EMPIRICAL RESEARCH INVOLVING AN ALTERNATIVE VARIANCE ESTIMATOR TO THE COLLAPSED STRATUM VARIANCE ESTIMATOR

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I. INTRODUCTION

In sample designs with one sample primary sample unit (PSU) selected per stratum, a collapsed stratum variance estimator is generally employed. Shapiro and Bateman [4] presented some theory and limited empirical evidence in support of the premise that a without replacement variance estimator produces an estimate of the variance with both smaller bias and smaller variance than a collapsed stratum estimator. This paper presents considerably more empirical evidence which, in general, confirms the empirical evidence of the earlier paper. A brief description of the variance estimators being compared and the concepts involved is given in this section. It is recommended, however, that [4] be read for more details. Section II presents the detailed empirical data in which one particular without replacement variance estimator is compared to the collapsed stratum variance estimator. Section III summarizes the paper, recommends that a without replacement variance estimator be used instead of a collapsed stratum variance estimator, and presents plans for future work in this area.

Surveys are frequently designed and conducted with one PSU selected per stratum. In such a design, no unbiased estimate of variance is possible. Generally, some form of the collapsed stratum variance estimator is employed. For the estimator, pairs of strata are formed for those strata comprised of more than one PSU. The aim is to pair strata with similar characteristics and approximately equal measures of size. The form of the estimator considered in this paper is intended for estimating the variance of simple unbiased estimates. However, the basic principles also apply when the collapsed strata are used with more sophisticated weighting in conjunction with replication or linearized variance estimation. Also, to keep things simple and eliminate extraneous concerns, it is assumed throughout this paper that a census is conducted within each sample PSU, i.e., there is no within PSU variance, and between PSU variance is equal to the total variance.

Let y_{ijk} = estimate for characteristic of interest for k^{th} PSU in j^{th} stratum within i^{th} pair of strata.

j takes on values of only 1 and 2 and thus (i,j) denotes a unique stratum.

Since a census of sample PSU's is being assumed, y_{ijk} is the estimate obtained from a census of the k^{th} PSU in stratum (i,j).

L_{ij} = the number of PSU's in the $(i,j)^{th}$ stratum.

N_{ijk} = the measure of size for the k^{th} PSU in $(i,j)^{th}$ stratum.

$N_{ij} = \sum_{k}^{L_{ij}} N_{ijk}$, $(i,j)^{th}$ stratum total.

$P_{ijk} = \frac{N_{ijk}}{N_{ij}}$ probability of selecting the k^{th} PSU in a single draw within the $(i,j)^{th}$ stratum.

We are interested in an estimated total,

$$\hat{y} = \sum_i \sum_j \frac{1}{P_{ijk}} y_{ijk} .$$

$P'_{ijk} = N_{ijk}/N_i$ probability of selecting the k^{th} PSU assuming a single draw within the i^{th} stratum.

$\pi_{ijk} = 2P'_{ijk}$ probability of selecting the k^{th} PSU from the i^{th} stratum assuming a Durbin selection of 2 PSU's in the stratum.

$$\pi_{i1k_1,2k_2} = \frac{2P'_{i1k_1}P'_{i2k_2}}{\lambda_i}\left[\frac{1}{1-2P'_{i2k_2}} + \frac{1}{1-2P'_{i1k_1}}\right]$$

joint probability of selecting the k_1 and k_2 PSU's in the i^{th} stratum assuming a Durbin selection method.

$$\lambda_i = 1 + \sum_j \sum_k \frac{P'_{ijk}}{1-2P'_{ijk}} .$$

The true one PSU per stratum variance of $\hat{y}$ is

$$VAR_T = \sum_i \sum_j \sum_k \frac{N_{ijk}}{N_{ij}}\left(\frac{y_{ijk}N_{ij}}{N_{ijk}} - y_{ij}\right)^2 \quad (1)$$

The usual form of the collapsed stratum variance estimator [4] is:

$$VAR_{CS} = \sum_i \left(\frac{2N_{i2}}{N_i}\frac{y_{i1k_1}N_{i1}}{N_{i1k_1}} - \frac{2N_{i1}}{N_i}\frac{y_{i2k_2}N_{i2}}{N_{i2k_2}}\right)^2 \quad (2)$$

where k_1 is the sample PSU in stratum (i,1) and k_2 is the sample PSU in stratum (i,2).

