} [1 - (1 - \hat{q}_{k})^{R}]^{-1} n_{\ell,k}$$
, and (12)

3)
$$\hat{f}_{\ell k} = n_{\ell}^{-1} [1 - (1 - \hat{q}_{k})^{R}] \hat{r}_{\ell}^{-1} n_{\ell \cdot k}$$
,

where

$$\mathbf{n}_{.rk} = \sum_{\ell=1}^{L} \ell_{rk}, \ \mathbf{n}_{\ell.k} = \sum_{r=1}^{R} \mathbf{n}_{\ell rk},$$

and

$$n_{..k} = \sum_{r=1}^{R} n_{.rk}$$

Observe that q_k is obtained by ignoring stratum boundaries and applying the formula for q_k given for simple random sampling.

An estimator for the mean of Y is obtained as follows: let $\bar{y}_{\ell k}$ be the sample mean of Y in the k^{th} category of the ℓ^{th} stratum computed using the responses from all calls. Assume that the probability that an individual responds on any particular call is independent of the Y value of the individual. An estimator of \bar{Y} is

$$\hat{\bar{\mathbf{Y}}}_{str} = \begin{bmatrix} \mathbf{L} & \mathbf{W}_{\ell} & \sum_{k=1}^{K} \hat{\mathbf{f}}_{\ell k} & \bar{\mathbf{y}}_{\ell K} \end{bmatrix}$$
(13)

whose asymptotic variance is

$$V(\hat{\bar{Y}}_{str}) = \sum_{\ell=1}^{L} \sum_{k=1}^{K} (W_{\ell} f_{\ell k})^{2} V(\bar{y}_{\ell k}) + Y'_{w} V(\hat{f}_{str}) Y_{w},$$
(14)

where $V(\bar{y}_{\ell k})$ is the variance of $\bar{y}_{\ell k}$, $Y'_{\ell w} = (W_1 \bar{Y}_{11}, \dots, W_1 \bar{Y}_{1K}, \dots, W_L \bar{Y}_{L1}, \dots, W_L \bar{Y}_{LK})$

 $\mathbf{\bar{Y}}_{\ell k}$ is the population mean of units in the \mathbf{k}^{th} category of the ℓ^{th} stratum,

$$\hat{f}_{str} = (\hat{f}_{11}, \hat{f}_{12}, \dots, \hat{f}_{LK}), \text{ and}$$

 $\overset{\mathsf{V}}{\overset{\leftarrow}{\hat{\mathsf{f}}}}_{\mathsf{str}}$ is the IK \times IK covariance matrix of $\overset{\mathsf{f}}{\overset{\leftarrow}{\mathsf{f}}}_{\mathsf{str}}$.

An estimator of $V(\hat{Y}_{str})$ is

$$\hat{\mathbf{V}}(\hat{\bar{\mathbf{Y}}}_{\mathtt{str}}) = \sum_{\ell=1}^{L} \sum_{k=1}^{K} (\mathbf{W}_{\ell} \hat{\mathbf{f}}_{\ell k})^{2} \hat{\mathbf{V}}(\bar{\mathbf{y}}_{\ell k}) + \hat{\bar{\mathbf{Y}}}_{w}' \hat{\hat{\mathbf{V}}}(\hat{\bar{\mathbf{f}}}_{\mathtt{str}}) \hat{\bar{\mathbf{Y}}}_{w}',$$
(15)

where

 $\hat{\mathsf{V}}(\bar{\mathsf{y}}_{\ell\mathsf{k}})$ is the estimated sample variance of $\bar{\mathsf{y}}_{\ell\mathsf{k}'}$

$$\hat{\bar{Y}}'_{w} = (w_{1}\bar{y}_{11}, w_{1}\bar{y}_{12}, \dots, w_{L}\bar{y}_{L1}, \dots, w_{L}\bar{y}_{LK}),$$
and

 $\hat{\mathbb{V}}(\hat{f}_{str})$ is the inverse of the estimated information matrix of \hat{f}_{str}

IV. STRATIFIED TWO STAGE SAMPLING.

We extend the analysis of the preceding sections in two ways. First, suppose the popu-

lation of PSUs is stratified into I strata, of sizes $N_1,\ N_2,\ldots,\ N_I$, and a simple random sample of n_i PSUs is selected in the i^{th} stratum, $i=1,2,\ldots,I$. Let the number of elements in the s^{th} PSU of the i^{th} stratum be $M_{is},\ i=1,2,\ldots,I;$ $s=1,2,\ldots,N_i$, and suppose a simple random sample of m_{is} elements is selected in this PSU. Second, suppose interest centers on the total proportion λ_K of elements in the k^{th} category, pooled over PSUs and strata.

