A COMPARISON OF METHODS TO MEASURE TREE LIMBS IN THE RANDOM PATH TECHNIQUE FOR ESTIMATING THE SET OF A FRUIT OR NUT TREE

Nicholas J. Ciancio, USDA
Max Cain, USDA

The random path procedure used by the California Crop and Livestock Reporting Service (CCLRS) to estimate the set of fruit and nut trees was developed by Raymond Jessen [3]. Briefly, a limb is chosen with probability proportional to its cross sectional area (CSA). This process is continued until a terminal limb, with a predetermined specific size CSA, is selected. Fruit counts are made at each stage along the randomly selected limb. An expansion representing all fruit on the tree is then calculated. For ease in measuring, a specially calibrated CSA tape measure is used.

For more detail, please refer to Figure 4.

A CSA trunk measurement is made but is not used in the final calculation for the set. This measurement is used as an indication for year of plant. Generally, the older the tree, the larger the trunk measurement. It is also used as an indication that the same tree is sampled each year since only small increases in trunk diameter are expected.

The first measurement used in set calculation is the first or primary split of branches above the trunk. This measurement is called stage one. The primary branches and all other splits are numbered, using a consistent technique. CCLRS uses a tagged corner of the orchard as a reference point. The limb pointing in the general direction of that reference corner is limb one. Remaining limbs are numbered consecutively in a clockwise direction.

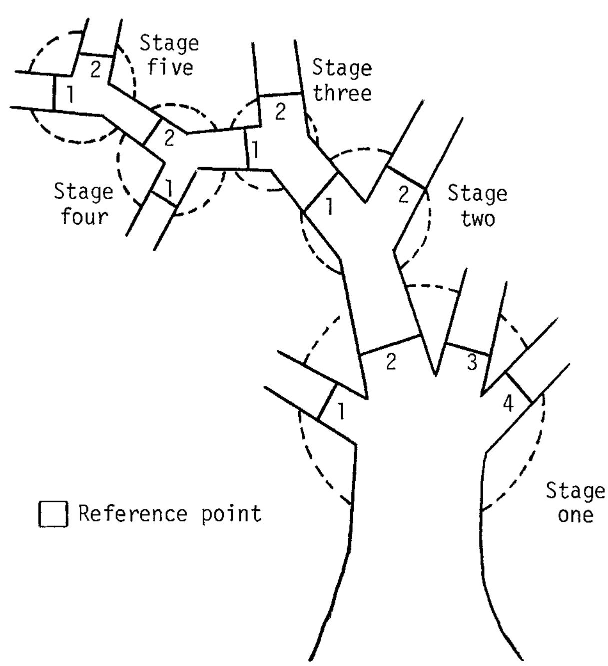
The measurement process uses a standard area of the limb for a CSA determination. The area is approximately one hand's width above the limb split for larger limbs, and two fingers' width on smaller limbs. If at that point there is an irregularity in the limb, vis., a pruning scar or a bulge, then the measurement is made above the irregularity. Using Figure 2, "Random Path Schedule" as an example, limb one of the first stage has a CSA measurement of 21.1, limb two a CSA of 22.8, limb three a CSA of 10.0, and limb four a CSA of 22.4. As the CSA are entered, a cumulative total is calculated. Therefore, the "total" CSA are 21.1, 43.9, 53.9, and 76.3. Actual CSA measurements are entered in the unshaded boxes, while the cumulative total CSA are entered in the shaded boxes. These measurements are in the column for stage one (primary).

Using these measurements, limb one has a probability of $\frac{21.1}{76.3}$ or 28 percent chance of selection, limb two has a $\frac{22.8}{76.3}$ or 30 percent chance of selection, limb three has a $\frac{10.0}{76.3}$ or 13 percent chance of selection, and limb four has a $\frac{22.4}{76.3}$ or 29 percent chance of selection. To aid the enumerator in selecting the proper limb, a set of random CSA measurements are provided at the bottom of the random path schedule. (There are 10 different sets of random path schedules, so that different random numbers are used for every tree. These 10 sets are replaced each year.)

