# SOME UNRESOLVED APPLICATION ISSUES IN RAKING RATIO ESTIMATION

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This paper provides a few initial results from a simulation study of raking ratio estimators. The particular aspects of raking examined here were in part suggested by our various "real life" experiences with raking, some of which are described in the previous paper [31].

## 1. SOME UNRESOLVED ISSUES

Despite the extensive literature on raking (see references in [31]), there remain a number of major unresolved issues. With one slight exception [35], for example, there has been no consideration of what happens to raking ratio estimators when the outside (or external) marginal totals are themselves subject to error. There also seems to have been virtually no study made of the properties of raking estimators when the sample (or interior) marginal totals are subject to misclassification or other response problems.

Perhaps the issue of most concern to us has been the performance of raking ratio estimators when used, as is commonly the case, in surveys (like the Current Population Survey [49]) which suffer from coverage errors. In particular, what kind of bias-variance tradeoffs can be expected (especially in small samples)?

The use of raking to make coverage adjustments is the focus of the simulation results provided here. Our attention will be confined to an examination of raking's impact on the mean square error when adjusting for coverage errors in samples of small to moderate size. We will assume basically that the coverage errors encountered are such that every class of individuals in the population is represented in the sample, but not necessarily in its proper proportion.

## 2. INITIAL SIMULATIONS

The simulation experiments we have conducted so far systematically vary six different factors. These factors, and how each was treated, are discussed briefly below, along with some of the hypotheses we wished to test.

Sample size.--Four different sample sizes were examined: n=50; n=100; n=400; and n=800. One of the hypotheses of interest here was that for the smaller samples (e.g., n=50 or n=100) there would actually be a marked deterioration in variance performance over the unraked estimator. (This turns out to be the case in at least some circumstances as we will see below.)

Number of levels of each marginal raked.--Attention was confined in the simulation to only the simplest form of univariate raking, i.e., the case where we successively rake a sample to known outside row and column totals. In the computations done for these Proceedings, we have looked at just 4x4 and 7x7 tables. 1/

We hypothesized that the variance performance of the 7x7 raked data would be inferior to that for the 4x4 case in small samples. We surmised, too, that it would become superior only in moderate to large samples where there was a very strong dependence between the raked and unraked information.

Whether the expected sample totals and outside marginal totals were equal.--Two alternatives were considered in the simulation (see figure 1):

(a) Unbiased.--The row and column outside marginal totals, or "controls," were equal to their corresponding expected sample counts.

(b) Biased.--The controls were taken to be different from the sample expected values.

The questions of obvious interest here are to what extent did the bias "correction" adversely affect variances, and at what point did it begin to reduce the mean square error?

Figure 1.--Expected Sample Marginals and Outside Controls for Sample Size n = 50

| Row or Column Class | Unbiased 7 x 7 | Unbiased 4 x 4 | Biased 7 x 7 | Biased 4 x 4 |
|---|---|---|---|---|
| First........... | 10.0 | 20.0 | 5.0 | 10.0 |
| Second.......... | 10.0 | | 5.0 | |
| Third........... | 7.5 | 15.0 | 5.0 | 12.5 |
| Fourth.......... | 7.5 | | 7.5 | |
| Fifth........... | 5.0 | 10.0 | 7.5 | 17.5 |
| Sixth........... | 5.0 | | 10.0 | |
| Seventh......... | 5.0 | 5.0 | 10.0 | 10.0 |

Note: For the larger sample sizes considered in the simulations the "controls" used were multiples (2,8 and 16) of those shown above (for n = 100, 400 and 800 respectively).

Extent of relationships within variables being raked. 2/--Three alternatives were considered (see figure 2):

(a) Totally unrelated.--The row and column variables were statistically independent.

(b) Totally related.--The row and column variables were the same (i.e., we set the column variable equal to that for the row). This is equivalent to employing a simple ratio estimator based on the rows.

(c) Partially related.--At random, one-fourth of the time, we made the column variable equal to that for the row.

We hypothesized that when the raked variables were totally unrelated, the adjustment would have a greater impact on the variance than for the partially or totally related settings.