Let x_{isrk} be the number of elements observed in the k^{th} category of the s^{th} PSU of the i^{th} stratum on the r^{th} call. Let x_{iso} be the number of selected elements in the s^{th} PSU of the i^{th} stratum unobserved after R calls. Let X_{isk} be the number of units in the k^{th} category of the sampled portion of the s^{th} PSU of the i^{th} stratum, and let X_{isk}^* be the total number of elements in the k^{th} category of the s^{th} PSU of the i^{th} stratum. For a given category k, the joint distribution of

$$(x_{\ldots 1k}, x_{\ldots 2k}, \ldots x_{\ldots Rk}, X_{\ldots k} - x_{\ldots k})$$
 is multinomial with parameters X $_{\mathbf{k}}$ and

$$\{ \Upsilon q_k, \ \Upsilon (1-q_k) \ q_k, \ldots, \ \Upsilon (1-q_k)^{R-1} \ q_k, \ 1-\Upsilon + \Upsilon (1-q_k)^R \}.$$

The total likelihood is the product multinomial

$$\prod_{k=1}^{K} (x_{..1k}!)^{-1} \dots (x_{..Rk}!)^{-1} ((x_{..k} - x_{...k}!)^{-1} x_{..k}! \dots (\tau q_k)^{K-1} x_{..k}! \dots (\tau q_k)^{K-1} x_{..k}! \dots (\tau (1 - q_k)^{R-1} q_k)^{K..Rk} (1 - \tau + \tau (1 - q_k)^{R})^{K..k^{-K}} \dots k$$
(16)

the maximization of which is a problem in non-linear programming. If, however, we ignore the requirement that $X_{...k'}$, $k=1,\ 2,...,$ K be integers, then the maximum likelihood estimators can be written in a form analogous to (3), (4) and (5).

An estimator for \bar{Y} , is given by

$$\hat{\bar{Y}}_{sep} = \frac{\sum_{i=1}^{I} n_{i}^{-1} N_{i} \sum_{s=1}^{n_{i}^{-1}} m_{is}^{-1} M_{is} \sum_{k=1}^{K} \hat{\phi}_{k}^{-1} x_{is.k} \bar{y}_{isk}}{\sum_{i=1}^{I} n_{i}^{-1} N_{i} \sum_{s=1}^{n_{i}^{-1}} m_{is}^{-1} M_{is} \sum_{k=1}^{K} \hat{\phi}_{k}^{-1} x_{is.k}}$$

where \bar{y}_{isk} is the sample mean of the Y-values in the k^{th} category of the s^{th} PSU of the i^{th} stratum. The variance of (17) can be written more compactly as

$$\left(\sum_{i=1}^{I} \sum_{s=1}^{N_{i}} M_{is} \right)^{-2} \left[\sum_{i=1}^{I} N_{i}^{2} (n_{i}^{-1} - N_{i}^{-1}) (N_{i} - 1)^{-1} \sum_{s=1}^{N_{i}} (Z_{is} - \bar{Z}_{i...})^{2} \right]$$

$$+ \sum_{i=1}^{I} n_{i}^{-1} N_{i} \sum_{s=1}^{I} \sum_{k=1}^{K} X_{isk}^{*} v(\bar{y}_{isk}) + \sum_{k=1}^{K} \phi_{k}^{-2} v(\phi_{k}) (Y_{...k} - \bar{Y}X_{...k}^{*})^{2}$$

$$+ \sum_{i=1}^{I} \ n_{i}^{-1} \ N_{i} \ \sum_{s=1}^{N_{i}} \ M_{is}^{2} \ (M_{is}^{-1} - M_{is}^{-1}) (M_{is} - 1)^{-1} \ \sum_{k=1}^{K} \ X_{isk}^{*} \ (\overline{Y}_{isk} - \overline{Y}_{is})^{2}$$

$$+ \sum_{i=1}^{I} n_{i}^{-1} N_{i} \sum_{s=1}^{N_{i}} \sum_{k=1}^{K} m_{is}^{-1} M_{is} (\tau_{\phi_{k}})^{-1} (1 - \tau_{\phi_{k}}) X_{isk}^{*} (\bar{Y}_{isk} - \bar{Y})^{2} \bigg]$$
(18)

where

$$Z_{is.} = Y_{is.} - \overline{Y} M_{is}$$
,

and

$$\bar{Z}_{i..} = N_i^{-1} \sum_{s=1}^{N_i} Z_{is}$$
.

