A SAMPLE SURVEY DESIGN TO ESTIMATE THE MEANS OF TWO OVERLAPPING SUBPOPULATIONS*

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1. Introduction

A common situation in survey sampling is for several organizations to collect information on a regular basis from the same segment of a population. In addition, some of the same units may actually be selected for use in two or more surveys, and the information to be obtained from them may be almost identical. When much of the required data for all the surveys could be collected simultaneously from the same set of sampling units, or from subunits of those units, this practice is statistically and cost inefficient.

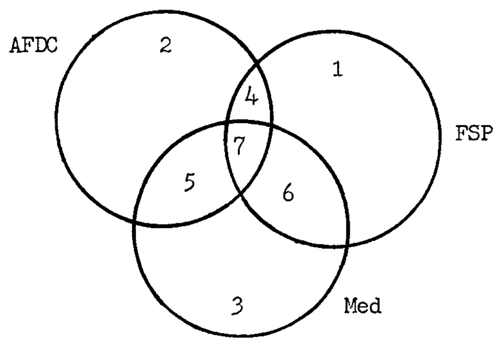
The procedure of simultaneously collecting data on the same unit for several surveys is sometimes referred to as integrated survey sampling. When a single population is composed of two or more overlapping subpopulations, independent surveys of the subpopulations can be integrated and the sample designs can be modified to either (1) reduce or eliminate multiple coverage in the overlap domain, or (2) to improve the estimates of the parameters of the individual subpopulations by advantageously combining information available from all the surveys. These features resemble some the features of the multiple frames techniques. However, while the multiple frames approach addresses the question of how to best sample a single population covered by two or more distinct, but overlapping, lists of units (frames), the integrated survey approach deals with the problems of sampling two or more distinct, but overlapping, populations or subpopulations, each covered by a single frame or list of units.

This investigation addresses the general problems of integrated survey sampling and estimation in the overlapping subpopulations context. For the special case of a single population covered by exactly two overlapping subpopulations, a basic integrated sample survey design is proposed which yields a more precise estimator of the means of the two individual subpopulations (combining all the available survey data) than estimators obtained with conventional survey designs. Alternatively, for fixed levels of precision, procedures are described for reducing the sample sizes of the two surveys in order to minimize coverage of the overlap domain, resulting in a more cost efficient system of surveys.

2. An Example of Overlapping Subpopulations

An example of overlapping subpopulations is found in the federal welfare system. The federal welfare system is most generally defined as the collection of federally funded family nutritional/income support programs, the agencies and their staffs which administer and govern those programs, and the population of all family units receiving at least one program benefit. Although there are a number of these programs in existence serving specific interests throughout the country, there are three major programs which collectively support the majority of "welfare recipients" nation-wide. It is these three programs which are of specific interest: Aid to Families with Dependent Children (AFDC), the basic welfare grant program administered by the Health Care Financing Administration (HFCA) of the U.S. Department of Health, Education, and Welfare (DHEW); the Food Stamp Program (FSP), administered by the Food and Nutrition Service (FNS) of the U.S. Department of Agriculture (USDA); and the Medicaid program (Med), administered by the Social Security Administration (SSA) of DHEW.

Every state directs these three programs within its geographic boundaries under federal statutes and with the support of public funds. In most cases the programs are managed independently of one another, either by different state agencies or by different staffs within the same agency, and each program supports its own participating constituency. In this regard the population of welfare recipients in each state actually consists of three uniquely defined and uniquely governed subpopulations. Under the collective regulations currently in force, it is possible, and highly likely, that a family unit eligible to participate in one program may also be eligible to participate in one or both of the other programs. Consequently, the subpopulations, though uniquely defined, inherently overlap. A schematic of this general overlapping subpopulations situation is provided in the following figure.