To select one of the four limbs, the enumerator proceeds to the random number table, and selects the first number less than or equal to 76.3 (total CSA measurement) in the first column of random numbers that has the same number of significant digits as the final cumulative CSA total. In this case, proceed to column 2, i.e., 3 significant digits. The first random number less than or equal to 76.3 is 23.3. The branch to be selected is that branch which makes the cumulative CSA exceed or equal the selected random number, i.e., branch #2.

A count of fruit is needed at each stage. These fruit are called intermediate fruit. The count is made as follows: (1) trunk count is all fruit from the ground to the point of measurement of all the limbs (primary) in stage one; (2) stage one fruit count is the count of fruit from the measurement of the selected stage one (branch #2) limb up to the measurement point of all limbs at the second stage; (3) stage 2 fruit count is the count of fruit from the measurement of the selected stage 2 limb (branch #1) up to the measurement of all limbs at the third stage; etc. (refer to Figure 3).

Figure 1: Illustration of stages and limb numbering of a fruit or nut tree.



RANDOM PATH SCHEDULE

Calif. Crop & Livestock Reporting Serv., P O Box 1258, Sacramento, Ca., Phone-916-445-3214

Survey Code	Year	County	Variety	Block No.	Tree No.	Sample No.	Checker Code	Update Code	Month/ Day	Block Status	Number Stages

Tags:	Block	Row	Tree	End / Start	Minutes	Team	Spacing	Trees	Acres	Trees/ Acre	Year of Plant
Found											
Replaced							X				

BLOCK STATUS CODE: 1 Sampled, 2 Wet, 3 Pulled Out, 4 Abandoned, 5 Sprayed, 6 Not Visited, 7 Refusal

	Branch	Trunk 0	Primary 1	2	3	4	5	6	7	8
1	CSA	71.0	21.1	9.0	8.4	4.1	0.8	.	.	.
	Total		21.1	9.0	8.4	4.1	0.8	.	.	.
2	CSA		22.8	13.1	7.2	3.3	1.1	.	.	.
	Total		43.9	22.1	15.6	7.4	1.9	.	.	.
3	CSA		10.0	.	.	.	.	.	.	.
	Total		53.9	.	.	.	.	.	.	.
4	CSA		22.4	.	.	.	.	.	.	.
	Total		76.3	.	.	.	.	.	.	.
5	CSA		.	.	.	.	.	.	.	.
	Total		.	.	.	.	.	.	.	.
6	CSA		.	.	.	.	.	.	.	.
	Total		.	.	.	.	.	.	.	.
7	CSA		.							
	Total		.							
8	CSA		.							
	Total		.							
9	CSA		.							
	Total		.							
10	CSA		.							
	Total		.							
Random Number			23.3	07.4	07.2	6.8	0.3	.	.	.
Selected Branch			2	1	1	2	1			
Fruit Count		6	5	6	2	4	19			
Check										

TABLE OF RANDOM NUMBERS

101.9	23.3	87.7	14.4	69.1	44.3	8.0	6.3	6.8
184.8	49.3	03.4	97.9	06.9	93.0	6.8	4.8	1.4
978.9	83.7	15.8	65.7	08.6	01.6	6.7	7.6	5.0
021.4	07.4	52.5	31.3	64.1	28.5	0.3	1.2	1.4
478.0	07.2	52.1	02.7	38.0	20.3	6.9	0.1	1.9
523.9	00.3	25.8	11.7	34.3	10.1	0.9	0.8	2.0
435.5	84.5	16.1	05.6	22.8	04.4	2.5	1.4	1.0
388.0	37.1	20.4	19.0	05.2	26.8	6.4	0.8	0.2
341.6	51.4	29.6	11.3	69.8	59.0	6.0	2.5	0.6
389.2	48.1	18.1	68.1	44.5	42.4	1.3	4.9	1.7
064.3	86.9	60.2	57.7	30.4	20.6	0.3	0.7	0.6
809.3	13.7	01.1	25.2	11.8	10.8	6.5	1.2	2.8
691.6	15.5	04.9	96.7	26.2	19.8	9.3	0.1	6.0