Figure 2 - Alternative relationships between row and column variables used in the raking

| Extent of Relationship | Row Variable $\tilde{X}_1$ | Column Variable $\tilde{X}_2$ |
|---|---|---|
| Totally Unrelated | $\tilde{X}_1 = X_1$ | $\tilde{X}_2 = X_2$ |
| Totally Related | $\tilde{X}_1 = X_1$ | $\tilde{X}_2 = X_1$ |
| Partially Related | $\tilde{X}_1 = X_1$ | $\tilde{X}_2 = \begin{cases} X_1 & X_4 < .25 \\ X_2 & X_2 \geq .25 \end{cases}$ |

Nature of variables whose mean square error we wish to reduce.--The basic structure of the experiment was to draw samples of vectors,

$$\underline{X}' = (X_1, X_2, X_3, X_4),$$

each component of which was an independent uniform random number on (0, 1). Two basic functions,

$$Y_h = g_h(\underline{X}) \qquad h = 1, 2,$$

were examined in the simulations. These were:

(a) bounded "uniform type" random variables $\{g_1(\underline{X})\}$ constructed essentially as linear combinations of some of the components of $\underline{X}$; and

(b) unbounded "exponential type" random variables $\{g_2(\underline{X})\}$ obtained as linear combinations of some of the components of $\underline{Z}$ where $\underline{Z}$ and $\underline{X}$ are related component for component by the (probability integral) transformation

$$X = 1 - \exp\left\{-(0.1) \text{ times } Z\right\}.$$

Naturally, greater variance effects, both increases and decreases, were anticipated for the unbounded "exponential type" variables rather than for the bounded "uniform" ones.

Degree of dependence between raked and unraked variables.--There were three types of dependence considered between the raked row and column variables and the "unraked" $\{Y_h\}$ variables. We have characterized these as "Complete Independence," "Complete Dependence," and "Partial Dependence." There are two versions of each form of dependence subject to whether a "uniform" or "exponential" type variable is being looked at. (See figure 3.)

Replication of experimental conditions.--The results to be discussed in the next section were based on 200 replications for the n=50 case and 100 replications for sample sizes n=100, n=400 and n=800. In every instance the raking was carried out until for all levels of the row and column marginal either

$$\left| \ln \frac{\text{outside total}}{\text{adjusted sample total}} \right| < .00001$$

or the process had proceeded for 100 cycles.

Generally, except for samples of size n=50, convergence occurred quickly even when the expected sample marginals differed from the outside totals being introduced. For the n=50 samples, two difficulties arose. First, in five or six cases, one (or more) row or column classes of the data were zero and, hence, raking could not be carried out unless some collapsing was done. (We discarded these samples before raking and they were not used in any of the comparisons.) Second, again because n=50 is so small, for the biased 7x7 case a substantial portion of the replications were ones where we proceeded the full 100 cycles, i.e., convergence did not occur. (These latter samples were, however, still used in the comparisons, and undoubtedly contributed to the poor variance performance of the raking estimator for n=50.)

Figure 3.--Alternative relationships between the raked and unraked variables

| Form of Dependence | Uniform-type variables | Exponential-type variables |
|---|---|---|
| Complete Independence | $Y = X_3$ | $Y = Z_3$ |
| Complete Dependence | $Y = 0.5+\sqrt{9/5}\,\{1/3(X_1-.5) + 2/3(X_2-.5)\}$ | $Y = 10+\sqrt{9/5}\,\{1/3(Z_1-10) + 2/3(Z_2-10)\}$ |
| Partial Dependence | $Y=0.5+\sqrt{36/14}\,\{1/3(X_1-.5) +1/6(X_2-.5)+1/2(X_3-.5)\}$ | $Y=10+\sqrt{36/14}\{1/3(Z_1-10) +1/6(Z_2-10)+1/2(Z_3-10)\}$ |

## 3. RESULTS OF INITIAL SIMULATIONS

In this section we will attempt to highlight, factor by factor, the simulation results obtained. We will focus our remarks (see tables 1 to 3) solely on the performance characteristics of the means of the unconstrained variables $\{Y_h\}$.

Sample size.--For the n=50 case, as expected, there was an increase in the variance caused by the raking. This was especially marked if an adjustment for coverage errors was being made. It is also interesting to note that the increase occurred quite generally, even sometimes when the raked and unraked variables were completely dependent.

For the larger sample sizes, some variance "price" continued to be paid when raking if the unconstrained mean was independent of the raked variables. When it was not independent, substantial benefits in reduced variance were achievable.

Number of marginal totals.--As hypothesized, for n=50 the variance performance of the 7x7 raked estimator was inferior to the 4x4 one. If the raked and unraked variables were independent, the 7x7 raked estimator continued to be inferior for larger samples. The difference decreased as "n" grew larger but did not disappear even for n=800. If the raked and unraked variables were dependent, then the performance of the 7x7 and 4x4 estimators followed no consistent pattern.