A consistent estimator of the variance given in Eq. (18) is

$$\left(\sum_{i=1}^{I}\frac{\mathbf{N}_{i}}{\hat{\mathbf{n}}_{i}}\sum_{s=1}^{\hat{\mathbf{n}}_{i}}\mathbf{M}_{is}\right)^{-2}\left[\sum_{i=1}^{I}\mathbf{N}_{i}^{2}\left(\mathbf{n}_{i}^{-1}-\mathbf{N}_{i}^{-1}\right)\left(\mathbf{n}_{i}-1\right)^{-1}\sum_{s=1}^{\hat{\mathbf{n}}_{i}}\left(\hat{z}_{is}\right)^{-2}\hat{z}_{i...}\right)^{2}$$

$$+ \sum_{i=1}^{I} \; n_{i}^{-1} \; N_{i} \; \sum_{s=1}^{n_{i}} \; \sum_{k=1}^{K} \; \hat{x}_{isk}^{*} \; v(\bar{y}_{isk})$$

$$+\sum_{k=1}^{K} \hat{\delta}_{k}^{-2} \hat{v}(\hat{\delta}_{k}) (\hat{Y}_{\ldots k} - \hat{Y}_{sep} \hat{X}_{\ldots k}^{*})^{2}$$

$$+ \sum_{i=1}^{I} \; n_{i}^{-1} \; N_{i} \sum_{s=1}^{n_{i}} \; M_{is}^{2} \; (m_{is}^{-1} - M_{is}^{-1}) (M_{is} - 1)^{-1} \; \sum_{k=1}^{K} \; x_{isk}^{*} \; (\bar{y}_{isk} - \bar{y}_{is})^{2}$$

$$+ \sum_{i=1}^{1} \, n_{i}^{-1} \, \, \mathbf{N}_{i} \, \sum_{s=1}^{n_{i}} \, \sum_{k=1}^{K} \, m_{is}^{-1} \, \, \mathbf{M}_{is} \, \, (\hat{\mathbf{r}}_{\delta_{k}})^{-1} \, \, (\mathbf{1} \, - \, \hat{\mathbf{r}}_{\delta_{k}}) \, \, \hat{\mathbf{x}}_{isk}^{*} \, \, (\bar{\mathbf{y}}_{isk} \, - \, \bar{\mathbf{y}}_{sep}) \, \, \right]$$

where

(17)

$$\hat{Z}_{is.} = \hat{Y}_{is.} - \hat{Y}_{sep} M_{is} ,$$

$$\hat{Z}_{i...} = n_i^{-1} \sum_{s=1}^{n_i} \hat{Z}_{is.} ,$$

$$\hat{Y}_{isk} = m_{is}^{-1} M_{is} \hat{\phi}_k^{-1} x_{is.k} \bar{y}_{isk} ,$$
and
$$\hat{X}_{isk}^* = m_{is}^{-1} M_{is} \hat{\phi}_k^{-1} x_{is.k} .$$

V. EXAMPLE

A mail survey of households in five communities of north central lowa was taken in 1975 to determine people's views of the community in which they lived. We use the variable "Age of Respondent" to partition the population into seven categories, and we wish to estimate the population mean of the variable "Number of Years in Community." An initial mailing was made to 1,023 households. After two additional mailings, a total of 787 units had responded. These data were analyzed as a simple random sample in Drew and Fuller (1980). The data were actually collected as five independent random samples from the towns of Clare, Clarion, Lehigh, Pocahontas and Stanhope. We treat the data as a stratified random sample of 787 observations in five strata. Size characteristics of the strata are given in Table 1.

TABLE 1 STRATUM SIZES, WEIGHTS, SAMPLE SIZES, NUMBER OBSERVED

STRATUM	CLARE	CLARION	LEHIGH	POCAHONTAS	STANHOPE
Size	66	1130	270	823	183
Weight	0.027	0.457	0.109	0.333	0.074
Sample Size	59	330	187	300	147
Number Observed	42	268	141	229	107

We first use model (9) to analyze the data. From Eq. (11), the estimates of q_k , k=1, $2, \ldots, K$ are the same as for the simple random

sampling case considered in Drew and Fuller (1980). These are:

$$\hat{g} = \begin{pmatrix} 0.376 & 0.518 & 0.464 & 0.462 & 0.627 & 0.594 & 0.313 \\ (0.108) & (0.065) & (0.064) & (0.046) & (0.048) & (0.057) & (0.088) \end{pmatrix}$$

where the estimated standard errors of the estimates are given in parentheses. The estimates and standard errors of Υ_{ℓ} , $\ell=1,\,2,\ldots,\,5$ are

$$\hat{\Upsilon} = \begin{pmatrix} 0.811 & 0.932 & 0.869 & 0.891 & 0.856 \\ (0.101) & (0.034) & (0.054) & (0.040) & (0.070) \end{pmatrix}$$

The values of $\hat{f}_{\ell k}$, $\ell = 1, 2, ..., 5$; k = 1, 2, ..., 7 and their standard errors are given in Table 2.