Definition of Overlapping Regions

1. FSP only
2. AFDC only
3. Med only
4. AFDC and FSP
5. AFDC and Med
6. FSP and Med
7. FSP, AFDC, and Med

Figure 1. Overlapping Subpopulations in the Federal Welfare System

Every six months each state is required to conduct a quality control survey of a sample of family units residing in the state and participating in each of the three major welfare programs. There are three independent surveys to conduct and three independent samples of fixed size from which to collect and analyze data (in most cases this requires the expertise of three separate, trained staffs). The purpose of these surveys is to validate the management practices of the state agencies directing the programs by determining the number of participating family units obtaining benefits in error. Of particular interest is the proportion of family units certified to participate in a given program, but which, because of oversight or fraud by the caseworker or recipient, are totally ineligible. States are subject to fiscal sanctions if their "ineligibility rates" in any of the three programs exceed established tolerances. Consequently there is considerable interest in integrating these sample surveys to reduce overall costs and/or to improve the estimates of the individual subpopulation characteristics of interest.

3. A Basic Overlapping Sample Surveys Design

Suppose a population of size Ω is composed of two overlapping subpopulations of sizes N and M, respectively; and suppose two independent sample surveys are conducted over the population, with each survey aimed at a particular subpopulation. The staffs of two distinct agencies or organizations conduct the surveys. Let the survey of subpopulation 1 (having size N) be designated as the primary survey (primary subpopulation, primary sample, etc.). Let the overlap domain be of size N_2 ($=M_2$), so that $N = N_1 + N_2$ and $M = M_1 + M_2$.

Although the subpopulations are known to overlap, it cannot be known prior to sampling which population elements fall in the overlap domain. Assume the sampling units for both surveys are the same size (i.e., same definition of sampling unit), and that both surveys obtain identical measurements on the units for the characteristc of interest.

Simple random samples of fixed size n and m, respectively, are selected for the two surveys. The units selected in each sample fall into two categories, or strata—those which belong to the overlap domain and those which do not (called "mixed" and "non-mixed" units, respectively). Assume no duplicate units are selected in the two samples owing to sampling the overlap domain twice.

The surveys are conducted by the two organizations. Within the context of each survey it is first determined for each sample unit whether or not it falls in the overlap domain. If a sample unit is determined to be a "mixed" unit, it is first surveyed with respect to membership in the subpopulation from which it was selected. Within the scope of this investigation, or subsequent to it, additional information is obtained on the characteristics of interest to the other survey. After the two surveys are completed, each of the two original samples is post-stratified into the two categories—"mixed" and "non-mixed" units. In this manner four subsamples of random size are formed: n_1 "non-mixed" units (with respect to the primary subpopulation) and n_2 "mixed" units in the primary sample; and m_1 "non-mixed" units (with respect to the second subpopulation) and m_2 "mixed" units in the second sample. Note that $n = n_1 + n_2$ and $m = m_1 + m_2$ (n_i, m_i, $i = 1,2$ all non-zero). The two subsamples of "mixed" units are two independent samples from the overlap domain. Improved estimates for the characteristics of interest in each subpopulation can be computed by advantageously combining the information available in the four subsamples.

4. Estimating the Mean of Either Subpopulation

Assume the total size of the overlap domain of the two subpopulations is known. Let $W_1 = N_1/N$ and $W_2 = N_2/N$, $N_1 + N_2 = N$, be the usual weights appropriate to stratified sampling. Let y be a characteristic of interest in the primary survey, and let y_{1hi} be the value of y on the ith unit in stratum h of the primary sample ($h = 1,2$). Then an unbiased estimate of $\bar{Y}_1$, the true mean of the primary subpopulation, obtained via proportional stratification of a single sample of size n, is given by

$$\bar{y}_1^{st} = \sum_{h=1}^{2} W_h \bar{y}_{1h}\ , \text{ where } \bar{y}_{1h} = \frac{1}{n_h} \sum_{i=1}^{n_h} y_{1hi}\ .$$

$$\text{Var}(\bar{y}_1^{st}) = \frac{(1-f)}{n} \sum_{h=1}^{2} W_h S_h^2\ ,$$

where $S_h^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (y_{1hi} - \bar{Y}_{1h})^2$ is the within-stratum variance, $f = n/N$ is the finite population correction factor (fpc), and $\bar{Y}_{1h}$ is the true mean of stratum h in the primary subpopulation.