Roughly speaking, the collapsed stratum variance estimator acts as if each pair of collapsed strata had actually been one stratum in the first place, and two sample PSU's had been selected with replacement with probability proportionate to the measure of size N_{ijk}. When the two paired strata have equal measures of size, the collapsed stratum variance estimator (2) is always an overestimate of variance under circumstances specified in [4].

The empirical results in this paper deal with the Yates-Grundy [6] without replacement variance estimator using Durbin probability.[1]

$$VAR_D=\sum_i\left(\frac{\pi_{i1k_1}\pi_{i2k_2}-\pi_{i1k_1,2k_2}}{\pi_{i1k_1,2k_2}}\right)\left(\frac{y_{i1k_1}}{\pi_{i1k_1}}-\frac{y_{i2k_2}}{\pi_{i2k_2}}\right)^2 \quad (3)$$

The empirical results, as well as the theory in the earlier paper, indicate that the Durbin estimator usually has a smaller bias than the collapsed stratum estimator. The empirical results are mixed with respect to which estimator has a smaller variance, but show the Durbin estimator highly likely to be subject to a smaller mean square error.

II. EMPIRICAL RESULTS

Empirical results for part of South Dakota were presented in [4]. Results are presented here for a more extensive empirical investigation performed in those parts of 14 additional States that are nonself-representing (NSR) in the 1976 expansion of the Current Population Survey (CPS). The stratification for the 1976 CPS expansion is described in [1]. The list of PSU's and the collapsed strata for South Dakota and 14 additional States is given in [5]. The collapsed pairs of strata were formed by pairing NSR strata within a State according to their mean estimates of unemployment rate, with no regard for the population of the strata. The 1970 Census data were used to form the unemployment estimates. Variances were estimated for the 1960 Census estimate of total unemployment. Thus, the same characteristic was used as the characteristic of interest and the basic stratification variable, but with a 10-year time difference to keep the correlation from being unreasonably high.

Table A compares the expected values of the Durbin variance estimator (Formula (3)), and the collapsed stratum variance estimator (Formula (2)) with the true variance for all 15 States. The relative bias of these estimates is also listed in this table.

Table A shows that for 12 out of 15 States, the Durbin estimator has smaller bias as compared to collapsed stratum estimator. It also shows that both estimators underestimated the variance in 3 States and Durbin has smaller bias for 2 of these States. In 4 States, the direction of bias was different for the two estimators. Durbin underestimated in all these States and has smaller bias in 2 of these 4 States. Table A further shows that the expected value of the collapsed stratum estimator is usually higher (13 out of 15 States) than the Durbin estimator. In summary, Durbin is clearly preferable with respect to bias in 10 of the 15 States, sometimes by quite a bit (e.g., Idaho and New Mexico). Durbin is relatively worst in Oklahoma where it has a -16% relative bias compared to + 8% for collapsed stratum: -16%, however, is not a large relative basis.

Two hundred sets of sample PSU's, each set containing one PSU from each stratum, were selected with probability proportionate to 1970 Census populations.[2] Each of the 200 selections was independent so that the same selection could be repeated more than once. For each set, three variance estimates--Durbin, collapsed stratum, and actual $(\text{deviation})^2$--were computed. The actual $(\text{deviation})^2$ for a particular sample is the "true error" for that sample. The formula used for the actual $(\text{deviation})^2$ for the K^{th} set of PSU's was

$$DEV_K^2=\left(\sum_i\sum_{j=1}^{2}\frac{y_{ijk}N_{ij}}{N_{ijk}}-Y\right)^2 \quad (4)$$

Table B compares the results of these statistics for the 200 samples combined. Column 4 and column 5 measure the closeness of Durbin and collapsed stratum from the "true error" respectively. For example, the first entry in column (5) was calculated by taking the absolute difference between the collapsed stratum variance estimate and the $(\text{deviation})^2$ for each of the 200 samples, and then taking the average of the 200 absolute differences.