Comments: ___

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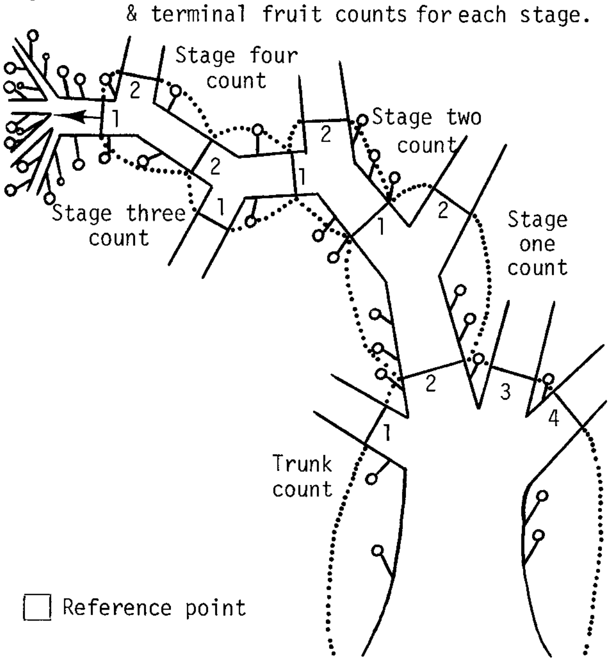
Figure 3: Illustrations of areas for intermediate & terminal fruit counts for each stage.



The intermediate fruit count must be tabulated at each stage, as the random selection process proceeds out the tree. In Figure 3, the trunk fruit count is 6 and stage one fruit is 5.

Since limb two was the selected stage 1 branch, continue out that limb to the next split of limbs, i.e., stage two (refer to Figure 1). The same process is applied as at stage one. Limb one is the limb, at the second stage, that points most closely to the reference point in the orchard. The rest of the limbs at that stage are numbered in a clockwise manner from limb one. CSA measurements, once again, are made approximately one hand's width from the split unless an irregularity is found.

Suppose limb one has a CSA measurement of 9.0, and limb two has a CSA measurement of 13.1. The cumulative CSA measurements are then 9.0 and 22.1 (refer to Figure 1). A random number less than or equal to 22.1 must be selected. Since the total cumulative CSA measurement is 22.1, i.e., three significant digits, proceed to the first column of random numbers with three significant digits. Start from the last random number used since we are once again in the same column as used for stage one. The first random number less than or equal to 22.1, excluding 00.0, is 07.4.

The first CSA total which makes the cumulative CSA total equal or exceed 07.4, is branch one. The stage two fruit count is six (refer to Figure 3).

The random limb process is continued until there are no limb splits with CSA measurements of at least 0.3 for Cling peaches, or when eight stages (branch splits) have been selected.

In our example, there are five stages. The last branch selected was branch one of the fifth stage. This is a terminal branch since there are not at least two branch splits with the specified minimum CSA measurements. All terminal fruit must then be counted from the point of measurement at the fifth stage out of the rest of the tree.

Any branch encountered in this entire measurement process that is less than the minimum CSA measurement is disregarded as a branch. However, all fruit on these limbs must be counted and included in the intermediate fruit totals.

The general expansion formula for the set is:

$$\begin{pmatrix}\text{Trunk}\\ \text{fruit}\\ \text{count}\end{pmatrix} + \begin{pmatrix}\dfrac{\text{Total cum. CSA stage one}}{\text{Selected branch CSA stage one}}\end{pmatrix} \times \begin{pmatrix}\text{Fruit}\\ \text{count}\\ \text{stage}\\ \text{one}\end{pmatrix} +$$

$$\begin{pmatrix}\dfrac{\text{Total cum. CSA stage one}}{\text{Selected branch CSA stage one}}\end{pmatrix} \times \begin{pmatrix}\dfrac{\text{Total cum. CSA stage two}}{\text{Selected branch CSA stage two}}\end{pmatrix} \times \begin{pmatrix}\text{Fruit}\\ \text{count}\\ \text{stage}\\ \text{two}\end{pmatrix} + \ldots +$$