If the sample of size n is post-stratified into n_1 and n_2 units, respectively, rather than proportionally stratified in advance, then the variance of the above estimator must be adjusted to reflect the randomness of n_h ($h = 1,2$). Hence,

$$\text{Var}(\bar{y}_1^{ps}|n_h) \doteq \sum_{h=1}^{2} \frac{W_h^2 S_h^2}{n_h} - \frac{1}{N} \sum_{h=1}^{2} W_h S_h^2\ .$$

The average value of $\text{Var}(\bar{y}_1^{ps})$ over all possible non-zero n_h must be obtained. Ignoring the case $n_h = 0$,

$$\text{Var}(\bar{y}_1^{ps}) = E_{n_h}[\text{Var}(\bar{y}_1^{ps}|n_h)] \doteq \frac{(1-f)}{n} \sum_{h=1}^{2} W_h S_h^2 + \frac{1}{n^2} \sum_{h=1}^{2} (1 - W_h) S_h^2\ ,$$

where $f = \frac{n}{N}$, given that $E(\frac{1}{n_h}) \doteq \frac{1}{nW_h} + \frac{1-W_h}{n^2 W_h^2}$.

Ignoring the fpc in the above equation,

$$\text{Var}(\bar{y}_1^{ps}) = E_{n_h}[\text{Var}(\bar{y}_1^{ps} \mid n_h)] \doteq \frac{1}{n} \sum_{h=1}^{2} W_h S_h^2 + \frac{1}{n^2} \sum_{h=1}^{2} (1 - W_h) S_h^2\ .$$

Now suppose additional information via a sample from the second subpopulation is available on stratum 2 (overlap domain). Let y_{1hi} and y_{2hi} be values of the characteristic y obtained on the ith units in strata h from subpopulations 1 and 2, respectively. A two-sample estimate of the mean, $\bar{Y}_1$, of the primary subpopulation for the characteristic y is given by

$$\bar{y}_1^{**} = W_1 \bar{y}_{11} + W_2[\beta \bar{y}_{12} + (1-\beta)\bar{y}_{22}],$$

where $\bar{y}_{1h} = \frac{1}{n_h} \sum_{i=1}^{n_h} y_{1hi}$ and $\bar{y}_{2h} = \frac{1}{m_h} \sum_{i=1}^{m_h} y_{2hi}$ are unbiased estimates of the means of the hth strata in subpopulations 1 and 2, respectively (a similar expression can be given for $\bar{y}_2^{**}$, a two-sample estimate of the mean, $\bar{Y}_2$, of the second subpopulation). $\bar{y}_1^{**}$ is unbiased for $\bar{Y}_1$, and its approximate variance is found similarly to $\text{Var}(\bar{y}_1^{ps})$. Ignoring the fpc's,

$$Var(\bar{y}_1^{**}|\beta) = W_1^2 \frac{S_1^2}{n_1} + W_2^2\beta^2 \frac{S_2^2}{n_2} + W_2^2(1-\beta)^2 \frac{S_2^2}{m_2} .$$

Now $Var(\bar{y}_1^{**}|\beta)$ is minimum when $\beta = \frac{n_2}{n_2 + m_2}$. Making this substitution in the above equation,

$$Var(\bar{y}_1^{**}|\beta = \frac{n_2}{n_2+m_2}) = W_1^2\frac{S_1^2}{n_1} + W_2^2S_2^2(\frac{1}{n_2+m_2}).$$

The average value of $Var(\bar{y}_1^{**}|\beta = \frac{n_2}{n_2+m_2})$ over all possible non-zero values of n_1, n_2, and m_2 must now be obtained.

$$Var(\bar{y}_1^{**}) = E_{n_2,m_2}[Var(\bar{y}_1^{**}|\beta = \frac{n_2}{n_2+m_2})] \doteq \frac{W_1S_1^2}{n} + \frac{W_2S_2^2}{n(1+\Delta)} + \frac{W_2S_1^2}{n^2} + \frac{(W_1+\Delta V_1)S_2^2}{n^2(1+\Delta)^3} ,$$

where $\Delta = \frac{mV_2}{nW_2}$, $E(\frac{1}{n_2+m_2}) \doteq \frac{1}{W_2n+V_2m} + \frac{nW_1W_2+mV_1V_2}{(W_2n+V_2m)^3}$,

and $V_1 = M_1/M$, $V_2 = M_2/M$. Analogous algebraic expressions may be obtained for the variances of multi-sample estimators of other subpopulation parameters.