Tables A and B show that the average Durbin estimate agrees quite closely to its expected value; the collapsed stratum estimate also agrees quite closely to its expected value, and the average actual $(\text{deviation})^2$ is quite close to the true variance. Most importantly, comparing columns (4) and (5) of table B show that the Durbin estimate tends to be somewhat closer to the actual $(\text{deviation})^2$ than collapsed stratum does. Of the 15 States, Durbin performs better in all but 2, sometimes by large amounts (e.g., column 4 is 63% of column 5 for Idaho). Collapsed stratum is only slightly preferable in the 2 States it is superior in. The difference of estimates from actual $(\text{deviation})^2$ were examined in detail for 5 States. These States were chosen such that they include one State for which both the Durbin and the collapsed stratum underestimated the variance, one State for which both estimators overestimated, one State for which only one estimator underestimated. The tallies for 200 sample estimates for these States were obtained to show how often a particular estimate was closer to actual $(\text{deviation})^2$. This was also identified by the direction of the difference and is summarized in table C.

Table C indicates that the Durbin estimator was usually closer to the actual $(\text{deviation})^2$, even for Mississippi where collapsed stratum has a smaller relative bias. Another important result that can be drawn from this table is that the collapsed stratum variance estimator is higher than the Durbin estimator most of the time.

Table D shows the estimated variances on the Durbin estimate and collapsed stratum estimate, and the absolute difference between Durbin and the DEV_K^2, and the absolute difference between collapsed stratum and the DEV_K^2. The formula used for the variance on the Durbin variance estimate is as follows:

$$VAR(\text{Durbin Estimate})=\sum_g^{200}\left(VAR_D(g)-\frac{\sum_g^{200}VAR_D(g)}{200}\right)^2\Bigg/199$$

Similar formulae were used for the other variances.

In 9 of the 15 States, Durbin has lower variance and the variance of the absolute difference of the Durbin estimate from the DEV_K^2 was smaller for 8 States.

Since both estimates were biased and the variances on one of the variance estimators were lower about half the time, the mean square errors (MSE) were computed for the two estimators. If the Durbin estimate has the lower bias and the lower variance in a State, then MSE for that State was not computed. Thus, MSE was computed only for 8 States, for which collapsed stratum estimate has either lower bias or lower variance, or both lower bias and variance. Table E summarizes the mean square errors of the two estimators by State.

The most important comparison in this table is between the mean square errors of the Durbin and collapsed stratum estimates. The collapsed stratum estimates have smaller MSE's for only 3 States; 2 of them within 10% of the Durbin. In the remaining 5 States, usually much lower mean square errors were observed for the Durbin estimator. In addition, of course, for all 7 States not included in table E, Durbin has smaller MSE.

In summary, the empirical evidence points unequivocally to Durbin being substantially preferable to collapsed stratum in most of the States. Durbin usually has smaller bias, it is generally closer to the DEV_K^2 and often has lower mean square error even if the collapsed stratum estimate has either lower bias or lower variance. The Durbin estimate has smaller variance about half the time. One disadvantage of the Durbin estimator is that it tends to underestimate more often than the collapsed stratum.

The results found here may not, of course, hold for all situations. But in general, we think that Durbin should be better than collapsed stratum with respect to its mean square error and its expected value. Particularly important are the table B results showing the Durbin estimator generally closer to the actual $(\text{deviation})^2$ than collapsed stratum for almost every State. As observed, it did not have smaller variance for about half of the States that we studied. Also in this empirical study, collapsing of strata within a State was done with regard to their mean estimates of unemployment rate and very little consideration was given to having similar strata populations. This could have been the reason that collapsed stratum and Durbin sometimes result in underestimates of variance (see [2]). In a number of States, however, this pairing could not have been improved upon greatly with respect to closer stratum size (within collapsed stratum).