To compensate for the variance increase that sometimes accompanied the use of more controls, there was an accompanying increase in the 7x7 estimator's ability to reduce the coverage bias. Except for n=50, in fact, when adjusting for coverage errors, the root mean square error of the 7x7 estimator was smaller than the corresponding 4x4 estimator. This was true even though the classifiers used were such that the ratios of outside control to expected sample total were the same before and after collapsing from seven classes to four (see figure 1).

Whether expected sample totals and outside "controls" were equal.--The variance impact of using raking as a coverage adjustment procedure can be clearly seen when we contrast its behavior to a raking estimator for which the outside totals and expected sample marginals were equal. Our simulation results show, for instance, that there was virtually always some increase in the variance when the controls differed from the expected sample size. This increase tended to be quite large when the unconstrained mean was independent of the raked variables. It diminished in importance in the partial dependence case and all but disappeared (for n>50) when there was complete dependence of the unconstrained mean on the raked variables.

Extent of relationship within variables being raked.--By and large, if the raked variables were not related to each other, then the raking had a greater impact on the variance. This impact could be either beneficial or adverse. If the unconstrained mean was independent of the raked variables, then the impact was adverse. On the other hand, if the unconstrained mean depended completely on the raked variables, then the greatest variance reductions were achieved.

Nature of variables being studied.--The same overall patterns we have been describing occurred for both the "uniform" and "exponential" type unconstrained means. There was some difference in the behavior of these two types of variables but it was a question of degree only. Substantially larger changes occurred in the "exponential" means than in the "uniform" ones when adjusting for bias. Conversely (contrary to our expectations), the variance impact of the raking tended to be smaller for "exponential" variables than for "uniform" ones (i.e., smaller relative increases or decreases occurred for the "exponential" cases, all other things being equal).

Degree of dependence between raked and unraked variables.--Perhaps the most important factor in deciding whether to use a raking ratio estimator is the degree of dependence anticipated between the raked and unraked variables. If there is little or no dependence, then raking just tends to increase the variance, especially in a very small samples or when attempting to correct for coverage errors. If there is a moderate amount of dependence, then raking can be quite beneficial. In fact, it can simultaneously reduce both the coverage bias and the sampling variance.

## 4. CONCLUSIONS AND AREAS FOR FUTURE STUDY

In a multi-purpose survey with many, many variables, we typically would have a situation in which a raking coverage adjustment materially reduced the mean square error of some variables. At the same time, however, the variance of the remainder would increase, possibly quite substantially, if the coverage errors were at all serious. It is hoped that the simulation results just described can aid practitioners in assessing the trade-offs involved in such settings. Obviously, though, this paper is just the beginning of our attempts to understand more about the performance of raking estimators when used to make coverage adjustments. For example, we need to go on and explore the behavior of alternative variance estimation procedures (as suggested by the discussant when we delivered this paper). We also want to see how much difference there is in the mean square error if we iterate only a few cycles instead of attempting to achieve complete convergence.

## ACKNOWLEDGEMENTS

The authors would like to extend their thanks to Ralph Folsom of the Research Triangle Institute who discussed this paper (and the previous one) at the San Diego meetings. Among our colleagues at SSA who made important contributions to this paper were Linda DelBene and Beth Kilss. The paper was typed by Joan Reynolds and Helen Kearney.

## FOOTNOTES

1/ When this paper was presented in San Diego, we also provided results for 5x5 and 6x6 tables. The patterns exhibited by all the simulations were roughly the same; hence, we have restricted ourselves just to the 4x4 and 7x7 tables, i.e., to the extremes.

2/ In our original paper we did not include this factor. The discussant suggested that at a minimum we compare the raked and simple ratio estimators as well as raked versus unraked estimators. We were happy to oblige.

TABLE 1.--RELATIVE PERCENT CHANGE IN UNCONSTRAINED MEAN BY NATURE OF DEPENDENCE, METHOD OF RAKING AND WHETHER SAMPLE DATA IS BIASED OR NOT