5. Comparison with Conventional Estimators

To the order of approximation used in computing the variance, it can be shown that $\bar{y}_1^{**}$ is uniformly more precise than $\bar{y}_1^{ps}$, the estimate of the mean of the primary subpopulation obtained with conventional single sample post-stratification. In addition, if

$$\frac{S_1^2}{S_2^2} + \frac{W_1}{W_2} < n^2W_2[E(\frac{1}{n_2}) - E(\frac{1}{n_2+m_2})] ,$$

then $\bar{y}_1^{**}$ is more precise than $\bar{y}_1^{st}$, the estimate of $\bar{Y}_1$ obtained with proportional stratification. An apoproximate condition is given by

$$S_1^2/S_2^2 < n .$$

The relative precision of $\bar{y}_1^{ps}$ to $\bar{y}_1^{**}$ was computed for each of a number of combinations of values of the parameters n, m, W_2, and V_2 in the expression for the variance of $\bar{y}_1^{**}$, and for three different combinations of the within-stratum variances, S_1^2 and S_2^2. Some of the results are shown in Tables I-III.

6. Two Sample Size Reduction Schemes Based on the Two-Sample Estimator

Given two overlapping subpopulations, the size of the sample for the survey of the primary subpopulation may be reduced without altering the sample size for the second survey by solving for n" in the equation

$$Var(\bar{y}_1^{**}|n'',m,W_2,V_2) = C.$$

In this equation C is the desired precision of the estimate $\bar{y}_1^{**}$, n" is the reduced primary sample size, and $\bar{m}$ is the size of the sample for the second survey. Table IV displays the sizes of n" to which n may be reduced for several combinations of values of the parameters n, m, W_2, and V_2, when C is taken to be the precision associated with conventional single-sample post-stratification (of a primary sample size n).

It is also possible to reduce both original sample sizes by simultaneously solving the two equations

$$Var(\bar{y}_1^{**}|n',m',W_2,V_2) = C$$

and

$$Var(\bar{y}_2^{**}|n',m',W_2,V_2) = D .$$

In these equations, $\bar{y}_1^{**}$ is the two-sample estimate of the mean of the first subpopulation, $\bar{y}_2^{**}$ is the two-sample estimate of the mean of the second subpopulation, n' and m' are the sizes to which n and m may be reduced, respectively, C is the desired precision for $\bar{y}_1^{**}$, and D is the desired precision for $\bar{y}_2^{**}$. Table V shows some pairs of reduced sample sizes obtained when C and D are taken to be the precision associated with estimating the means of the two subpopulations using conventional single-sample post-stratification (of samples of sizes n and m, respectively).

The second procedure described above yields a uniformly smaller combined total sample size for the surveys of two overlapping subpopulations than does the first procedure. Both procedures allow fewer sample units to be physically surveyed than would ordinarily be required, using conventional single-sample survey designs, to maintain a desired level of precision for estimates of the subpopulation parameters of interest. Accomplishment of this goal depends on the sharing of information among survey organizations as specified in the survey design previously described. Specific sample survey situations will dictate a choice between the two procedures.

7. Summary of Findings

In the context of two overlapping surveys, an estimate of the mean of either subpopulation, $\bar{y}^{**}$, can be obtained by combining information in samples of size n and m, respectively, selected from the two subpopulations. $\bar{y}_1^{**}$, the estimate of the mean of the subpopulation of primary interest, is always more precise than $\bar{y}_1^{ps}$, the estimate of the mean obtained by post-stratifying the single sample of size n. The difference in the precision of $\bar{y}_1^{**}$ and the precision of $\bar{y}_1^{ps}$ is greatest for any combination of stratum variances when m is large relative to the size of n, and W_2 and V_2 are both large and about the same size. Even larger gains in precision are to be obtained using $\bar{y}_1^{**}$ if $S_2^2 > S_1^2$.