To investigate the effect of variation in stratum size within a collapsed pair, 2 strata in Mississippi were restratified such that they were similar in their size (N_{i1} = 248,780; N_{i2} = 263,189). These 2 strata were in the same collapsed pair and previously had larger differences (N_{i1} = 185,051; N_{i2} = 326,918) in their sizes than the strata of any other pair in the State. Similar restratification for 2 strata in a pair was also carried out for Minnesota but the difference between the populations of the paired strata was only reduced from 138,036 to 103,775. For both States, the collapsed restratified strata in a pair still had similar mean estimates of unemployment rate. Table F presents results for these 2 States. Table F indicates that the Durbin estimator performed better than the collapsed stratum estimator in terms of bias and mean square error after restratification.

Table G gives an overall summary of the comparison for each State. For 3 States, Durbin is better in all respects[3], while for 12 States the results are mixed. In no State is collapsed stratum better in all respects. If variance of the absolute deviations of estimates from actual $(\text{deviation})^2$ is disregarded, then Durbin is better for 5 States in all respects and the results are mixed for the remaining States.

III. CONCLUSION

The purpose of this paper is to compare the "Durbin" estimator to the collapsed stratum variance estimator. The empirical study indicates that one can slightly reduce the chance of underestimating the variance by using the collapsed stratum variance estimator but the user has to pay in terms of a relatively much higher bias and mean square error and of usually being farther from the actual $(\text{deviation})^2$.

Based on this study, we recommend that the "Durbin" variance estimator be used for a one PSU per stratum sample design, except when the sizes of strata in collapsed pairs differ greatly (see the discussion near the end of Section II), since this increases the likelihood that Durbin will result in an underestimate of variance.

An important decision to make in designing a sample survey is whether or not to stratify PSU's beyond a two PSU per stratum selection method. Historically, a major drawback in stratifying to a point where only one PSU is selected from a stratum is the inability to get an unbiased variance estimate. This may no longer be a strong argument against the Durbin estimator since it often has relatively small bias and mean square error as shown by this empirical study. Table H gives bias averaged over all 15 States, and over 14 States (excluding Idaho as an outlier, since the enormous relative bias for Idaho tends to dominate the average). The absolute bias for Durbin is less than ½ that of collapsed stratum. Taking into account the sign of the bias, Durbin compares even more favorably.

A great deal of further investigation can be done on this topic. (1) More theoretical work can be done to determine under exactly what circumstances Durbin can be expected to have a smaller bias and not be an underestimate. (2) The question that Hartley, Rao, and Kiefer [2] have raised about the collapsed stratum estimator sometimes being an underestimate needs to be studied in regards to its implications for the suitability of the Durbin estimator. (3) All study so far has been for one stage selection--theoretical, and perhaps empirical, work is needed for two stage selection. (4) Other types of data, for different statistics, should be examined to see if the empirical results found here hold more generally. In particular, a larger number of strata should be dealt with so that the variance estimators will be more stable. (5) Additional without replacement estimators should be compared--those suggested to us are Murthy's estimator [3], Hartley-Rao-Kiefer estimator [2], and the collapsed stratum estimator with the use of a finite population factor. (6) Finally, comparisons could be made assuming a super population model. We intend to work in most of these areas, but would encourage others also.

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FOOTNOTES

[1]In this paper this variance estimator will be referred to simply as the Durbin estimator.

[2]The empirical results for Arkansas, Mississippi, Oklahoma and South Dakota were also produced for 500 samples. These results were similar to those that were obtained from 200 samples.

[3]Underestimation is considered an undesirable property of the variance estimator.