(Continued on next page)

$$\left(\frac{\text{Total cum. CSA stage one}}{\text{Selected branch CSA stage one}}\right) \times \ldots \times \left(\frac{\text{Total cum. CSA terminal stage}}{\text{Selected branch CSA terminal stage}}\right) \times \left(\text{Terminal fruit count}\right) = \text{set per tree}$$

For our example the expanded set would be:

$$6 + \left(\frac{76.3}{22.8}\right)(5) + \left(\frac{76.3}{22.8}\right)\left(\frac{22.1}{9.0}\right)(6) +$$

$$\left(\frac{76.3}{22.8}\right)\left(\frac{22.1}{9.0}\right)\left(\frac{15.6}{8.4}\right)(2) + \left(\frac{76.3}{22.8}\right)\left(\frac{22.1}{9.0}\right)\left(\frac{15.6}{8.4}\right)\left(\frac{7.4}{3.3}\right)(4)$$

$$+ \left(\frac{76.3}{22.8}\right)\left(\frac{22.1}{9.0}\right)\left(\frac{15.6}{8.4}\right)\left(\frac{7.4}{3.3}\right)\left(\frac{1.9}{0.8}\right)(19)$$

$$6 + 17 + 49 + 31 + 137 + 1{,}544 = 1{,}784$$

Certain refinements were introduced in an effort to improve the current random path procedures. One refinement includes using various sections of the limb to take the CSA measurement in order to obtain the best section of the limb that represents the bearing surface from which the next split of limbs extend. As mentioned previously, the current procedure is approximately one hand's width above the limb split. If there is an irregularity then the measurement is made above the irregularity. One alternative to the present system was to measure the CSA one hand's width above the limb split disregarding any irregularities. The second alternative was to make all branch measurements one hand's width below the next split of limbs; in effect the measurements were taken where the branch was slightly smaller.

Another refinement was to vary the size of the terminal branch. For this project 0.5 and 0.8 CSA were used besides the current minimum CSA of 0.3. The project was conducted on 3 different varieties of Cling peach trees; Loadels, Peaks, and Guame.

The objectives of the project are to test 1) the best section of the limb to take CSA measurements, 2) the optimum terminal branch measurements and, 3) the effect of variety on an estimate of the set.

PROCEDURE:
For this project nine (9) Cling peach trees were stripped of their fruit and measurements were made at the three (3) selected portions of the limb. Tables 1, 2, and 3 show the summarized results for the average estimated sets for the 9 trees for the three methods of measuring the CSA with a .3, .5, and .8 terminal branch.

TABLE 1: All measurements one hand's width above the previous split disregarding abnormalities.

Tree	Actual Set	.3 Terminal	Difference
1	1,363	1,505	+142
2	1,621	2,162	+541
3	956	1,062	+106
4	905	995	+ 90
5	1,036	1,222	+186
6	1,824	2,465	+641
7	1,818	2,014	+196
8	1,517	2,076	+559
9	1,474	1,895	+421
Average	1,391	1,711	+320

Tree	.5 Terminal	Difference	.8 Terminal	Difference
1	1,328	- 35	1,179	-184
2	1,736	+115	1,554	- 67
3	894	- 62	840	-116
4	861	- 44	829	- 76
5	939	- 97	911	-125
6	1,944	+120	1,641	-183
7	1,761	- 57	1,529	-289
8	1,672	+155	1,450	- 77
9	1,554	+ 80	1,340	-134
Average	1,410	+ 19	1,252	-139

TABLE 2: Current procedure for measuring.

Tree	Actual Set	.3 Terminal	Difference
1	1,363	1,537	+ 174
2	1,621	2,063	+ 442
3	956	1,032	+ 76
4	905	995	+ 90
5	1,036	1,183	+ 147
6	1,824	2,341	+ 517
7	1,818	1,825	+ 7
8	1,517	2,687	+1,170
9	1,474	1,897	+ 423
Average	1,391	1,729	+ 338

Tree	.5 Terminal	Difference	.8 Terminal	Difference
1	1,293	- 70	1,153	-210
2	1,705	+ 84	1,537	- 84
3	841	-115	790	-166
4	858	- 47	826	- 79
5	911	-125	883	-153
6	1,886	+ 62	1,607	-217
7	1,654	-164	1,392	-426
8	2,170	+653	1,883	+366
9	1,524	+ 50	1,296	-178
Average	1,427	+ 36	1,264	-127

TABLE 3: All measurements one hand's width below the next split.