Therefore, if (1) the two subpopulations are about the same size and are substantially overlapped, (2) the size of the sample from the second subpopulation exceeds the size of the primary sample, and (3) the overlap domain is the most variable stratum, then $\bar{y}_1^{**}$ should be used to estimate the primary subpopulation mean. Otherwise $\bar{y}_1^{ps}$ is about as precise as $\bar{y}_1^{**}$, and the conventional single-sample estimator is recommended if the additional administrative costs of operating in the overlapping surveys mode are substantial (analogous results can be obtained regardless which subpopulation is chosen as the primary subpopulation). It can be demonstrated that these recommendations apply for small values of n and m as well as for large values, though the results noted above are likely to be more pronounced when n and m are both relatively large (for example, 100 or larger). In addition, for almost all choices of n, m, W_2, V_2, S_1^2, and S_2^2, $\bar{y}_1^{**}$ is also more precise than $\bar{y}_1^{st}$, the estimate of the mean obtained by proportional allocation of the single sample of size n among the strata of the primary subpopulation.

If the precision of the two-sample estimator, $\bar{y}_1^{**}$, is pre-specified, then the combined total sample size for the surveys of the two overlapping subpopulations can be reduced using either of two different procedures. The survey situation may allow only one of the sample sizes to be reduced, while the other remains fixed; or it may allow both sample sizes to be reduced simultaneously. The percent reduction in combined total sample size is greatest when both sample sizes are reduced simultaneously.

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TABLE I

Relative Precision of $\bar{y}_1^{ps}$ to $\bar{y}_1^{**}$, Large Sample Sizes, $S_2^2 = S_1^2$

	n = 3m				n = 2m				n = m				$n = \frac{1}{2}m$				$n = \frac{1}{3}m$			
$W_2 \backslash V_2$	.2	.4	.6	.8	.2	.4	.6	.8	.2	.4	.6	.8	.2	.4	.6	.8	.2	.4	.6	.8
.2	1.18	1.21	1.22	1.12	1.15	1.19	1.21	1.22	1.11	1.15	1.18	1.19	1.07	1.11	1.14	1.15	1.05	1.09	1.11	1.13
.4	1.31	1.43	1.48	1.52	1.25	1.36	1.43	1.47	1.15	1.25	1.32	1.36	1.09	1.15	1.21	1.25	1.06	1.11	1.15	1.19
.6	1.42	1.66	1.81	1.92	1.31	1.52	1.66	1.77	1.18	1.31	1.43	1.52	1.09	1.18	1.25	1.31	1.06	1.12	1.18	1.23
.8	1.51	1.91	2.23	2.48	1.36	1.66	1.92	2.14	1.19	1.36	1.52	1.66	1.10	1.19	1.28	1.36	1.06	1.13	1.19	1.25

TABLE II

Relative Precision of $\bar{y}_1^{ps}$ to $\bar{y}_1^{**}$, Large Sample Sizes, $S_2^2 = 2S_1^2$

	n = 3m				n = 2m				n = m				$n = \frac{1}{2}m$				$n = \frac{1}{3}m$			
$W_2 \backslash V_2$	.2	.4	.6	.8	.2	.4	.6	.8	.2	.4	.6	.8	.2	.4	.6	.8	.2	.4	.6	.8
.2	1.33	1.40	1.43	1.44	1.28	1.36	1.40	1.42	1.20	1.29	1.33	1.36	1.12	1.20	1.25	1.29	1.09	1.15	1.20	1.23
.4	1.52	1.75	1.87	1.96	1.40	1.61	1.75	1.84	1.23	1.40	1.52	1.61	1.13	1.23	1.32	1.40	1.09	1.17	1.23	1.30
.6	1.60	1.99	2.28	2.49	1.43	1.75	2.00	2.20	1.23	1.43	1.60	1.75	1.12	1.23	1.33	1.43	1.08	1.17	1.23	1.30
.8	1.61	2.13	2.59	2.99	1.42	1.80	2.14	2.45	1.22	1.42	1.61	1.80	1.11	1.22	1.32	1.42	1.07	1.14	1.22	1.28