| ITEM | DEGREE OF DEPENDENCE BETWEEN RAKED VARIABLES AND UNCONSTRAINED MEAN | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | COMPLETE INDEPENDENCE | | | COMPLETE DEPENDENCE | | | PARTIAL DEPENDENCE | | |
| | RAKED MARGINALS | | SIMPLE RATIO ADJUSTMENT | RAKED MARGINALS | | SIMPLE RATIO ADJUSTMENT | RAKED MARGINALS | | SIMPLE RATIO ADJUSTMENT |
| | UNRELATED | RELATED | | UNRELATED | RELATED | | UNRELATED | RELATED | |
| | PART I - 'UNIFORM-TYPE' VARIABLES | | | | | | | | |
| UNBIASED CASE | | | | | | | | | |
| 7×7 RAKED DATA BY SAMPLE SIZE | | | | | | | | | |
| 50........... | 0.8542 | 1.0635 | 0.2240 | -2.3136 | -2.0679 | 0.7365 | -0.5805 | -0.1573 | 1.1651 |
| 100........... | 0.1211 | 0.2206 | 0.0289 | 0.1410 | 0.4796 | 0.4330 | 0.5066 | 0.6921 | 0.5130 |
| 400........... | 0.1203 | 0.1235 | 0.1438 | 0.1758 | 0.6536 | 0.3799 | 0.5189 | 0.6611 | 0.5956 |
| 800........... | -0.0014 | -0.0235 | 0.0253 | 0.1794 | 0.6550 | 0.4298 | 0.4208 | 0.5443 | 0.5163 |
| 4×4 RAKED DATA BY SAMPLE SIZE | | | | | | | | | |
| 50........... | 0.2748 | 0.4339 | -0.0080 | -0.0945 | -0.0260 | 0.5403 | 0.0800 | 0.4736 | 0.7513 |
| 100........... | -0.0468 | 0.1888 | -0.0297 | 0.2760 | 0.5813 | 0.1742 | 0.3324 | 0.6160 | 0.3150 |
| 400........... | 0.0673 | 0.0806 | 0.0643 | 0.3408 | 0.7570 | 0.2856 | 0.4498 | 0.5835 | 0.4244 |
| 800........... | -0.0211 | -0.0070 | 0.0135 | 0.3340 | 0.7714 | 0.3424 | 0.3730 | 0.5163 | 0.4037 |
| BIASED CASE | | | | | | | | | |
| 7×7 RAKED DATA BY SAMPLE SIZE | | | | | | | | | |
| 50........... | 0.3377 | 0.7194 | 0.1969 | 33.6511 | 24.4543 | 13.9715 | 21.1873 | 19.9535 | 17.3114 |
| 100........... | 0.0968 | 0.3035 | -0.0325 | 41.4355 | 28.8568 | 13.8952 | 25.3351 | 21.7162 | 16.9212 |
| 400........... | -0.3694 | -0.1404 | 0.0183 | 41.5663 | 29.5871 | 13.9881 | 25.0019 | 21.5794 | 17.0000 |
| 800........... | -0.6183 | -0.4002 | -0.1175 | 41.5615 | 29.7025 | 14.0438 | 24.7989 | 21.4066 | 16.9085 |
| 4×4 RAKED DATA BY SAMPLE SIZE | | | | | | | | | |
| 50........... | -0.0285 | 0.1362 | -0.1143 | 37.6979 | 27.1578 | 13.2422 | 22.8496 | 20.4130 | 16.2613 |
| 100........... | -0.2712 | 0.0143 | -0.0826 | 39.7580 | 28.1971 | 13.0916 | 23.9825 | 20.7308 | 16.0911 |
| 400........... | -0.4144 | -0.1836 | 0.0404 | 39.9650 | 28.6269 | 13.2998 | 23.9736 | 20.7322 | 16.2547 |
| 800........... | -0.5504 | -0.3188 | -0.0018 | 39.9702 | 28.7340 | 13.3653 | 23.8665 | 20.6577 | 16.2436 |
| | PART II - 'EXPONENTIAL-TYPE' VARIABLES | | | | | | | | |
| UNBIASED CASE | | | | | | | | | |
| 7×7 RAKED DATA BY SAMPLE SIZE | | | | | | | | | |
| 50........... | 1.3634 | 1.4368 | 0.4899 | -2.7747 | -2.6520 | 1.5400 | -0.6020 | -0.2711 | 2.1269 |
| 100........... | 0.1989 | 0.1502 | -0.0586 | 0.5045 | 0.7967 | 0.2276 | 0.5165 | 0.5434 | 0.2449 |
| 400........... | 0.1886 | 0.2208 | 0.1949 | 0.4704 | 1.0457 | 0.2623 | 0.5880 | 0.7766 | 0.5126 |
| 800........... | 0.0165 | -0.0281 | 0.0394 | 0.4884 | 1.0333 | 0.3851 | 0.4448 | 0.5609 | 0.4271 |
| 4×4 RAKED DATA BY SAMPLE SIZE | | | | | | | | | |
| 50........... | 0.3756 | 0.4388 | 0.1071 | -0.0216 | -0.4005 | 1.3722 | -0.0115 | 0.2325 | 1.7486 |
| 100........... | 0.0017 | 0.2128 | -0.0780 | 0.6481 | 0.8924 | -0.0588 | 0.3490 | 0.5738 | 0.1018 |
| 400........... | 0.1093 | 0.1595 | 0.0819 | 0.6575 | 1.1933 | 0.1901 | 0.5201 | 0.7190 | 0.3479 |
| 800........... | 0.0069 | 0.0262 | 0.0416 | 0.6844 | 1.2164 | 0.3189 | 0.4417 | 0.6174 | 0.3598 |
| BIASED CASE | | | | | | | | | |
| 7×7 RAKED DATA BY SAMPLE SIZE | | | | | | | | | |
| 50........... | 0.1573 | 0.9231 | 0.7479 | 56.9549 | 41.2274 | 24.3113 | 35.1305 | 32.7998 | 29.4224 |
| 100........... | 0.7540 | 0.8549 | 0.0790 | 69.8573 | 47.9980 | 23.0271 | 42.4058 | 35.8049 | 27.7306 |
| 400........... | -0.0001 | 0.4224 | 0.2679 | 70.2186 | 49.4622 | 23.4715 | 42.0293 | 36.1222 | 28.1199 |
| 800........... | -0.4199 | -0.0895 | 0.0870 | 70.2537 | 49.6454 | 23.6101 | 41.6778 | 35.7550 | 28.0209 |
| 4×4 RAKED DATA BY SAMPLE SIZE | | | | | | | | | |
| 50........... | -0.1166 | 0.2331 | 0.1852 | 62.2009 | 44.2236 | 23.2652 | 37.1619 | 33.0016 | 28.0104 |
| 100........... | 0.2176 | 0.3769 | 0.0732 | 66.7069 | 46.8666 | 21.7250 | 40.0787 | 34.1922 | 26.4520 |
| 400........... | -0.1266 | 0.2678 | 0.2689 | 67.2370 | 47.7921 | 22.2779 | 40.1911 | 34.6547 | 26.8656 |
| 800........... | -0.2940 | -0.0157 | 0.2488 | 67.3775 | 48.0134 | 22.4516 | 40.1132 | 34.5158 | 26.9112 |