Tree	Actual Set	.3 Terminal	Difference
1	1,363	2,335	+972
2	1,621	2,140	+519
3	956	1,006	+ 50
4	905	888	- 17
5	1,036	1,150	+114
6	1,824	2,266	+442
7	1,818	2,061	+243
8	1,517	1,924	+407
9	1,474	2,334	+860
Average	1,391	1,789	+398

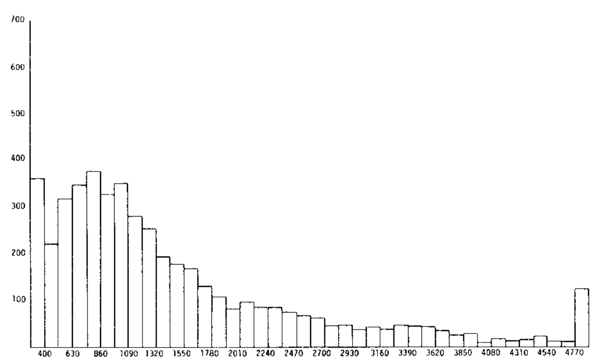
Tree	.5 Terminal	Difference	.8 Terminal	Difference
1	2,228	+865	2,074	+711
2	1,816	+195	1,677	+ 56
3	854	-102	810	-146
4	775	-130	707	-198
5	904	-132	851	-185
6	1,815	- 9	1,493	-331
7	1,746	- 72	1,460	-358
8	1,572	+ 55	1,331	-186
9	2,068	+594	1,775	+301
Average	1,531	+140	1,354	- 37

Table 4 indicates that the relative differences by terminal branch measurements show greater variations than the differences by the point at which the measurements are made. The smallest relative difference would indicate the ideal terminal measurement and point of measurements.

TABLE 4. Average difference from the actual set for the 3 methods with .3, .5, and .8 Terminal CSA.

Terminal CSA	Current Procedures	All Measurements One Hand's Width Above Previous Split	All Measurements One Hand's Width Below the Next Split
.3	+338	+320	+398
.5	+ 36	+ 19	+ 97
.8	-139	-127	- 37

Figure 4: Histogram of Sets



Nonparametric statistical tests were used to test the effects of the various measurement methods and terminal branch CSA's since normal data was not obtained (refer to Figure 4). For ease in testing, one way nonparametric analysis of variance was used to test the differences in measurement methods, terminal branch CSA and variety estimates at an error (α) of .05/3=.0167.

For testing the differences of more than 2 groups, the Kruskal-Wallis test was used. The basic model is assumed to be $X_{ij} = \mu + \alpha_j + e_{ij}$, where $i = 1,\ldots, n_j$, $j = 1, 2, 3$, μ is the overall mean, α_j is treatment j effect and $\sum_{j=1}^{3} \alpha_j=0$.

The errors are assumed to be independent and identically distributed.

The data consists of $N=\sum_{j=1}^{3} n_j=4857$ observations with n_j observations from the j-th treatment, j=1, 2, 3.

Treatments 1	2	3
X_{11}	X_{12}	X_{13}
X_{21}	X_{22}	X_{23}
$\vdots$	$\vdots$	$\vdots$
$X_{n_1 1}$	$X_{n_2 2}$	$X_{n_3 3}$

We want to test the hypothesis H_0: $\alpha_1 = \alpha_2 = \alpha_3$ vs. H_A: α's are not all equal. All the sets were ranked jointly from least to greatest. Let r_{ij} be the rank of the X_{ij} element.

Then $R_j = \sum_{i=1}^{nj} r_{ij}$, $R._j = \frac{R_j}{n_j}$, and $R.. = \frac{\sum_{j=1}^{3} R._j}{3}$

Average ranks were used in case of equal sets. The H statistic is

$H= \frac{12}{N(N+1)} \sum_{j=1}^{3} n_j (R._j - R..)^2$ which has an asymatic χ^2 distribution based on 3-1=2 degrees of freedom.