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Table A. EXPECTED VALUE OF TWO VARIANCE ESTIMATORS COMPARED TO TRUE VARIANCE

State	True Variance (1)	Expected Value of Durbin (2)	Expected Value of Collapsed Stratum (3)	|(1)-(2)| (4)	|(1)-(3)| (5)	Percent Relative Bias of Estimator $\left(\frac{\text{Expected-True}}{\text{True}}\right) \times 100$ Durbin	Collapsed Stratum
Arkansas (6)*	2,835,595	3,287,745	4,969,829	452,150	2,134,235	15.95	75.27
Idaho (3)	266,159	1,337,708	2,003,741	1,071,549	1,737,582	402.60	652.84
Iowa (6)	2,817,127	3,841,805	4,868,176	1,024,678	2,051,049	36.37	72.81
Kansas (5)	2,159,731	2,504,695	3,141,347	344,964	981,616	15.97	45.45
Minnesota (4)	11,031,963	8,270,689	7,297,599	2,761,274	3,734,364	-25.03	-33.85
Mississippi (5)	5,002,061	4,084,544	4,632,847	917,517	369,213	-18.34	-7.38
Montana (3)	1,115,708	1,010,248	1,214,181	105,460	98,473	-9.45	8.83
Nebraska (4)	1,030,793	1,147,678	1,259,757	116,884	228,964	11.34	22.21
New Mexico (4)	681,444	801,474	1,341,342	120,030	659,898	17.61	96.84
North Dakota (5)	794,006	743,440	1,037,522	50,566	243,517	-6.37	30.67
Oklahoma (4)	3,133,143	2,620,341	3,369,006	512,803	235,863	-16.37	7.53
Oregon (4)	3,315,178	2,429,252	2,354,265	885,926	960,913	-26.72	-28.99
South Dakota (6)	453,416	513,618	722,863	60,202	269,447	13.28	59.43
West Virginia (3)	2,427,192	2,298,296	3,478,642	128,896	1,051,450	-5.31	43.32
Wyoming (3)	105,845	116,594	170,667	10,749	64,822	10.16	61.24

*Number of collapsed strata in a State is given in parenthesis after the State's name.

Table B. MEAN SAMPLE ESTIMATES OF THE DURBIN VARIANCE ESTIMATE, THE COLLAPSED STRATUM VARIANCE ESTIMATE, AND THE ACTUAL DEVIATION SQUARED

State	Durbin Estimate (1)	Collapsed Stratum Estimate (2)	Actual (Deviation)[2] (3)	$\frac{1}{G}\sum_{g}^{G}\|(1_g)-(3_g)\|$ (see note 1) (4)	$\frac{1}{G}\sum_{g}^{G}\|(2_g)-(3_g)\|$ (see note 2) (5)	Percent Ratio (4)/(5) (6)
Arkansas	3,099,321	4,752,881	3,055,956	3,032,925	3,866,875	78.43
Idaho	1,287,817	1,961,914	242,187	1,091,757	1,741,939	62.67
Iowa	3,957,311	4,962,643	2,683,927	3,270,499	3,878,140	84.33
Kansas	2,435,849	3,066,646	2,131,677	2,218,711	2,506,019	88.54
Minnesota	7,773,099	6,896,191	10,475,599	8,923,482	8,873,567	100.56
Mississippi	3,661,284	4,282,747	5,035,403	5,258,259	5,444,332	96.58
Montana	1,013,756	1,210,466	1,022,239	1,141,941	1,249,558	91.47
Nebraska	1,068,233	1,185,975	1,088,810	1,166,689	1,218,518	95.75
New Mexico	756,791	1,305,699	684,055	707,292	1,102,580	64.15
North Dakota	742,799	1,025,402	812,036	864,045	992,708	87.04
Oklahoma	2,753,206	3,465,591	2,890,794	2,914,494	3,202,179	91.02
Oregon	2,719,007	2,377,125	3,863,056	3,446,226	3,407,513	101.14
South Dakota	519,183	739,262	483,016	511,895	648,834	78.89
West Virginia	2,225,667	3,203,288	2,418,193	2,486,259	3,130,730	79.41
Wyoming	109,005	154,646	118,149	143,999	171,731	83.85

[1]The absolute values of the differences between the Durbin estimate and the actual deviation squared averaged over the sets of sample PSU's.