TABLE 2.--RELATIVE PERCENT CHANGE IN STANDARD ERROR OF UNCONSTRAINED MEAN BY NATURE OF DEPENDENCE, METHOD OF RAKING AND WHETHER SAMPLE DATA IS BIASED OR NOT

| ITEM | DEGREE OF DEPENDENCE BETWEEN RAKED VARIABLES AND UNCONSTRAINED MEAN | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | COMPLETE INDEPENDENCE | | | COMPLETE DEPENDENCE | | | PARTIAL DEPENDENCE | | |
| | RAKED MARGINALS | | SIMPLE RATIO ADJUSTMENT | RAKED MARGINALS | | SIMPLE RATIO ADJUSTMENT | RAKED MARGINALS | | SIMPLE RATIO ADJUSTMENT |
| | UNRELATED | RELATED | | UNRELATED | RELATED | | UNRELATED | RELATED | |
| | | | | PART I - 'UNIFORM-TYPE' VARIABLES | | | | | |
| UNBIASED CASE | | | | | | | | | |
| 7×7 RAKED DATA BY SAMPLE SIZE | | | | | | | | | |
| 50........... | 28.0885 | 23.9756 | 17.9181 | -3.9858 | 1.2444 | 7.5032 | 14.0071 | 8.7258 | 7.9716 |
| 100........... | 9.7653 | 10.6406 | 7.5243 | -83.0342 | -40.8634 | -14.2655 | -19.4723 | -16.7612 | -17.1507 |
| 400........... | 5.1158 | 4.4126 | 1.4203 | -84.0396 | -50.7409 | -14.5894 | -19.7920 | -20.9620 | -22.1189 |
| 800........... | -0.0951 | 1.4903 | 0.2960 | -85.3595 | -50.4629 | -15.3240 | -28.7347 | -24.2963 | -23.0474 |
| 4×4 RAKED DATA BY SAMPLE SIZE | | | | | | | | | |
| 50........... | 12.8368 | 12.2242 | 4.5984 | -3.9215 | 0.7623 | 2.9159 | 1.7825 | 5.9882 | 2.3144 |
| 100........... | 5.0609 | 6.1514 | 3.2822 | -69.5525 | -38.5023 | -18.1085 | -20.3243 | -17.7683 | -18.2403 |
| 400........... | 3.2681 | 2.6544 | 0.9653 | -68.3077 | -42.4664 | -14.4232 | -20.3250 | -21.7030 | -21.8655 |
| 800........... | -0.4122 | 0.6659 | 0.0211 | -73.3581 | -47.7633 | -15.1337 | -27.3129 | -23.2203 | -22.6919 |
| BIASED CASE | | | | | | | | | |
| 7×7 RAKED DATA BY SAMPLE SIZE | | | | | | | | | |
| 50........... | 81.2761 | 71.0930 | 43.3960 | 40.3490 | 39.2768 | 20.3811 | 60.9456 | 47.4610 | 21.2695 |
| 100........... | 66.5678 | 53.2557 | 26.6596 | -80.6731 | -0.6257 | -3.2865 | 21.2143 | 15.2029 | -3.7944 |
| 400........... | 50.2068 | 34.0251 | 23.2212 | -84.5035 | -10.7395 | 5.8094 | 14.5921 | 3.3799 | -10.2437 |
| 800........... | 46.4116 | 26.7202 | 11.5421 | -85.5066 | -23.7086 | -3.7813 | 7.3590 | -1.9310 | -11.7206 |
| 4×4 RAKED DATA BY SAMPLE SIZE | | | | | | | | | |
| 50........... | 70.6965 | 57.6538 | 26.5811 | 8.3320 | 22.1949 | 14.5839 | 36.4610 | 31.1395 | 11.1017 |