H is replaced by H' where average ranks are used, therefore

$H' = \frac{H}{1-\left(\sum_{j=1}^{g} T_j / [N^3-N]\right)}$ where g is the number of tied groups and $T_j=(t_j^3-t_j)$, with t_j the size of tied group j.

If an overall difference in treatments is found then a distribution-free multiple comparisons test for a large sample approximation with unequal sample sizes is to decide

$$\alpha_m \neq \alpha \quad \text{if} \quad |R._m - R._n| \geq Z(\alpha/[k(k-1)])\left(\left[\frac{N(N+1)}{12}\right]\left[\frac{1}{n_m}+\frac{1}{n_n}\right]\right)^{\frac{1}{2}}$$

Tables 5, 6, and 7 show a summary of the number of sets and their average ranking by treatment for measurement methods, terminal branch CSA and variety.

TABLE 5: Measurement Method Summary

	Treatment number (j)	n_j	$R._j$
All measurements one hand's width above previous split w/o regard to irregularities	1	1623	5817

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TABLE 5: Measurement Method Summary (continued)

	Treatment number (j)	n_j	$R._j$
Current Procedures	2	1629	5852
All measurements one hand's width below the next split of limbs	3	1605	5781

TABLE 6: Terminal Branch CSA Summary

.3 Terminal Branch CSA	1	1619	6401
.5 Terminal Branch CSA	2	1619	5709
.8 Terminal Branch CSA	3	1619	5341

TABLE 7: Variety Summary

Loadel Variety	1	1677	6179
Peak Variety	2	1293	4699
Guame Variety	3	1887	6263

The calculations for the H' statistics are as follows:

Measurement Method

1. H_0: $\alpha_1=\alpha_2=\alpha_3$ vs. H_A: not all α's are equal.

$$H'=\frac{\frac{12}{(4857)(4858)}\left[1623(5817-5817)^2+1629(5852-5817)^2+1605(5781-5817)^2\right]}{1-.00001}$$

$= 2.07$

$H' \sim \chi^2(2) \doteq 9.210 \Rightarrow$ No difference in the measurement methods.

Similarly H' is calculated for terminal branch CSA and variety.

2. H' (terminal branch CSA) = 1124 $\Rightarrow$ significant differences.

3. H' (variety) = 477 $\Rightarrow$ significant differences.

Since only terminal branch CSA and variety had significant differences, multiple comparisons were made on the original sets. For terminal branch CSA H_0: .3=.5 H_A: .3≠.5

The test:

$$|6401-5709|\geq 2.65\sqrt{\frac{(4857)(4858)}{12}\left(\frac{1}{1619}+\frac{1}{1619}\right)}$$

$692\geq 130.6 \Rightarrow$

Reject $H_0 \Rightarrow$ .3 is different from .5 terminal branch CSA.

Similarly .3 is different from .8 and .5 is different from .8.

For variety the multiple comparisons showed that the Loadel and Peak estimates of set were different as was Peak and Guame but there were no significant differences between Loadel and Guame.

A nonparametic two-way analysis of variance is applied to the various factors so that further exploration of the data can be made.

For the two-way layout the data is assumed to consist of n k observations, with one observation from each of k treatments in each of n blocks. For simplicity, the median of each cell was used as that observation.

The model is assumed to be $X_{ij} = \mu+\alpha_i+\beta_j+e_{ij}$, $i=1,2,3$, $j=1,2,3$ where μ is the unknown overall mean, α_i is the block i effect, β_j is treatment j effect $\sum_{i=1}^{3}\alpha_i=0$ and $\sum_{j=1}^{3}\alpha_j=0$. The errors are independent and identically distributed.

To test H_0: $\beta_1=\beta_2=\beta_3$ within each block, rank the 3 observations from least to greatest. Let r_{ij} denote the rank of X_{ij}. Set $R_j=\sum_{i=1}^{3} r_{ij}$, $R._j=\frac{R_j}{n}$ and $R..=\frac{K+1}{2}$.