[2]The absolute values of the differences between the collapsed stratum estimate and the actual deviation squared averaged over the sets of sample PSU's.

Table C. Frequency Distribution of Variance Estimates Closer to Actual (Deviation)²

State	Estimate Closer to Actual (Deviation)² — Out of All 200 Samples		When Only Durbin[1] Is Lower than DEV_K^2		When Both Are Lower Than DEV_K^2		When Both Are Greater Than DEV_K^2		Collapsed Stratum Estimate Higher Than Durbin
	Durbin	Collapsed Stratum	Durbin	Collapsed Stratum	Durbin	Collapsed Stratum	Durbin	Collapsed Statum	
Mississippi	106 (53.0%)	94 (47.0%)	5	3	10	67	91	24	166 (83.0%)
Montana	120 (60.0%)	80 (40.0%)	3	4	2	73	115	3	195 (97.5%)
North Dakota	125 (62.5%)	75 (37.5%)	6	8	1	65	118	2	197 (98.5%)
Oklahoma	122 (61.0%)	78 (39.0%)	3	9	8	59	111	10	182 (91.0%)
West Virginia	113 (56.5%)	87 (43.5%)	6	9	8	56	99	22	170 (85.0%)

[1]Whenever Durbin estimate was higher than actual (deviation)², the collapsed stratum estimate was also higher in all the samples in these States. But if only one estimate was lower than actual (deviation)², it happened to be Durbin all the time.

Table D. Comparison of the Variance of Variance Estimates
(All Numbers x 10⁹)

State	Variance of — Durbin Estimate (1)	Collapsed Stratum Estimate (2)	$\lvert Durbin-DEV_K^2\rvert$ (3)	$\lvert Collapsed\ Stratum-DEV_K^2\rvert$ (4)	Ratio of Variance — (1)/(2) (5)	(3)/(4) (6)
Arkansas	2,217	3,906	7,289	7,193	56.76	101.33
Idaho	1,398	1,021	1,229	900	136.92	136.56
Iowa	6,721	5,939	7,873	7,781	113.17	101.18
Kansas	2,624	3,116	5,316	5,149	84.21	103.24
Minnesota	46,256	38,117	113,420	117,971	121.35	96.14
Mississippi	9,253	9,134	42,132	39,998	101.30	105.34
Montana	618	809	1,178	1,211	76.39	97.27
Nebraska	1,194	1,177	1,590	1,556	101.44	102.19
New Mexico	561	1,190	470	757	47.14	62.09
North Dakota	325	475	768	742	68.42	103.50
Oklahoma	3,232	4,023	9,947	10,056	80.34	98.92
Oregon	4,571	2,328	19,951	25,511	196.35	78.21
South Dakota	106	195	287	292	54.36	98.29
West Virginia	3,523	5,246	6,331	6,405	67.16	98.84
Wyoming	10	14	19	20	71.43	95.00

Table E. Comparison of the Mean Square Error (MSE) of Variance Estimates
(All MSE x 10⁹)

State	Mean Square Error of — Durbin Estimate (1)	Collapsed Stratum Estimate (2)	Estimate With Lower MSE (3)	Percent Ratio of MSE (1)/(2) (4)
Idaho	2,547	4,040	Durbin	63.04
Iowa	7,771	10,146	Durbin	76.59
Minnesota	53,880	52,062	Col. Strat.	103.49
Mississippi	10,095	9,270	Col. Strat.	108.90
Montana	630	819	Durbin	76.92
Nebraska	1,207	1,230	Durbin	98.13
Oklahoma	3,495	4,079	Durbin	85.68
Oregon	5,356	3,251	Col. Strat.	164.75

NOTE: For Mississippi, collaped stratum variance estimate has lower bias and lower variance, and for Montana and Oklahoma it has lower bias only. For other States, it has lower variance.