TABLE III

Relative Precision of $\bar{y}_1^{ps}$ to $\bar{y}_1^{**}$, Large Sample Sizes, $S_2^2 = \frac{1}{2}S_1^2$

	n = 3m				n = 2m				n = m				$n = \frac{1}{2}m$				$n = \frac{1}{3}m$			
$W_2 \backslash V_2$	.2	.4	.6	.8	.2	.4	.6	.8	.2	.4	.6	.8	.2	.4	.6	.8	.2	.4	.6	.8
.2	1.09	1.10	1.11	1.11	1.08	1.10	1.10	1.11	1.06	1.08	1.09	1.10	1.04	1.06	1.07	1.08	1.03	1.05	1.06	1.07
.4	1.17	1.22	1.25	1.26	1.14	1.20	1.23	1.25	1.09	1.14	1.18	1.20	1.05	1.09	1.12	1.14	1.04	1.07	1.09	1.11
.6	1.26	1.38	1.45	1.50	1.20	1.32	1.40	1.45	1.12	1.21	1.27	1.32	1.06	1.12	1.17	1.21	1.04	1.08	1.12	1.15
.8	1.37	1.63	1.81	1.95	1.28	1.49	1.66	1.79	1.15	1.28	1.40	1.50	1.08	1.15	1.22	1.28	1.05	1.10	1.15	1.20

TABLE IV

Reduced Primary Sample Sizes for the Surveys of Two Overlapping Subpopulations

			$S_2^2 = S_1^2$				$S_2^2 = 2S_1^2$				$S_2^2 = \frac{1}{2}S_1^2$			
n	m	$W_2 \backslash W_2$	.2	.4	.6	.8	.2	.4	.6	.8	.2	.4	.6	.8
200	200	.2	178	171	168	166	162	150	145	142	188	184	182	181
		.4	170	154	146	140	155	130	117	110	182	173	167	164
		.6	165	142	127	117	155	121	100	87	176	161	151	144
		.8	162	131	108	91	157	119	89	68	170	146	129	117
	400	.2	171	166	164	163	150	142	139	137	184	181	180	179
		.4	154	140	134	131	130	110	102	98	173	164	160	158
		.6	142	117	105	99	122	87	74	67	161	144	136	131
		.8	131	91	73	64	119	68	48	40	146	117	102	94
	600	.2	168	164	162	161	145	139	137	136	182	180	179	178
		.4	146	134	130	127	117	102	96	94	167	160	157	155
		.6	127	105	97	92	100	74	65	61	151	136	130	126
		.8	108	73	61	56	89	48	38	34	129	102	91	85
	800	.2	166	163	161	161	142	137	136	135	181	179	178	178
		.4	140	131	127	125	110	98	94	92	164	158	155	154
		.6	117	99	93	90	87	67	61	58	144	131	126	123
		.8	91	64	56	52	69	40	34	31	117	94	85	81
	1000	.2	165	162	161	160	140	136	135	134	180	179	178	178
		.4	137	129	126	124	105	95	92	90	162	156	154	153
		.6	110	95	90	88	79	64	59	57	140	128	124	122
		.8	80	59	53	50	56	36	32	30	108	89	82	79
	1200	.2	164	161	161	160	139	136	134	134	180	178	178	178
		.4	134	127	125	123	102	94	91	89	160	155	153	152
		.6	105	93	89	87	74	61	57	56	136	126	122	121
		.8	73	56	51	49	48	34	30	29	102	85	80	77
	1400	.2	163	161	160	160	138	135	134	134	179	178	178	178
		.4	132	126	124	123	100	92	90	89	159	154	153	152
		.6	102	91	88	86	70	60	56	55	133	125	121	120
		.8	68	54	50	48	43	32	30	28	97	83	78	76
	1600	.2	163	161	160	160	137	135	134	133	179	178	178	178
		.4	131	125	123	122	98	92	89	88	158	154	152	152
		.6	99	90	87	85	67	58	56	54	131	123	121	119
		.8	64	52	49	47	40	31	29	28	94	81	77	75
	1800	.2	162	161	160	160	137	134	134	133	179	178	178	177
		.4	130	125	123	122	96	91	89	88	157	153	152	152
		.6	97	89	86	85	65	57	55	54	130	122	120	119
		.8	61	51	48	47	38	30	28	28	91	80	76	74
400	200	.2	371	357	348	343	351	325	311	302	384	376	372	369
		.4	365	340	322	309	349	310	282	261	379	364	354	346
		.6	362	331	305	284	352	309	273	243	374	353	336	322
		.8	361	325	291	262	356	314	274	238	368	339	314	292
	400	.2	357	343	336	333	325	302	291	286	376	369	365	363
		.4	340	309	292	281	310	261	235	220	364	346	335	329
		.6	331	284	253	233	309	243	200	174	353	322	302	289
		.8	325	262	214	181	314	238	177	135	339	292	257	232
	600	.2	348	336	331	329	311	291	284	279	372	365	362	361
		.4	322	292	277	269	282	235	215	204	354	335	326	321
		.6	305	253	226	210	273	200	165	147	336	302	284	272
		.8	292	214	168	143	274	177	121	94	314	257	223	202
	800	.2	343	333	329	326	302	286	279	276	369	363	361	359
		.4	309	281	269	262	261	220	204	196	346	329	321	316
		.6	284	233	210	198	243	174	147	134	322	289	272	263
		.8	262	181	143	125	238	135	94	78	292	232	202	185
	1000	.2	339	330	327	325	296	282	277	274	367	362	259	358
		.4	299	274	263	258	246	211	198	191	340	324	317	313
		.6	267	220	200	190	219	158	136	126	311	279	265	256
		.8	236	158	129	115	205	110	81	70	273	215	189	175
	1200	.2	336	329	326	324	291	279	275	273	365	361	359	358
		.4	292	269	290	255	235	204	193	188	335	321	314	311
		.6	253	210	194	185	200	147	130	122	302	272	259	252
		.8	214	143	120	109	177	94	73	65	257	202	180	168
	1400	.2	334	327	325	323	288	277	274	272	364	360	358	357
		.4	286	265	257	253	227	200	190	185	332	318	312	309