TABLE 2 ESTIMATES OF $f_{\ell k}$, ℓ = 1, 2,..., 5, k = 1, 2,..., 7 WITH ESTIMATED STANDARD ERRORS IN PARENTHESES

AGE	CLARE	CLARION	LEHIGH	POCAHONTAS	STANHOPE
15-24	0.027	0.009	0.098	0.064	0.073
	(0.040)	(0.022)	(0.037)	(0.019)	(0.039)
25-34	0.188	0.102	0.104	0.169	0.125
	(0.088)	(0.016)	(0.033)	(0.024)	(0.045)
35-44	0.198	0.154	0,225	0.150	0.141
	(0.091)	(0.021)	(0.047)	(0.024)	(0.048)
45-54	0.219	0.180	0.148	0.161	0.150
	(0.090)	(0.020)	(0.037)	(0.022)	(0.052)
55-64	0.132	0.192	0.195	0.138	0.143
	(0.074)	(0.020)	(0.042)	(0.021)	(0.046)
65-74	0.112	0.143	0.112	0.124	0.145
00 7.	(0.069)	(0.018)	(0.036)	(0.020)	(0.046)
75+	0.124	0.130	0.118	0.194	0.223
7.55	(0.056)	(0.023)	(0.036)	(0.031)	(0.066)

The mean number of years in community is given by stratum and category in Table 3. For the data in Tables 1, 2 and 3, we have

$$\hat{\bar{Y}}_{str} = 30.61$$
(0.98)

As one would expect, $\hat{\overline{Y}}_{\mbox{str}}$ has a smaller estimated standard error than the simple random sampling estimator

$$\hat{\bar{Y}} = \sum_{k=1}^{K} \hat{f}_{k} \bar{y}_{k} = 31.40$$
(1.14)

given by Eq. (6), and whose estimated standard error is given by Eq. (8). The usual estimator of \bar{Y} for a stratified sample is

$$\sum_{\ell 1}^{\rm L} \ \, {\rm W}_{\ell} \bar{{\rm y}}_{\ell} = \underset{(0.83)}{30.08} \ \, , \label{eq:constraint}$$

where y_{ϱ} is the simple mean of the observations in the ℓ^{th} stratum. Note that $\hat{\overline{Y}}$ is larger than the usual estimator because of the estimated low response probability of old people, but $\hat{\overline{Y}}$ necessarily has a larger standard error because of its estimation of the response probabilities.

By letting $M_{is} = 1$, for all PSUs, the stratified two stage estimator (17) can be used here, giving:

$$\hat{\bar{Y}}_{sep} = 30.61$$
 (0.90)

TABLE 3
MEAN NUMBER OF YEARS IN COMMUNITY BY
TOWN AND AGE CATEGORY, WITH
ESTIMATED STANDARD ERRORS IN PARENTHESES

AGE	CLARE	CLARION	LEHIGH	POCAHONTAS	STANHOPE
15-24	2.000	9,391	13.167	4.615	8.429
	(0.000)	(1.901)	(2.744)	(1.756)	(2.635)
25-34	8.375	13.786	11.933	10,900	11.857
ì	(3.295)	(2.255)	(3.200)	(1.617)	(2.820)
35-44	18.500	19.250	26.000	18.059	14.733
	(5.435)	(2.360)	(2.124)	(2.149)	(2.766)
45-54	31.100	28.547	31,130	28.756	18.278
	(6.575)	(2.101)	(4.022)	(2.848)	(3.273)
55-64	35.500	38.107	40.793	35.686	33.177
	(9.447)	(2.184)	(3.594)	(3.392)	(4.947)
65-74	55.200	43.775	41.188	34.233	49.647
	(8.924)	(3.405)	(5.043)	(4.227)	(3.381)
75+	62.000	53.692	66.385	52.531	59.842
	(10.033)	(4.208)	(4.805)	(4.740)	(5.471)

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