| 100........... | 53.3898 | 43.5045 | 19.4036 | -68.2847 | -2.9539 | -8.2129 | 15.4499 | 9.5050 | -8.7511 |
| 400........... | 47.3938 | 34.5974 | 22.6489 | -72.1091 | -12.1244 | 5.5602 | 13.5652 | 2.6844 | -11.4072 |
| 800........... | 40.0275 | 21.7419 | 9.4810 | -74.0936 | -26.5986 | -4.0235 | 6.9843 | -5.9180 | -16.0150 |
| | | | | PART II - 'EXPONENTIAL-TYPE' VARIABLES | | | | | |
| UNBIASED CASE | | | | | | | | | |
| 7×7 RAKED DATA BY SAMPLE SIZE | | | | | | | | | |
| 50........... | 30.9734 | 30.6219 | 18.2296 | -5.1910 | -0.9298 | 16.4981 | 14.2248 | 10.4409 | 18.0942 |
| 100........... | 9.9486 | 9.5417 | 7.8944 | -63.3927 | -38.3794 | -15.5653 | -16.8576 | -15.3555 | -17.7314 |
| 400........... | 5.8796 | 4.7602 | 1.7130 | -68.8586 | -44.1341 | -13.1318 | -17.3002 | -17.1112 | -18.7531 |
| 800........... | -0.6070 | 0.1307 | -0.0867 | -68.2078 | -48.4882 | -12.8178 | -20.3352 | -18.3959 | -17.1665 |
| 4×4 RAKED DATA BY SAMPLE SIZE | | | | | | | | | |
| 50........... | 10.8215 | 10.3200 | 8.1118 | -4.9458 | -0.1392 | 12.2000 | 0.6703 | 4.4949 | 15.4620 |
| 100........... | 4.3724 | 4.7549 | 2.7422 | -60.3050 | -38.2203 | -19.4866 | -20.2154 | -17.8034 | -19.8590 |
| 400........... | 3.2221 | 2.9617 | 1.0779 | -65.7444 | -42.3684 | -13.3866 | -19.2563 | -18.8287 | -19.1187 |
| 800........... | -1.4029 | -0.7319 | -0.1658 | -62.7791 | -46.9729 | -13.6757 | -19.4135 | -17.5520 | -17.1020 |
| BIASED CASE | | | | | | | | | |
| 7×7 RAKED DATA BY SAMPLE SIZE | | | | | | | | | |
| 50........... | 71.8517 | 69.0369 | 43.8727 | 71.9420 | 60.4433 | 49.4973 | 63.5599 | 53.2207 | 46.1719 |
| 100........... | 65.4997 | 53.6405 | 29.4283 | -15.5542 | 25.0653 | 3.2552 | 30.1020 | 24.1653 | 1.5467 |
| 400........... | 49.1219 | 38.6396 | 29.5335 | -17.2178 | 21.2627 | 19.0808 | 23.3482 | 18.0932 | 4.1554 |
| 800........... | 32.0277 | 16.8909 | 5.2030 | -21.1441 | -7.4373 | 6.9432 | 24.5550 | 9.0054 | -3.1899 |
| 4×4 RAKED DATA BY SAMPLE SIZE | | | | | | | | | |
| 50........... | 62.7472 | 59.3478 | 31.1049 | 47.8924 | 49.0686 | 43.7109 | 42.2690 | 43.9682 | 41.9225 |
| 100........... | 53.7795 | 45.2523 | 22.3463 | -14.5031 | 20.8135 | -1.2840 | 26.7356 | 21.4924 | -1.6753 |
| 400........... | 44.4716 | 37.2036 | 28.7954 | -17.6611 | 16.3669 | 17.4082 | 19.8896 | 16.1590 | 3.7769 |
| 800........... | 27.0033 | 12.1416 | 2.8674 | -16.2339 | -7.5887 | 6.7783 | 24.7786 | 7.7126 | -6.0839 |