TABLE V

Reduced Sample Sizes for the Surveys of Two Overlapping Subpopulations

			$S_1^2 = S_2^2 = S_3^2$				$2S_1^2 = S_2^2 = 2S_3^2$				$\frac{1}{2}S_1^2 = S_2^2 = \frac{1}{2}S_3^2$			
n	m	$W_2 \backslash V_2$	.2	.4	.6	.8	.2	.4	.6	.8	.2	.4	.6	.8
200	200	.2	180 180	173 173	169 170	167 169	166 166	154 163	148 166	144 169	189 189	185 183	183 178	182 173
		.4	173 173	160 160	152 152	146 147	163 154	143 143	128 145	118 150	183 185	175 175	170 165	168 154
		.6	170 169	152 152	140 140	132 132	166 148	145 128	125 125	106 134	178 183	165 170	157 157	153 141
		.8	169 167	147 146	132 132	121 121	169 144	150 118	134 106	112 112	173 182	154 168	141 153	134 134
	400	.2	172 374	167 371	165 369	163 368	152 360	144 362	140 366	138 370	185 385	182 381	180 376	180 371
		.4	157 362	143 354	136 351	133 349	137 340	114 343	105 350	100 357	174 378	165 369	161 361	159 351
		.6	147 355	122 344	110 342	103 341	133 326	96 330	79 344	70 354	163 375	147 362	139 352	134 339
		.8	139 350	101 340	79 344	68 348	135 315	84 321	54 348	43 362	150 372	122 359	107 351	99 341
	600	.2	169 572	164 570	163 569	162 568	146 558	140 562	138 566	136 570	183 584	180 580	180 576	179 571
		.4	148 558	136 553	131 551	128 550	121 537	104 545	98 553	95 558	168 575	161 567	158 559	156 551
		.6	130 550	108 545	99 545	95 545	108 523	77 539	67 551	63 559	153 570	138 559	131 551	128 540
		.8	114 545	77 547	63 552	58 555	102 510	53 544	40 561	35 569	132 568	105 559	93 553	68 545
	800	.2	167 771	163 769	162 769	161 768	143 757	138 762	136 766	135 770	182 783	180 779	179 776	179 771
		.4	142 756	132 752	128 751	126 750	112 737	99 746	95 754	92 759	165 773	159 766	156 759	155 750
		.6	119 748	101 746	94 746	91 747	92 726	69 744	62 754	59 761	146 767	133 758	127 750	125 740
		.8	96 745	66 752	57 756	53 758	76 720	42 754	35 766	32 772	119 766	96 760	87 754	83 747
	1000	.2	165 971	163 969	162 968	161 968	141 957	137 962	136 967	135 970	181 983	179 979	179 975	178 971
		.4	138 955	130 952	127 951	125 951	107 938	96 947	93 954	91 960	162 972	157 965	155 958	154 950
		.6	112 947	97 947	91 947	89 948	82 930	65 947	60 956	57 962	141 966	129 958	125 950	123 941
		.8	83 948	60 955	54 958	51 960	60 931	37 959	32 968	30 973	110 965	90 960	83 955	80 948
400	200	.2	374 172	362 157	355 147	350 139	360 152	340 137	326 133	315 135	385 185	378 174	375 163	372 150
		.4	371 167	354 143	344 122	340 101	362 144	343 114	330 96	321 84	381 182	369 165	362 147	359 122