Compute: $S=\frac{12n}{k(k+1)}\sum_{j=1}^{3}(R._j-R..)^2$

At the α level of significance reject H_0 if $S\geq s(\alpha,3,3)$ where $s(\alpha,3,3)$ satisfies the equation $P_0\{S\geq s(\alpha,3,3)\}=\alpha$.

Once again if a difference is found then multiple comparisons will be used to determine specific differences taking into account the other factors present.

To test: H_0: $\beta_x=\beta_y$ H_A: $\beta_x\neq\beta_y$ then reject H_0 if $|R_x-R_y|\geq r(\alpha,3,3)$ where $r(\alpha,3,3)$ satisfies the equation $P_0\{|R_x-R_y|<r(\alpha,3,3),\ x=1,2,\ y=3\}=1-\alpha$.

Tables 8, 9, and 10 show rankings of the various factor.

TABLE 8: Ranks of Measurement Method vs. Terminal Branch CSA

		TERMINAL BRANCH CSA		
		1	2	3
MEASUREMENT METHOD	1	3	2	1
	2	3	2	1
	3	3	2	1

TABLE 9: Ranks of Measurement Method vs. Variety

		MEASUREMENT METHOD		
		1	2	3
VARIETY	1	3	2	1
	2	3	2	1
	3	2	3	1

TABLE 10: Ranks of Terminal Branch CSA vs. Variety

		TERMINAL BRANCH CSA		
		1	2	3
VARIETY	1	3	2	1
	2	3	2	1
	3	3	2	1

A sample calculation is applied to TABLE 10:

Therefore $R_1=3+3+3=9$

$R_2=2+2+2=6$

$R_3=1+1+1=3$

and $R._1=9/3=3$

$R._2=6/3=2$

$R._3=3/3=1$

$R..=\frac{3+1}{2}=2$

To test: H_o: $\beta_1=\beta_2=\beta_3$ H_A: not all β's are equal

$$S=\frac{12(3)}{(4)(3)}[(3-2)^2+(2-2)^2+(1-2)^2]$$

$$=6$$

$S(.028,3,3)=6\Rightarrow$

There is a significant difference of the terminal branch CSA accounting for variety.

A similar test was performed where the block was terminal branch CSA and the treatment was variety. The test showed no significant difference in variety taking into account the terminal branch CSA.

For the multiple comparison tests the only significant difference occurred between the .3 and .8 CSA. Otherwise no differences were found.

There was also a significant difference in terminal branch CSA taking into account the measurement method. The multiple comparisons test showed the only significant difference occurred between 0.3 and 0.8 terminal CSA but they were not different with the 0.5 CSA. There was no difference in measurement method taking into account the variety.

CONCLUSIONS:

Summarizing, we found that the measurement method had no effect by itself but the terminal branch CSA and variety did. Meanwhile, the terminal branch CSA showed differences taking into account the variety and also measurement method but variety had no significant differences while taking into account terminal branch CSA and measurement method.

For a one year project some conclusions and recommendations can be drawn from the data.

1. Since there is no significant difference in measurement method it is recommended that current procedure be maintained. This will enable enumerators who have worked on the survey for many years to maintain that certain degree of consistency in measuring limbs.

2. A change in terminal branch CSA to 0.5 is recommended. This change will eventually enhance the quality of the data. Since significant differences were found in all three terminal branch CSA, the 0.5 should be used because it showed the best absolute difference between estimated and actual set. This 0.5 CSA will also help the enumerators because the Cling Peach Survey and Almond Survey are overlapping and the terminal CSA for almonds is also 0.5, therefore, no confusion when sampling a tree as to whether the terminal branch CSA is 0.3 or 0.5

3. A similar project will be employed during the current harvest season to check on the validity of the current results. Cross checks can then be applied from one year to the next.

And 4. A possible sample allocation by variety along with year of plant and zone be employed since differences in variety estimates of sets were found. This is not as critical as the other 3 observations since the variety effect "washed out" when terminal branch CSA was taken into effect.

A special note of thanks is extended to everyone who helped on this research project including Marvin Stowe for the use of his orchards and all members of the Tree Fruit and Nut Section of the California Crop and Livestock Reporting Service.

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