TABLE 3.--RELATIVE PERCENT CHANGE IN SYMMETRIC 'TWO SIGMA' LOWER CONFIDENCE LIMIT OF UNCONSTRAINED MEAN BY NATURE OF DEPENDENCE, METHOD OF RAKING AND WHETHER SAMPLE DATA IS BIASED OR NOT

| ITEM | DEGREE OF DEPENDENCE BETWEEN RAKED VARIABLES AND UNCONSTRAINED MEAN | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | COMPLETE INDEPENDENCE | | | COMPLETE DEPENDENCE | | | PARTIAL DEPENDENCE | | |
| | RAKED MARGINALS | | SIMPLE RATIO ADJUSTMENT | RAKED MARGINALS | | SIMPLE RATIO ADJUSTMENT | RAKED MARGINALS | | SIMPLE RATIO ADJUSTMENT |
| | UNRELATED | RELATED | | UNRELATED | RELATED | | UNRELATED | RELATED | |
| | | | | PART I - 'UNIFORM-TYPE' VARIABLES | | | | | |
| UNBIASED CASE | | | | | | | | | |
| 7×7 RAKED DATA BY SAMPLE SIZE | | | | | | | | | |
| 50........... | ¯4.2584 | ¯3.2377 | ¯3.0977 | ¯1.9602 | ¯2.7677 | ¯0.6933 | ¯3.5770 | ¯1.9820 | ¯0.2331 |
| 100........... | ¯1.1178 | ¯1.1180 | ¯0.9340 | 11.6578 | 6.2042 | 2.4682 | 3.4181 | 3.2356 | 3.0872 |
| 400........... | ¯0.1888 | ¯0.1419 | 0.0648 | 5.3461 | 3.8089 | 1.2989 | 1.8609 | 2.0898 | 2.0965 |
| 800........... | 0.0023 | ¯0.0833 | 0.0146 | 4.3310 | 3.1360 | 1.1943 | 1.6969 | 1.6315 | 1.5476 |
| 4×4 RAKED DATA BY SAMPLE SIZE | | | | | | | | | |
| 50........... | ¯2.0834 | ¯1.7794 | ¯0.8728 | 0.7141 | ¯0.1926 | 0.0383 | ¯0.2698 | ¯0.6592 | 0.4302 |
| 100........... | ¯0.7030 | ¯0.5772 | ¯0.4552 | 9.9447 | 5.9930 | 2.7057 | 3.3427 | 3.2951 | 3.0191 |
| 400........... | ¯0.1308 | ¯0.0787 | 0.0085 | 4.5554 | 3.4106 | 1.1886 | 1.8225 | 2.0561 | 1.8972 |
| 800........... | ¯0.0056 | ¯0.0336 | 0.0132 | 3.9106 | 3.1270 | 1.0936 | 1.5848 | 1.5552 | 1.4145 |
| BIASED CASE | | | | | | | | | |
| 7×7 RAKED DATA BY SAMPLE SIZE | | | | | | | | | |
| 50........... | ¯14.8566 | ¯12.4916 | ¯7.9128 | 32.2359 | 21.3223 | 12.6171 | 13.0204 | 14.3031 | 16.4983 |
| 100........... | ¯8.4424 | ¯6.4990 | ¯3.4616 | 58.3432 | 32.9391 | 16.2743 | 25.9356 | 22.6654 | 19.9401 |
| 400........... | ¯3.4990 | ¯2.2545 | ¯1.4175 | 49.3061 | 32.0629 | 14.4902 | 25.6897 | 22.7820 | 18.8001 |
| 800........... | ¯2.4766 | ¯1.4718 | ¯0.5782 | 47.7286 | 32.2947 | 14.9089 | 25.5622 | 22.4281 | 18.1616 |
| 4×4 RAKED DATA BY SAMPLE SIZE | | | | | | | | | |
| 50........... | ¯13.3055 | ¯10.6614 | ¯5.1257 | 43.9031 | 28.2065 | 12.9587 | 20.0537 | 18.2096 | 17.3212 |