		.6	369 165	351 136	342 110	344 79	366 140	350 105	344 79	348 54	376 180	361 161	352 139	351 107
		.8	366 163	349 133	341 103	348 68	370 138	357 100	354 70	362 43	371 180	351 159	339 134	341 99
	400	.2	360 360	346 347	339 340	335 337	333 333	309 327	296 331	289 337	378 378	370 366	367 356	364 345
		.4	347 346	320 320	304 304	293 294	327 309	286 286	256 289	236 300	366 370	350 350	341 330	335 306
		.6	340 339	304 304	280 280	263 264	331 296	289 256	250 250	212 268	356 367	330 341	315 315	306 281
		.8	337 335	294 293	264 263	241 241	337 289	300 236	268 212	223 223	345 364	306 335	281 306	268 268
	600	.2	351 553	338 543	333 539	330 536	316 524	295 525	286 532	281 539	373 574	366 563	363 554	362 543
		.4	328 533	299 511	284 502	275 496	297 490	249 483	224 496	211 509	356 562	338 542	329 524	324 503
		.6	315 522	260 492	241 480	223 475	297 468	231 451	185 474	159 497	340 557	309 530	292 506	282 478
		.8	306 515	241 479	195 472	163 478	304 454	231 420	156 460	109 506	320 553	270 524	238 500	219 474
	800	.2	345 749	334 741	330 738	327 736	306 720	288 724	281 732	277 739	370 771	364 761	361 753	360 743
		.4	314 725	286 708	273 701	266 698	273 681	229 686	209 700	200 713	348 757	331 738	323 721	318 702
		.6	293 711	244 689	219 684	205 683	265 653	191 662	156 689	139 709	326 749	294 724	278 703	268 678
		.8	277 702	199 682	156 689	133 697	269 631	167 644	106 700	83 727	298 745	242 719	212 702	195 682
	1000	.2	341 947	331 940	328 937	326 936	299 918	284 924	278 932	275 939	367 969	362 960	360 952	359 942
		.4	304 920	277 906	266 901	260 899	256 877	216 888	201 903	193 915	342 953	326 935	319 920	315 901
		.6	275 904	227 889	206 887	195 887	237 847	168 872	142 898	130 915	315 944	283 921	268 902	260 879
		.8	250 894	170 889	136 900	120 907	235 821	125 873	86 917	72 936	279 939	222 918	195 904	181 888
600	200	.2	572 169	558 148	559 130	545 114	558 146	537 121	523 103	510 102	584 183	575 168	570 153	568 132
		.4	570 164	553 136	545 108	547 77	562 140	545 104	539 77	544 53	580 180	567 161	559 138	559 105
		.6	569 163	551 131	545 99	552 63	566 138	553 98	551 67	561 40	576 180	559 158	551 131	563 93
		.8	568 162	550 128	545 96	555 68	570 136	558 95	559 63	569 35	571 179	551 156	540 128	545 88
	400	.2	553 351	533 328	522 315	515 306	524 316	490 297	468 297	454 304	574 373	562 356	557 340	553 320
		.4	543 338	511 299	492 268	479 241	525 295	483 249	451 231	420 231	563 366	542 338	530 309	524 270