Table F. Results for Minnesota and Mississippi After Restratification

	Minnesota	Mississippi		Minnesota	Mississippi
True Variance (1)	10,886,352	4,467,677	Variance[1] of		
Expected Value of			Durbin (4)	45,387	7,545
Durbin (2)	8,187,905	4,099,833	Collapsed Stratum (5)	44,250	12,579
Collapsed Stratum (3)	7,668,569	5,366,847	$\lvert$Durbin-$DEV_K^2\rvert$	133,495	24,878
Relative Bias Durbin	-24.79%	-8.23%	$\lvert$Collapsed Stratum-$DEV_K^2\rvert$	137,879	24,643
Relative Bias Collapsed Stratum	-29.56%	20.13%	Variance Ratio (4)/(5)	102.57%	59.98%
Mean of			MSE[1] of Durbin	52,669	7,680
$\lvert$Durbin-$DEV_K^2\rvert$	10,472,676	4,809,192	MSE[1] of Collapsed Stratum	54,605	13,387
$\lvert$Collapsed Stratum-$DEV_K^2\rvert$	10,481,895	5,411,321	Estimate with Lower MSE Ratio of MSE's	Durbin 96.45%	Durbin 57.37%

[1]Variances and mean square errors (MSE) are number x 10^9.

Table G. Summary Table of Results for Two Estimates

State	Name of Estimate With: Expected Value Closer to True Value	Empirical Value Closer to DEV_K^2	Smaller Variance	Smaller Variance of Absolute Deviations	Smaller MSE
Arkansas	Durbin	Durbin	Durbin	Col. Strat.	Durbin
Idaho	Durbin	Durbin	Col. Strat.	Col. Strat.	Durbin
Iowa	Durbin	Durbin	Col. Strat.	Col. Strat.	Durbin
Kansas	Durbin	Durbin	Durbin	Col. Strat.	Durbin
Minnesota	Durbin[1]	Col. Strat.	Col. Strat.	Durbin	Col. Strat.
Mississippi	Col. Strat.[1]	Durbin	Col. Strat.	Col. Strat.	Col. Strat.
Montana	Col. Strat.[2]	Durbin	Durbin	Durbin	Durbin
Nebraska	Durbin	Durbin	Col. Strat.	Col. Strat.	Durbin
New Mexico	Durbin	Durbin	Durbin	Durbin	Durbin
North Dakota	Durbin[2]	Durbin	Durbin	Col. Strat.	Durbin
Oklahoma	Col. Strat.[2]	Durbin	Durbin	Durbin	Durbin
Oregon	Durbin[1]	Col. Strat.	Col. Strat.	Durbin	Col. Strat.
South Dakota	Durbin	Durbin	Durbin	Durbin	Durbin
West Virginia	Durbin[2]	Durbin	Durbin	Durbin	Durbin
Wyoming	Durbin	Durbin	Durbin	Durbin	Durbin
<u>Results for Two States After Restratification</u>					
Minnesota	Durbin[1]	Durbin	Col. Strat.	Durbin	Durbin
Mississippi	Durbin[2]	Durbin	Durbin	Col. Strat.	Durbin

[1]Both Durbin and collapsed stratum underestimated.

[2]Only Durbin underestimated.

TABLE H. Average Relative Bias of Two Variance Estimators Compared to True Variance

Average Absolute Bias (%)				Average Bias (%)			
All States		Excluding Idaho		All States		Excluding Idaho	
Durbin	Collapsed Stratum	Durbin	Collapsed Stratum	Durbin	Collapsed Stratum	Durbin	Collapsed Stratum
41.0	83.1	15.1	42.4	26.6	73.7	-.2	32.4