| 100........... | ¯7.1648 | ¯5.5727 | ¯2.5859 | 54.7180 | 32.5105 | 16.0415 | 25.2259 | 22.3668 | 19.7114 |
| 400........... | ¯3.3727 | ¯2.3358 | ¯1.3586 | 46.8456 | 31.1288 | 13.7750 | 24.6614 | 21.9247 | 18.0824 |
| 800........... | ¯2.1538 | ¯1.1905 | ¯0.3765 | 45.5062 | 31.4195 | 14.2093 | 24.6054 | 21.8209 | 17.6555 |
| | | | | PART II - 'EXPONENTIAL-TYPE' VARIABLES | | | | | |
| UNBIASED CASE | | | | | | | | | |
| 7×7 RAKED DATA BY SAMPLE SIZE | | | | | | | | | |
| 50........... | ¯9.2866 | ¯9.0605 | ¯5.8907 | ¯1.7770 | ¯3.3631 | ¯4.6363 | ¯6.5696 | ¯4.5825 | ¯4.2997 |
| 100........... | ¯2.0639 | ¯2.0296 | ¯1.9045 | 17.1430 | 10.9978 | 4.3400 | 5.1674 | 4.8057 | 5.0569 |
| 400........... | ¯0.4132 | ¯0.2591 | 0.0343 | 8.1043 | 6.0205 | 1.7371 | 2.5781 | 2.7666 | 2.6559 |
| 800........... | 0.0625 | ¯0.0398 | 0.0487 | 6.0102 | 5.0138 | 1.4463 | 2.0524 | 2.0275 | 1.7882 |
| 4×4 RAKED DATA BY SAMPLE SIZE | | | | | | | | | |
| 50........... | ¯3.3816 | ¯3.1153 | ¯2.7720 | 2.0117 | ¯0.5084 | ¯3.0988 | ¯0.2860 | ¯1.4830 | ¯3.7708 |
| 100........... | ¯1.0127 | ¯0.8414 | ¯0.7326 | 16.5198 | 11.0771 | 5.0001 | 5.8538 | 5.4931 | 5.4451 |
| 400........... | ¯0.2198 | ¯0.1368 | ¯0.0234 | 7.9691 | 5.9899 | 1.6850 | 2.7203 | 2.8938 | 2.5137 |
| 800........... | 0.1109 | 0.0821 | 0.0569 | 5.7855 | 5.0899 | 1.4438 | 1.9778 | 2.0231 | 1.7107 |
| BIASED CASE | | | | | | | | | |
| 7×7 RAKED DATA BY SAMPLE SIZE | | | | | | | | | |
| 50........... | ¯25.6297 | ¯23.5760 | ¯14.7632 | 50.7664 | 33.2929 | 13.9117 | 23.6881 | 24.5806 | 22.6809 |
| 100........... | ¯14.2732 | ¯11.3964 | ¯6.7328 | 92.0979 | 53.9695 | 28.1756 | 45.6994 | 38.9207 | 34.7398 |
| 400........... | ¯5.1939 | ¯3.6185 | ¯2.8264 | 79.8462 | 52.5672 | 23.9549 | 44.1076 | 38.1280 | 30.7860 |
| 800........... | ¯2.8133 | ¯1.3420 | ¯0.2904 | 77.6003 | 54.2336 | 24.9498 | 43.0024 | 37.8244 | 30.4355 |
| 4×4 RAKED DATA BY SAMPLE SIZE | | | | | | | | | |
| 50........... | ¯22.7273 | ¯21.0292 | ¯10.9359 | 68.1091 | 42.2231 | 14.8229 | 35.1064 | 28.5877 | 22.4109 |
| 100........... | ¯12.2139 | ¯10.0385 | ¯5.0963 | 87.8534 | 53.6507 | 27.7164 | 43.6505 | 37.5918 | 33.9814 |
| 400........... | ¯4.8420 | ¯3.6375 | ¯2.7473 | 76.5852 | 51.2524 | 22.8141 | 42.4497 | 36.7123 | 29.4343 |
| 800........... | ¯2.3075 | ¯0.9125 | 0.0556 | 74.0982 | 52.4827 | 23.7114 | 41.2995 | 36.5894 | 29.4638 |